

3-178

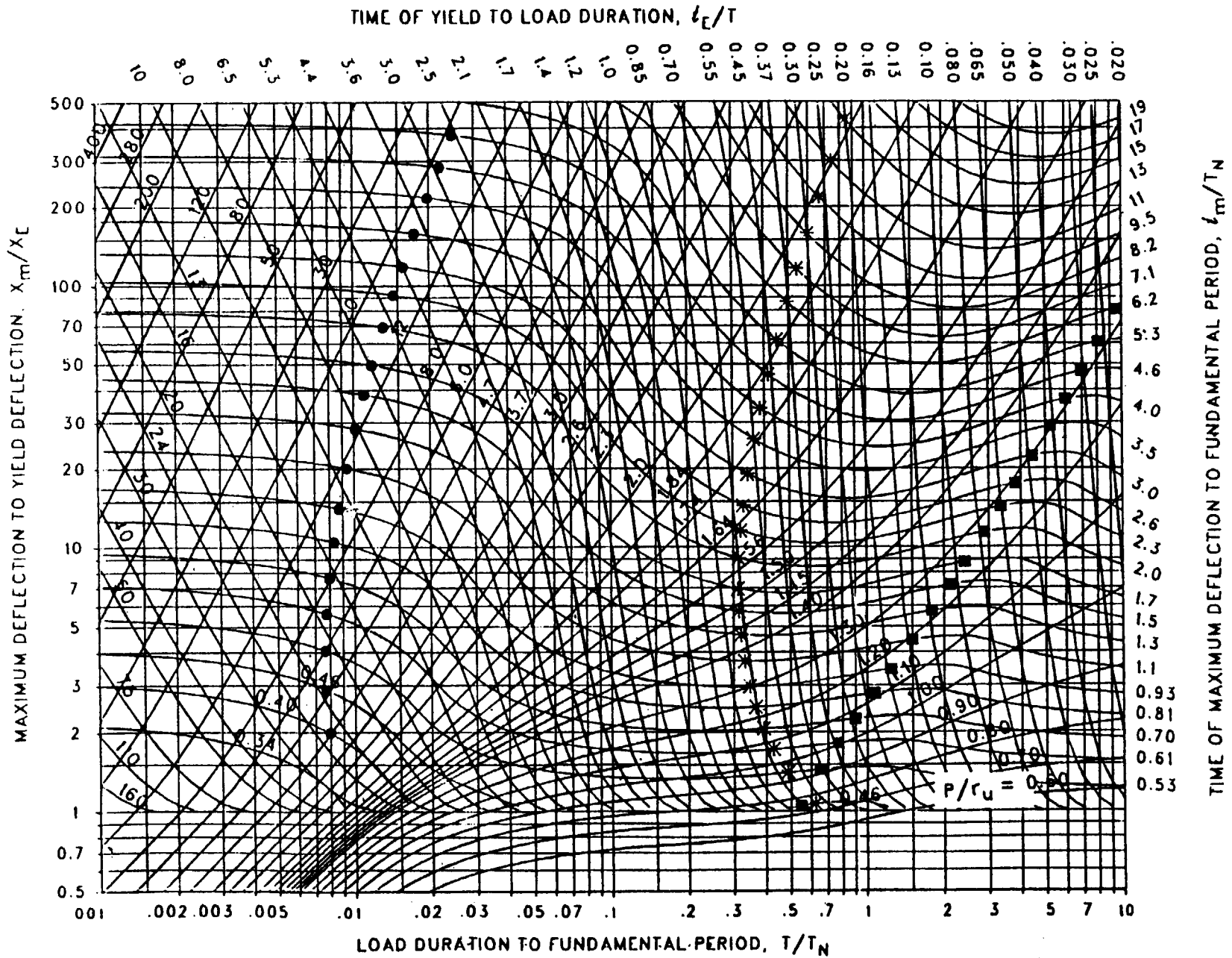


Figure 3-120 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.648$ ,  $C_2 = 30$ )

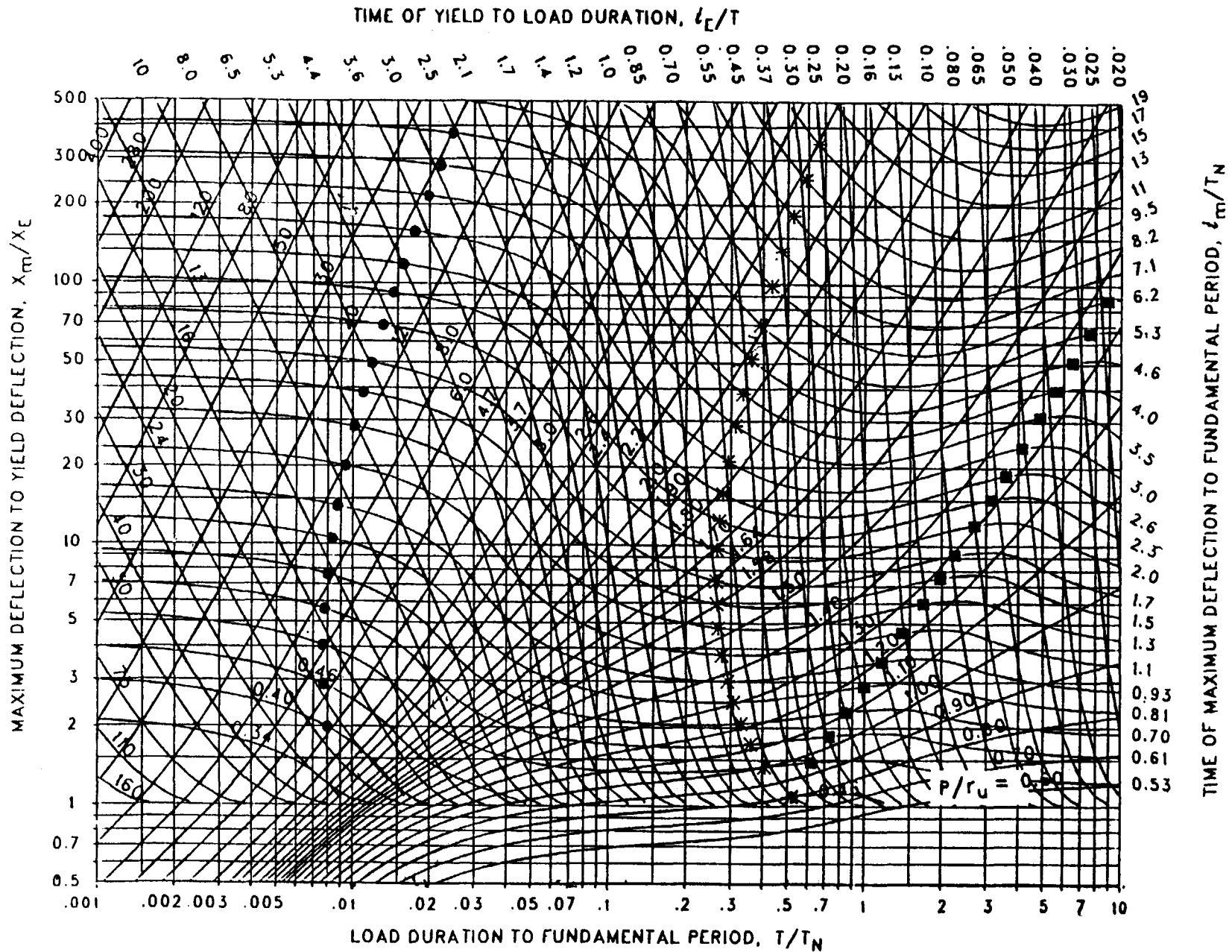


Figure 3-121 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.619$ ,  $C_2 = 30$ )

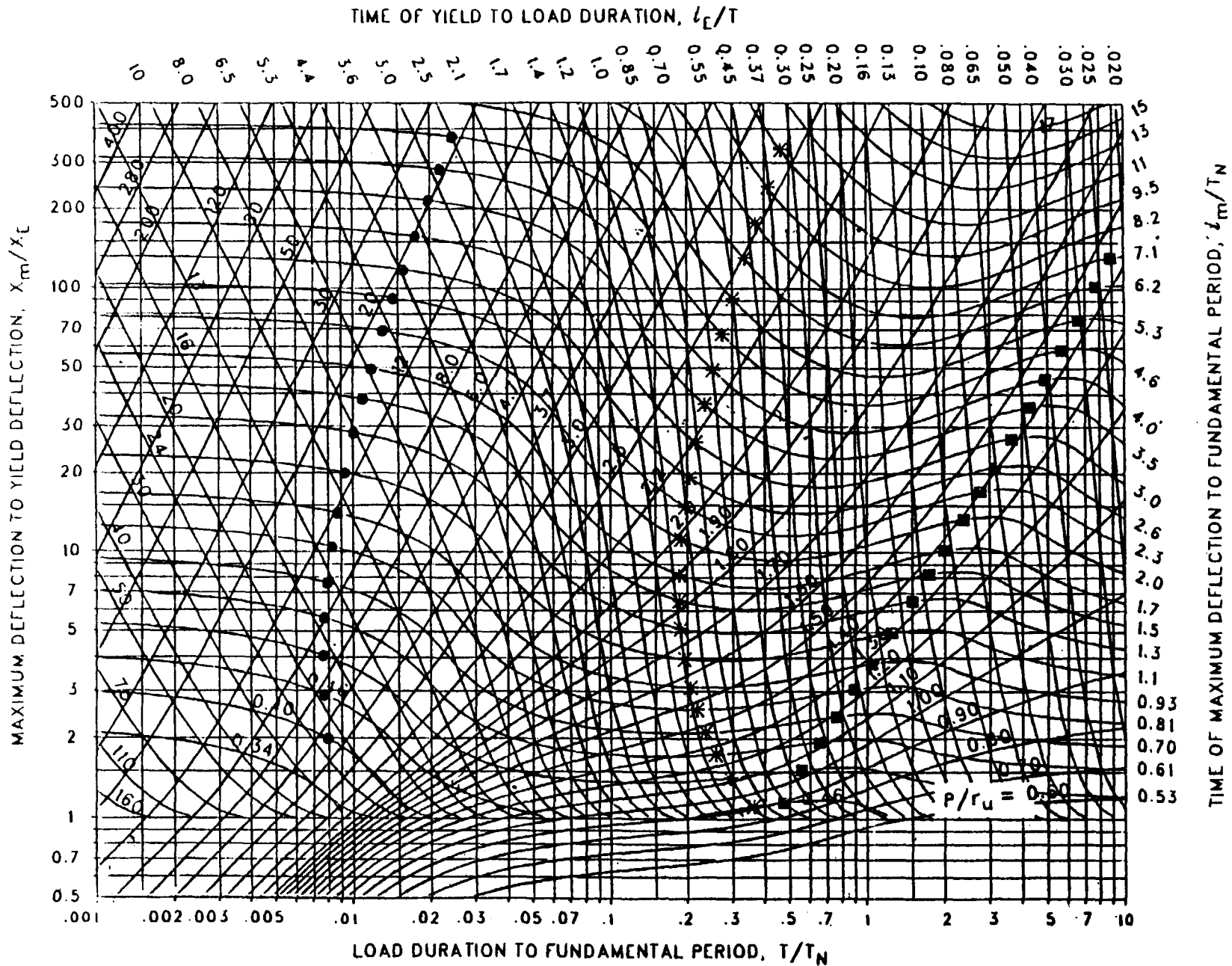


Figure 3-122 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.562$ ,  $C_2 = 30$ )

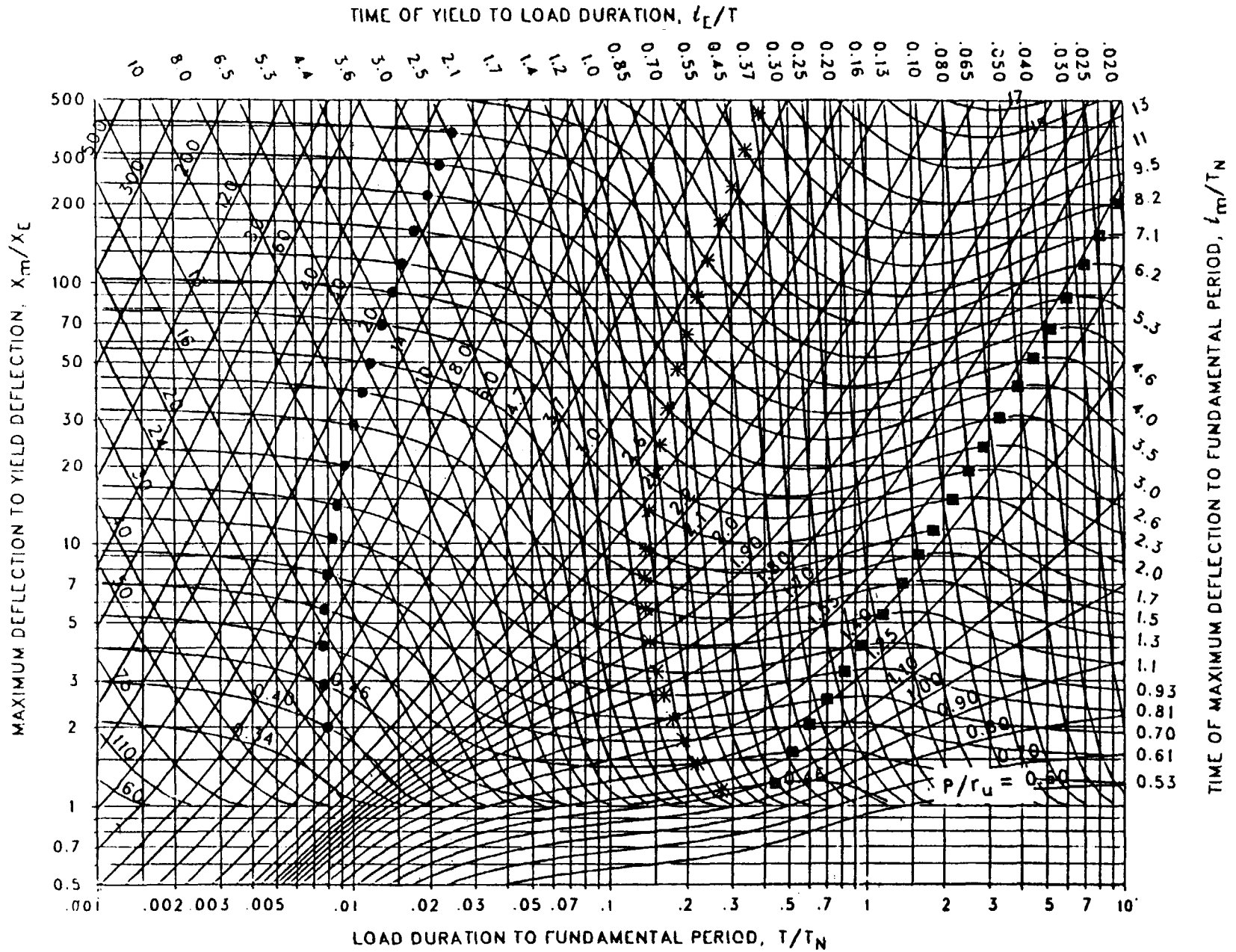


Figure 3-123 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.511$ ,  $C_2 = 30$ )

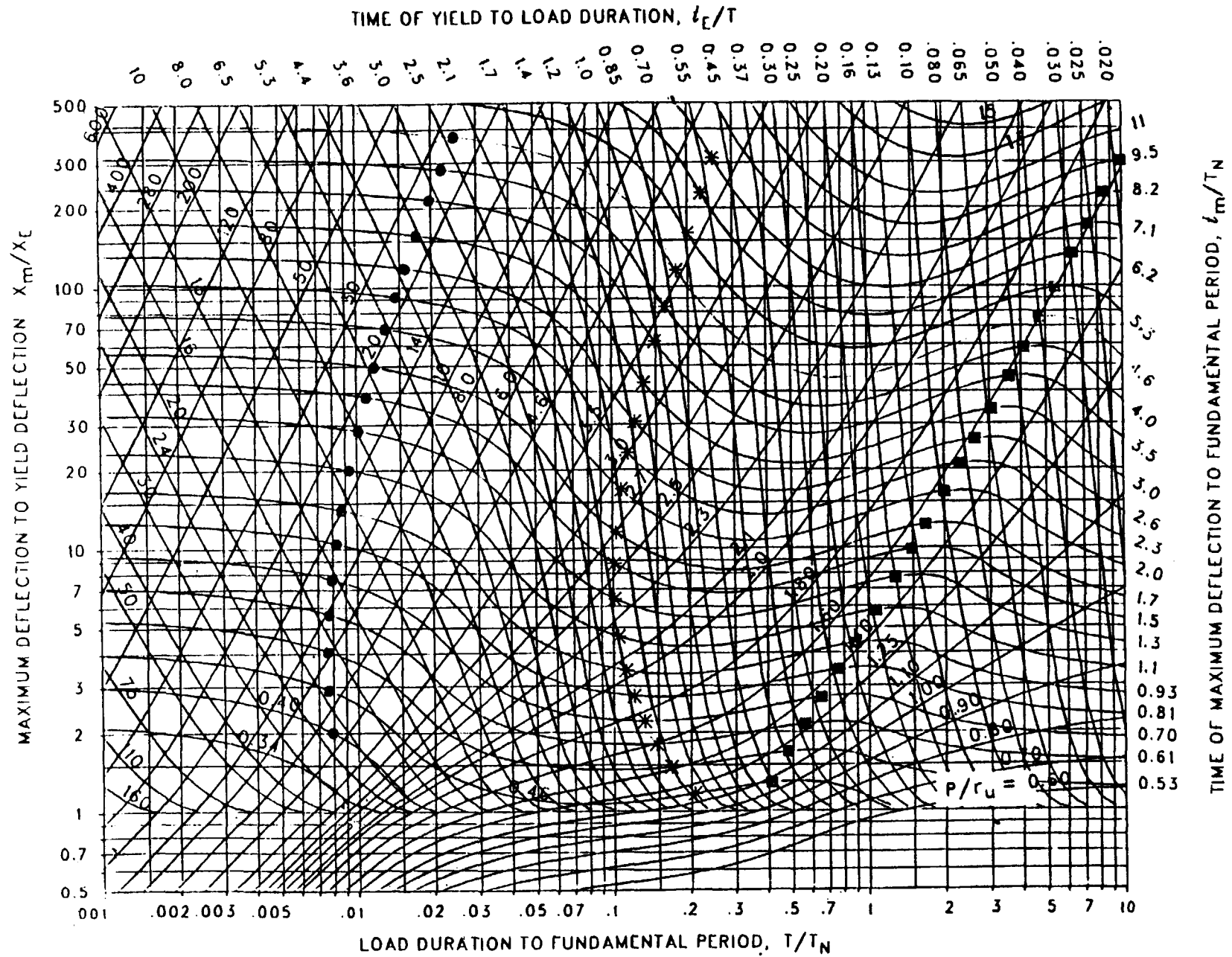


Figure 3-124 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.464$ ,  $C_2 = 30$ )

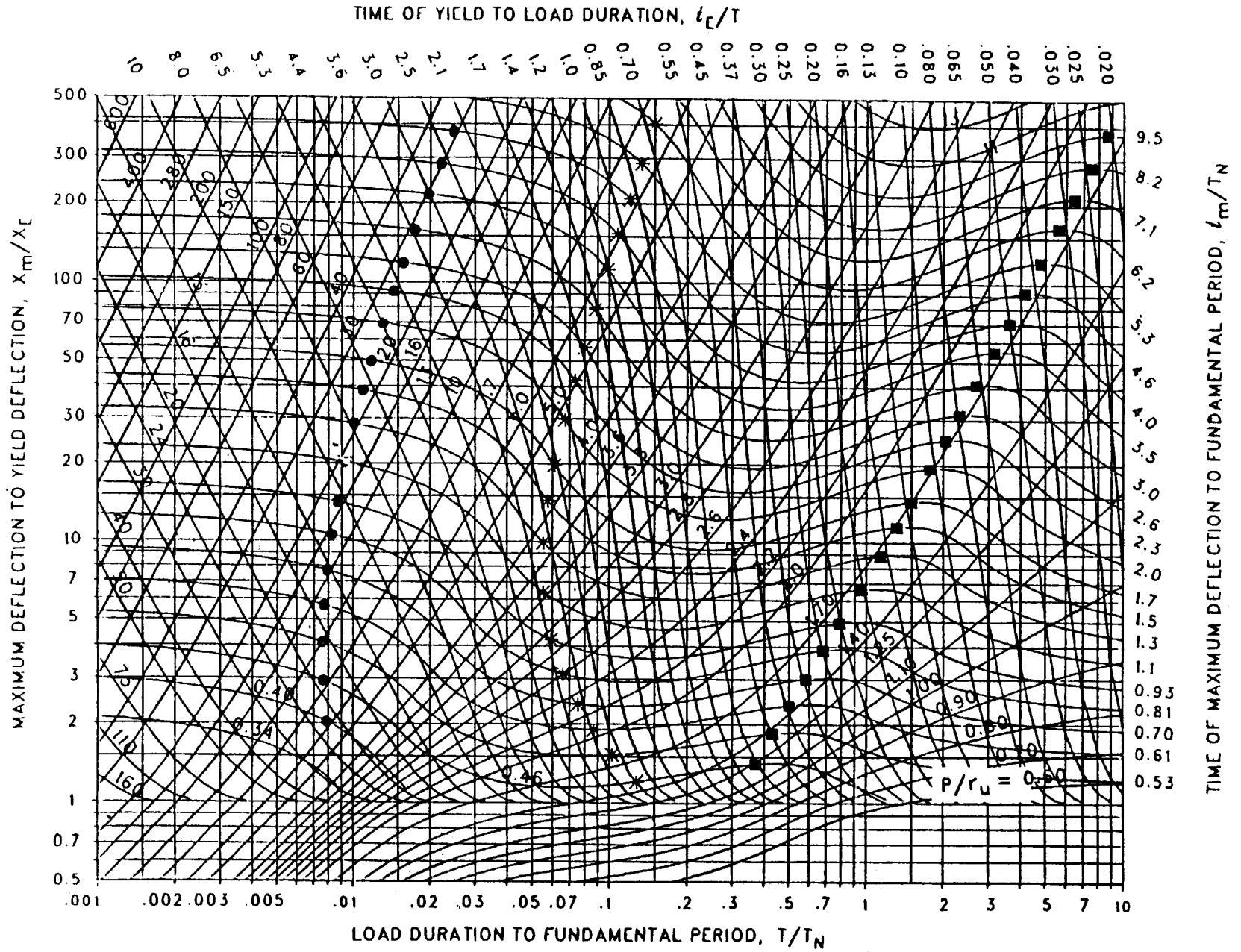


Figure 3-125 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.383$ ,  $C_2 = 30$ )

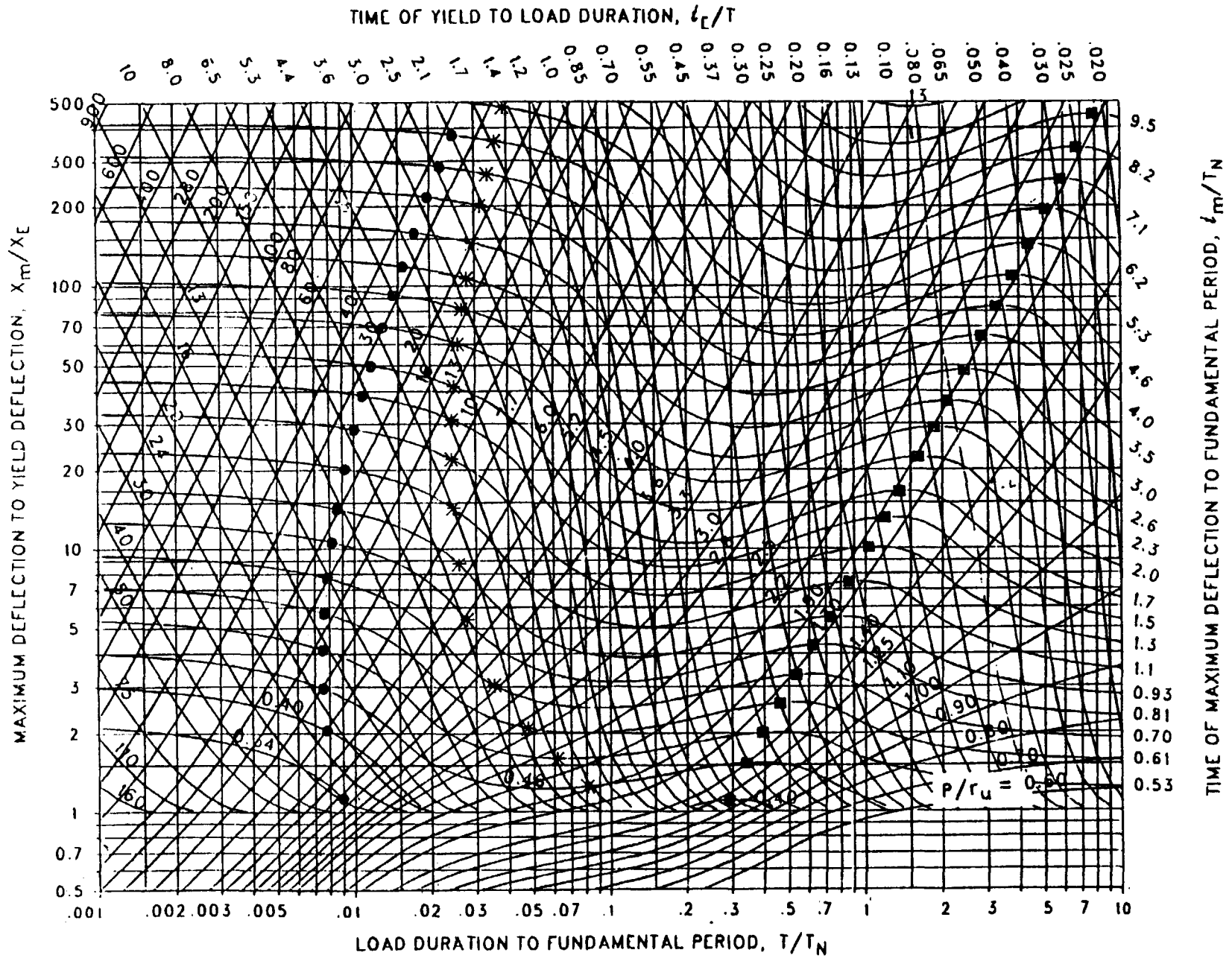


Figure 3-126 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.316$ ,  $C_2 = 30$ )

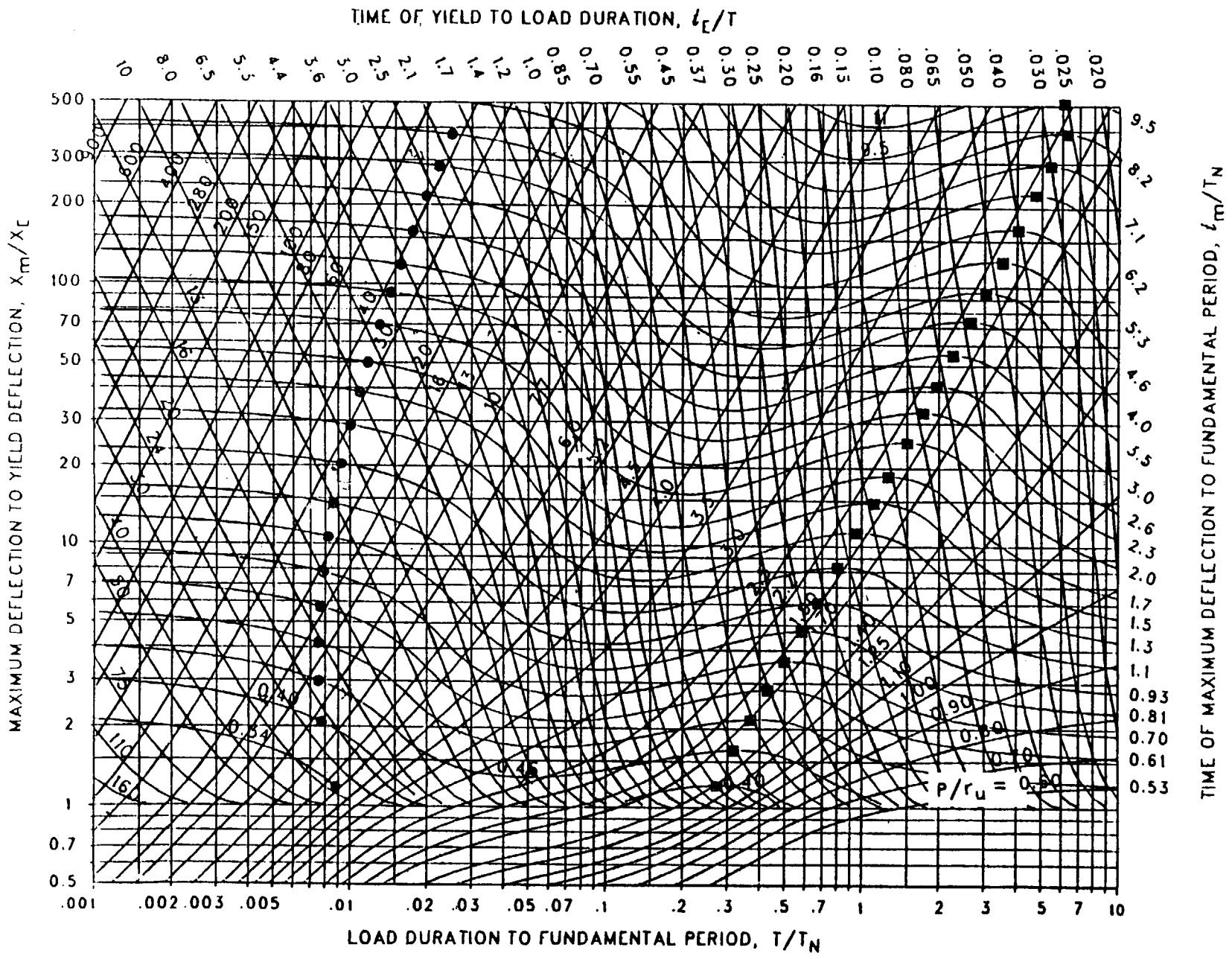


Figure 3-127 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.261$ ,  $C_2 = 30$ )



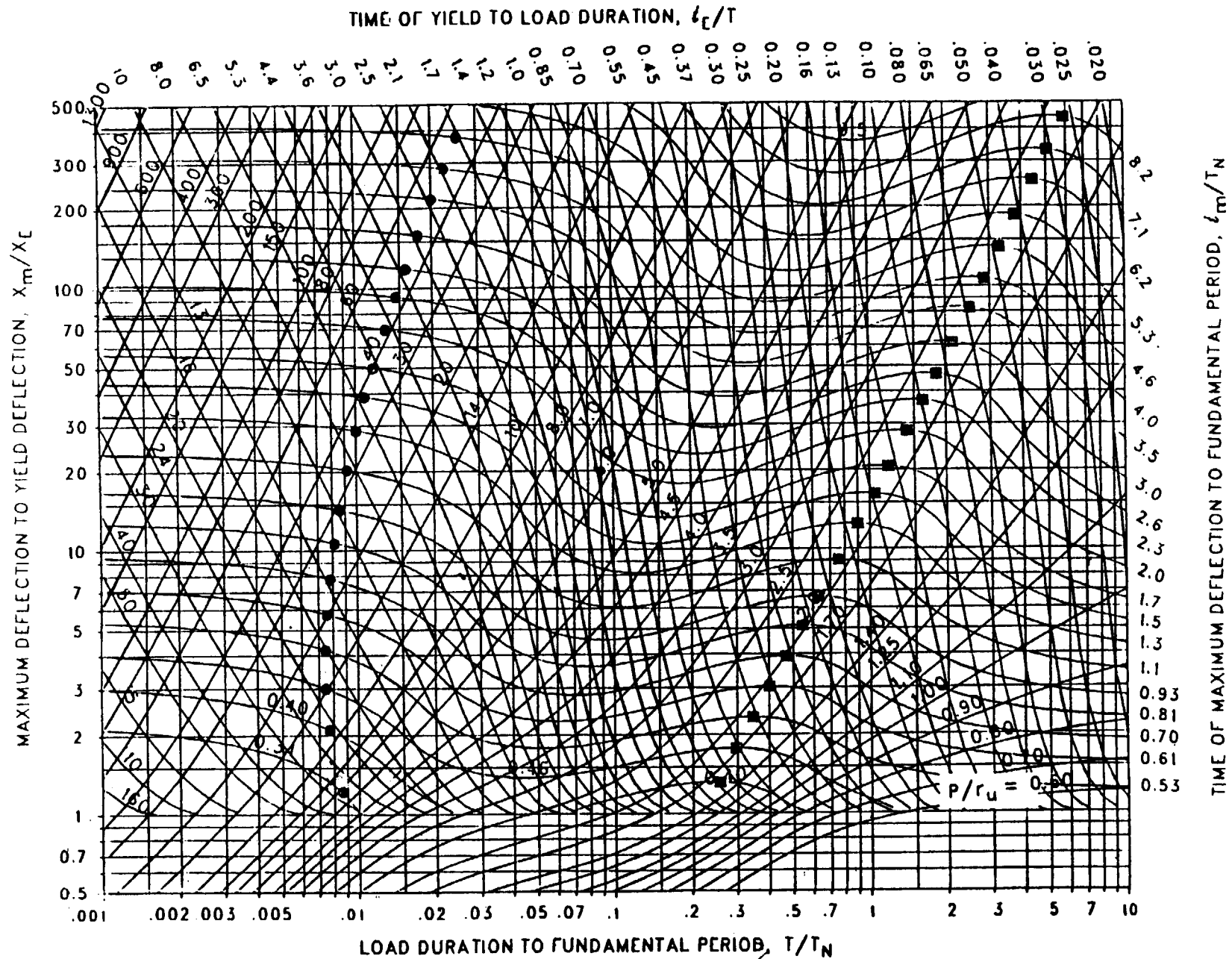


Figure 3-128 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.215$ ,  $C_2 = 30$ )

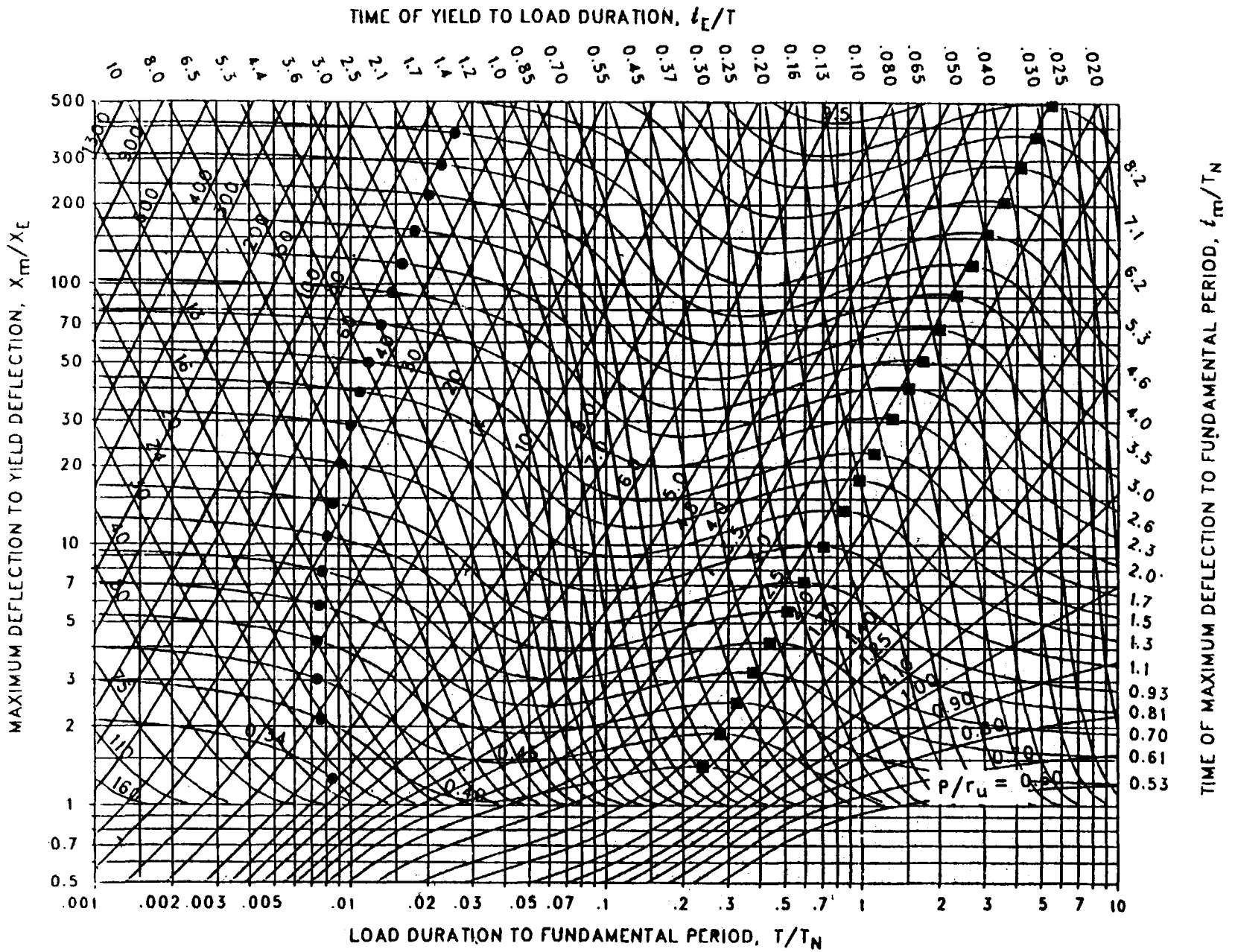


Figure 3-129 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.178$ ,  $C_2 = 30$ )

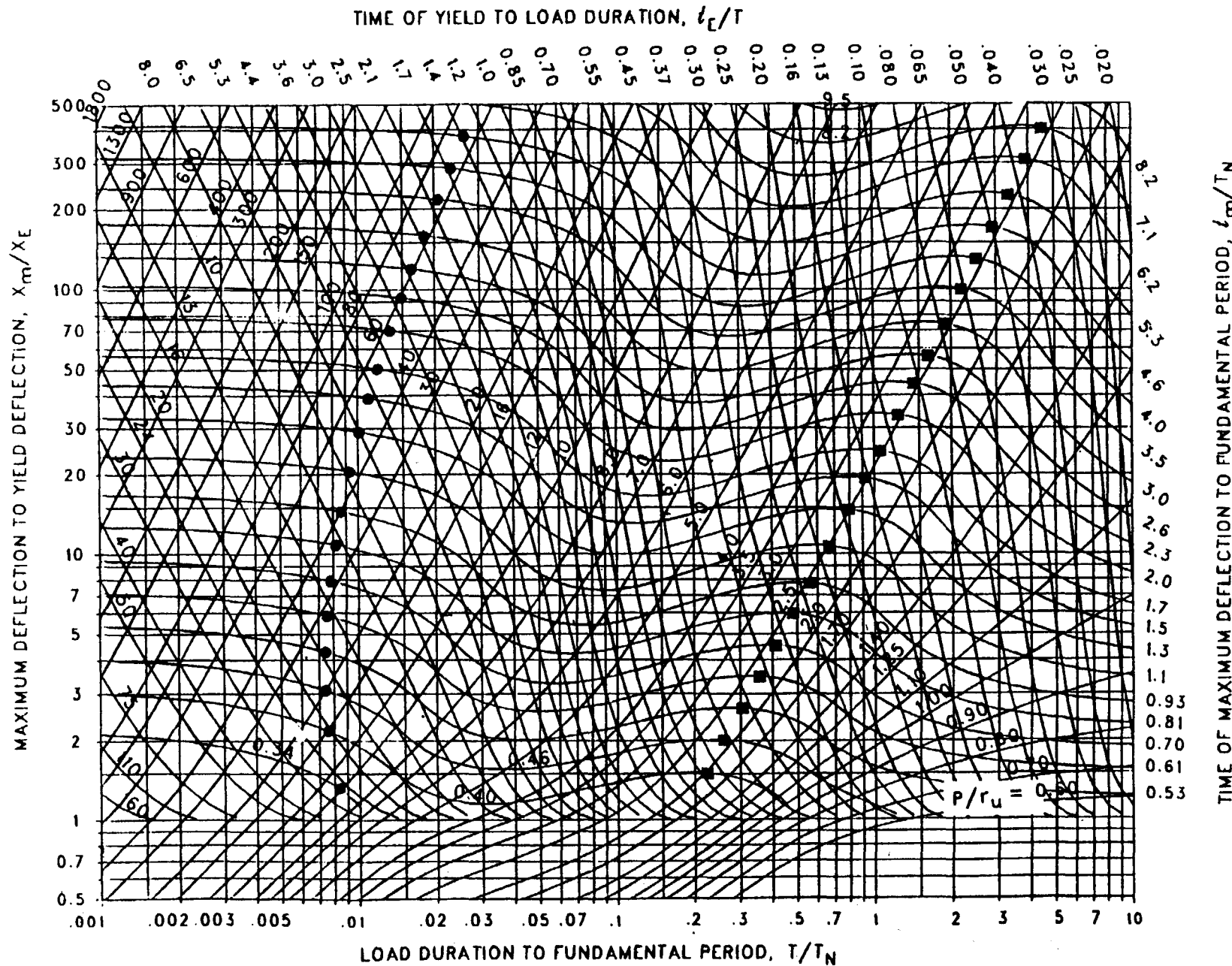


Figure 3-130 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.147$ ,  $C_2 = 30$ )

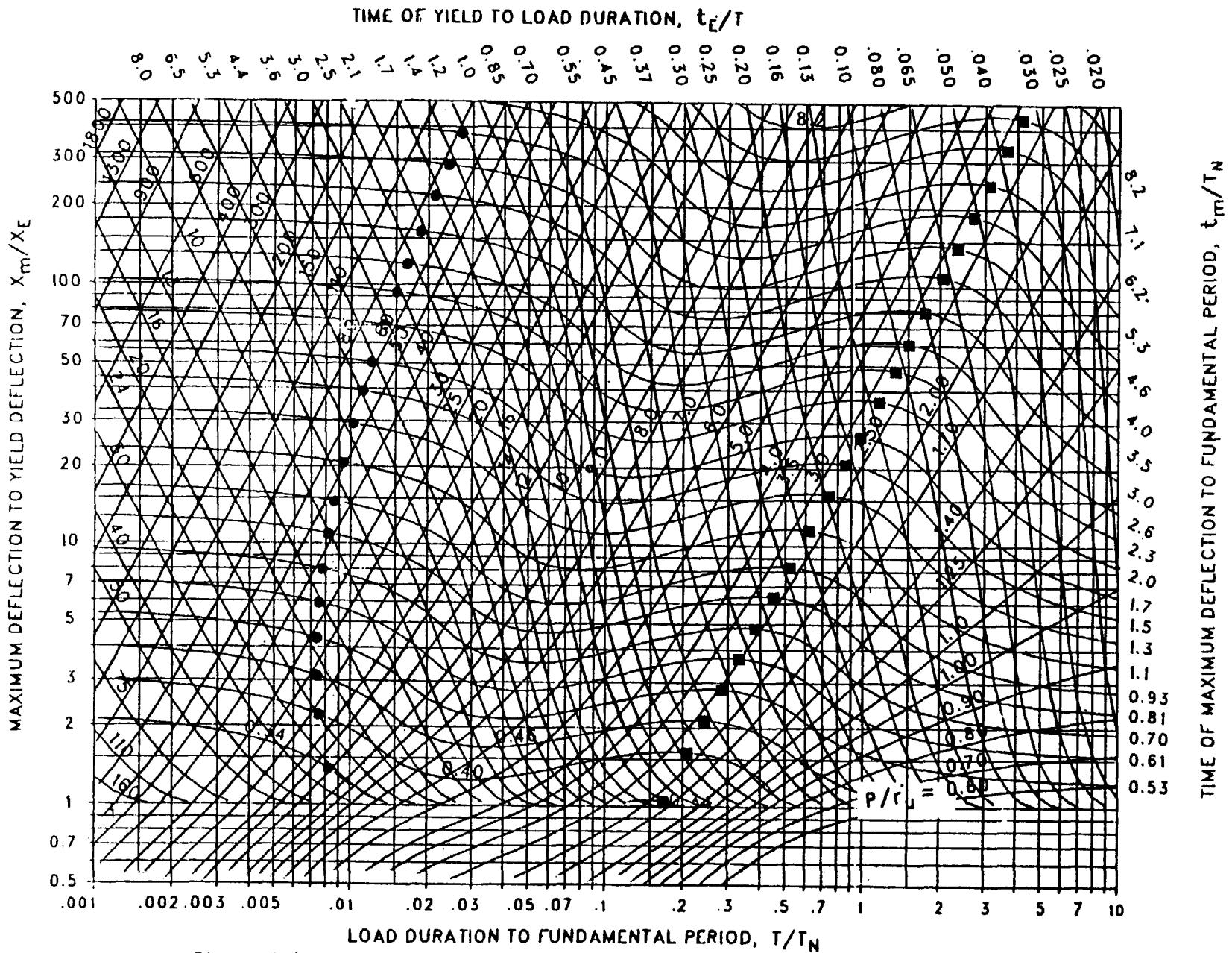


Figure 3-131 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.121$ ,  $C_2 = 30$ )

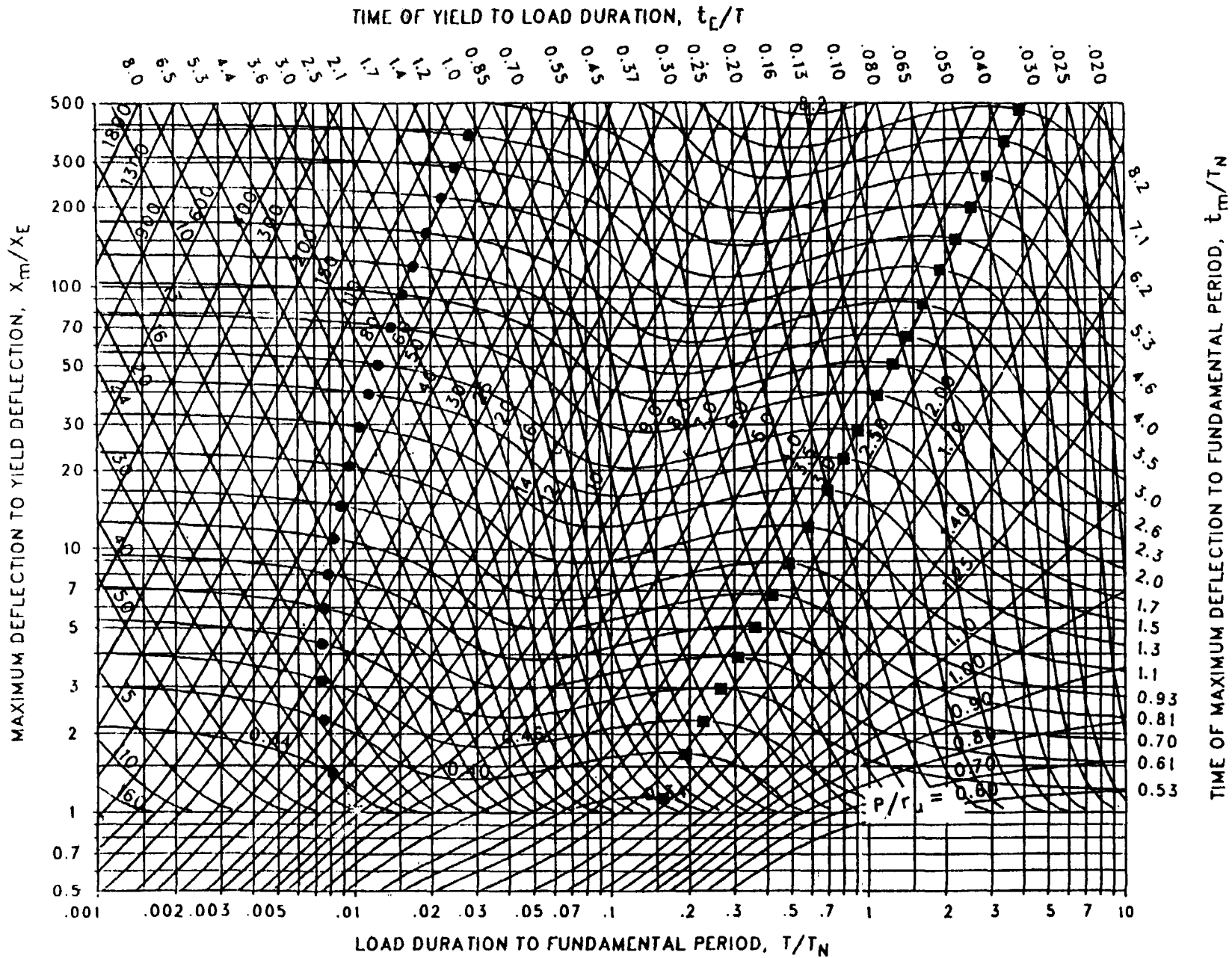


Figure 3-132 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.100$ ,  $C_2 = 30$ )

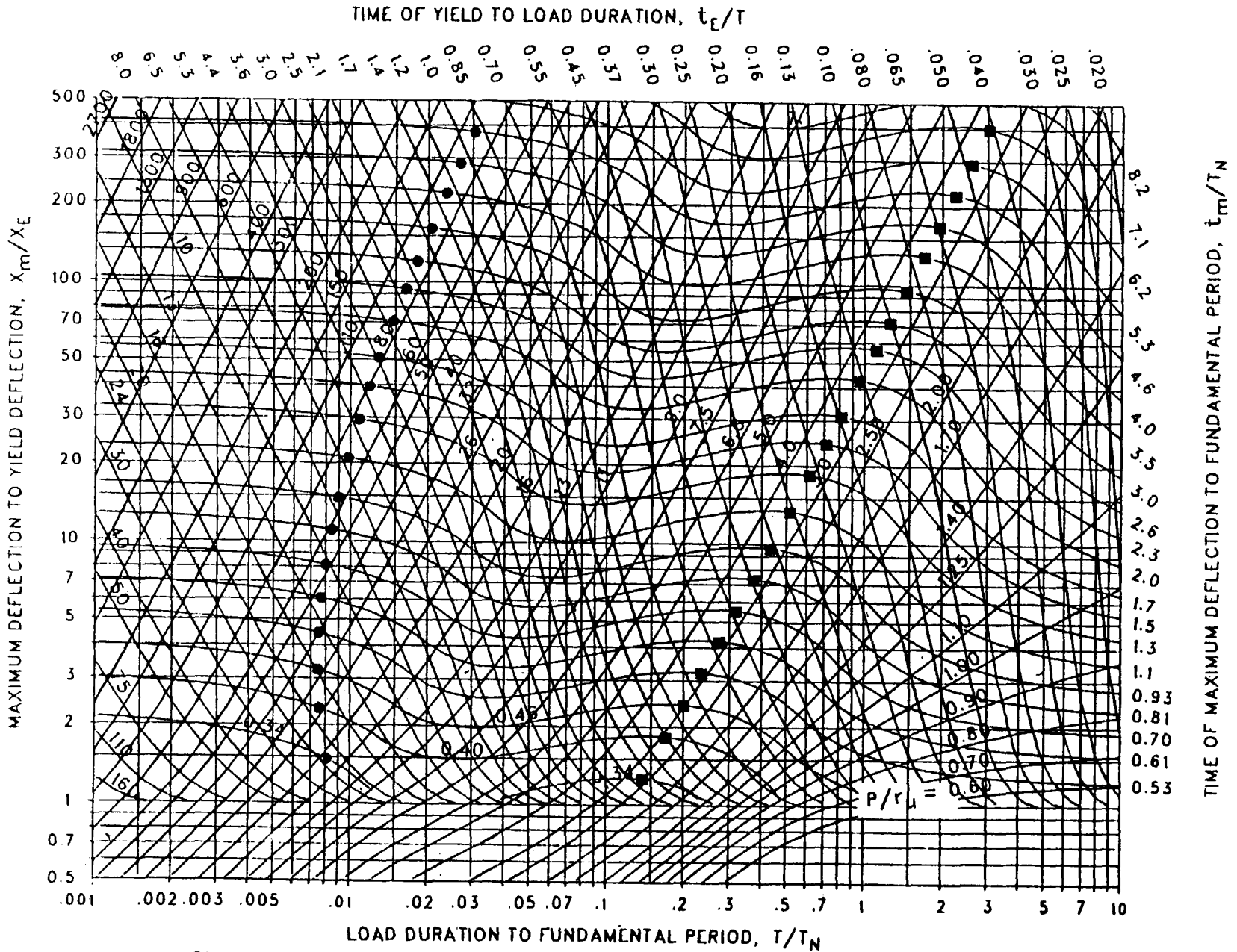


Figure 3-133 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.075$ ,  $C_2 = 30$ )

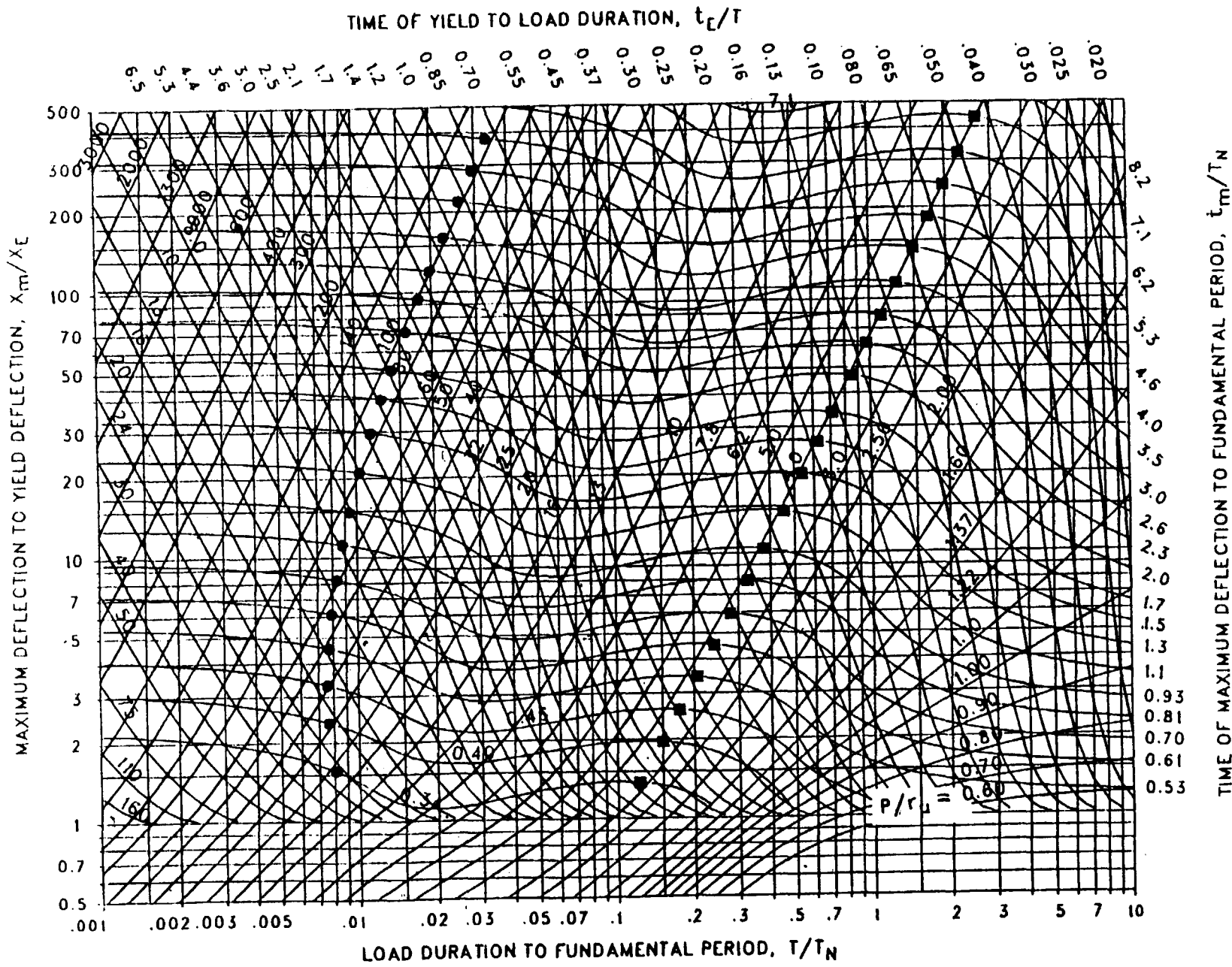


Figure 3-134 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.056$ ,  $C_2 = 30$ )

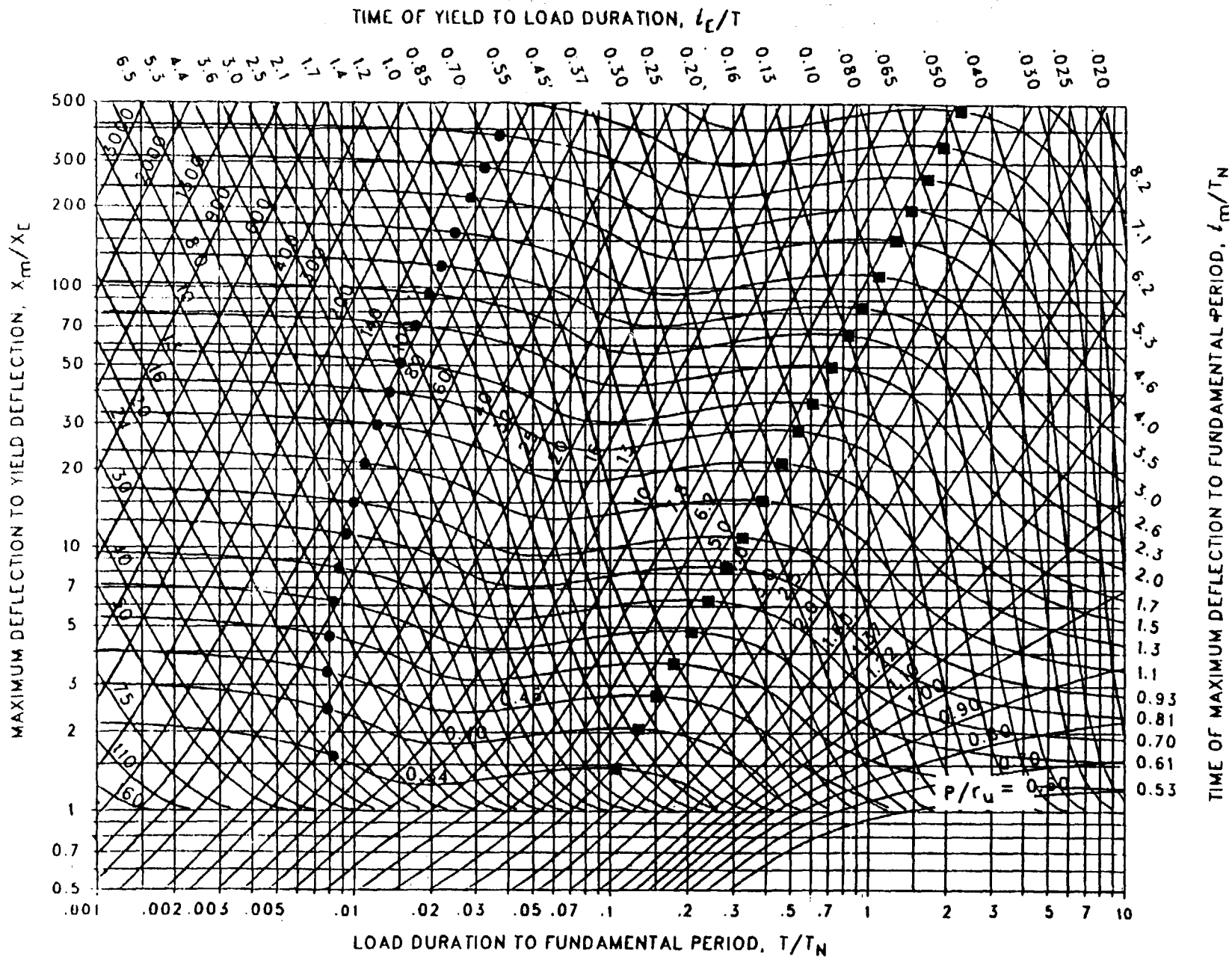


Figure 3-135 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.042$ ,  $C_2 = 30$ )



76T-8

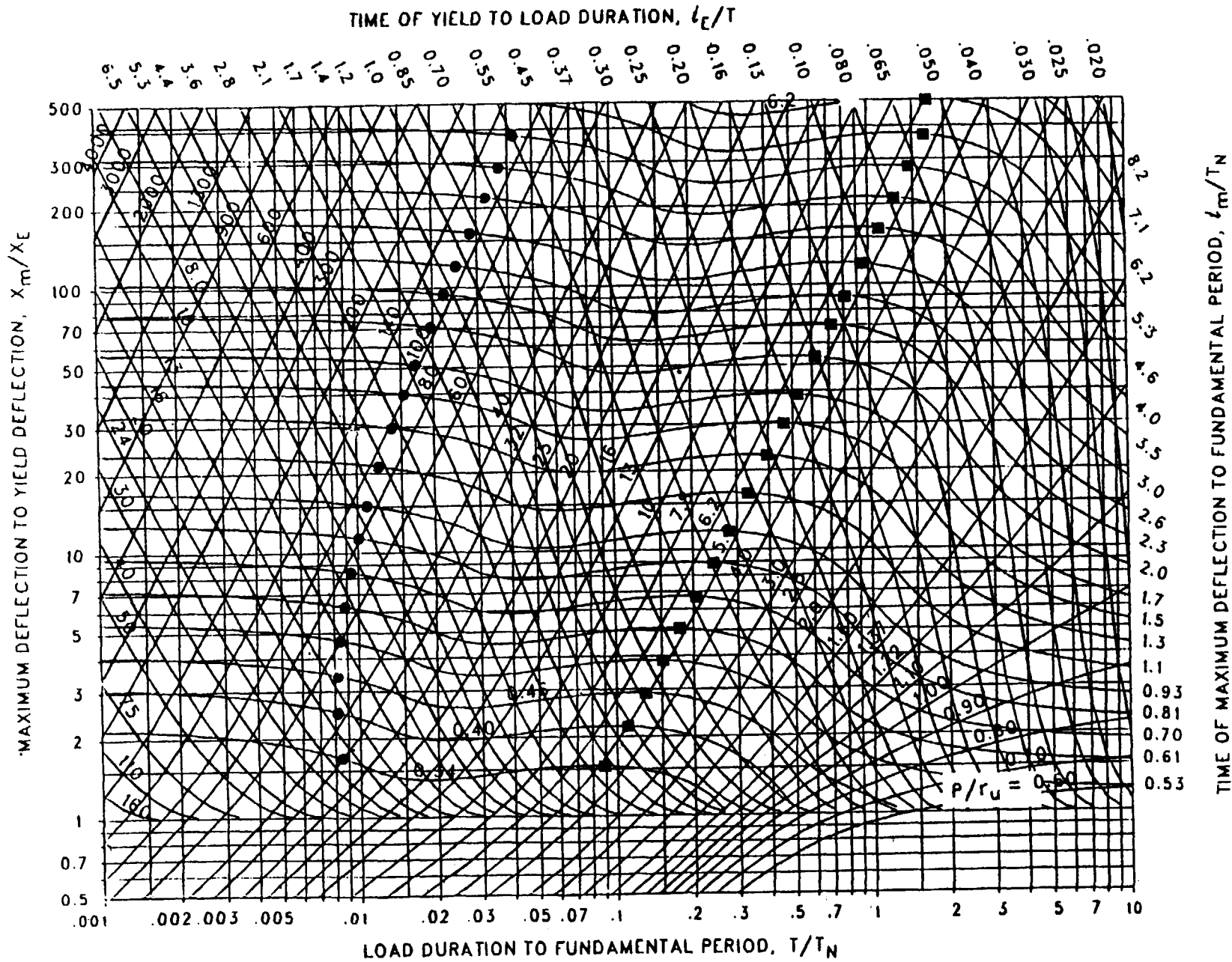


Figure 3-136 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.032$ ,  $C_2 = 30$ )

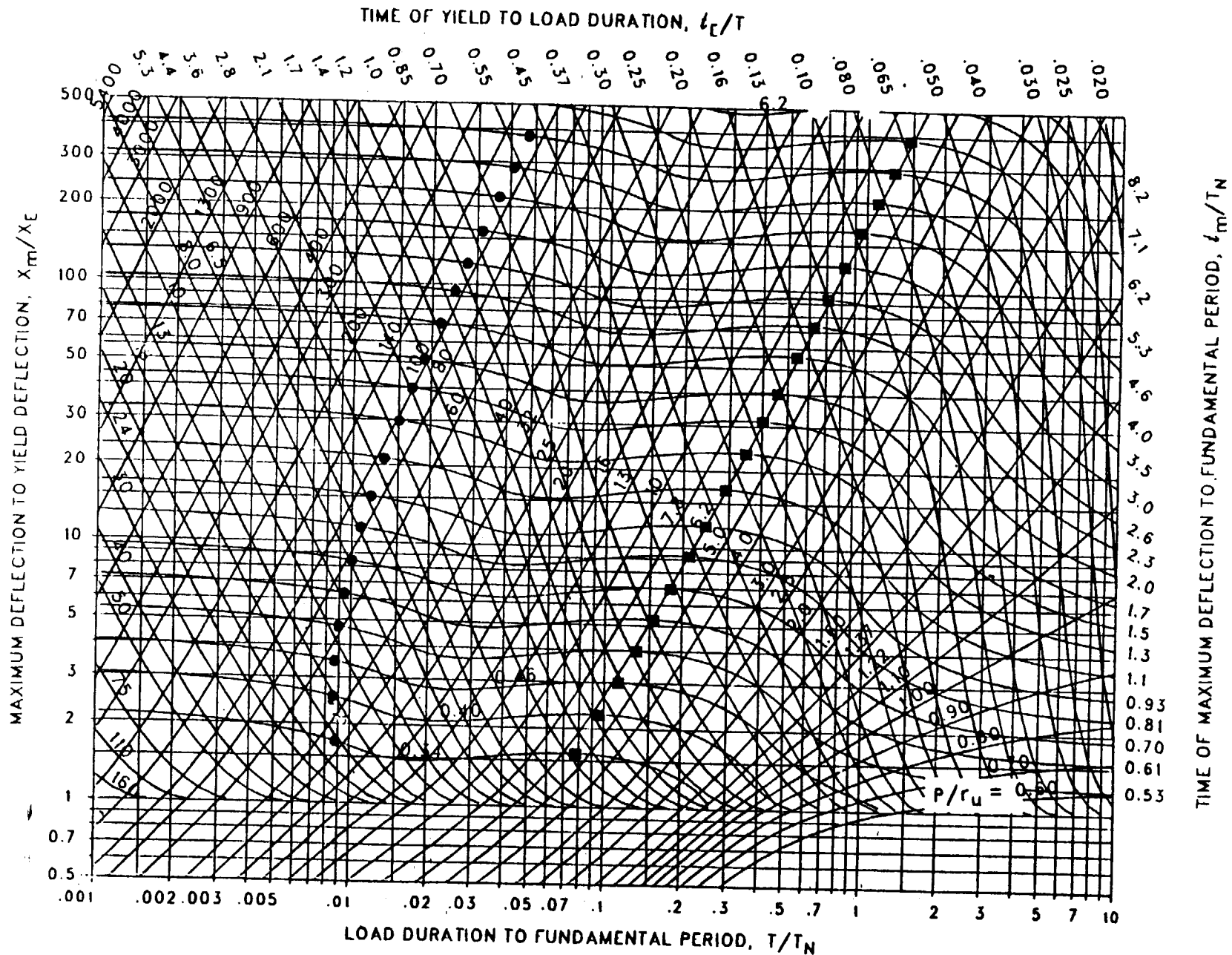


Figure 3-137 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.026$ ,  $C_2 = 30$ )

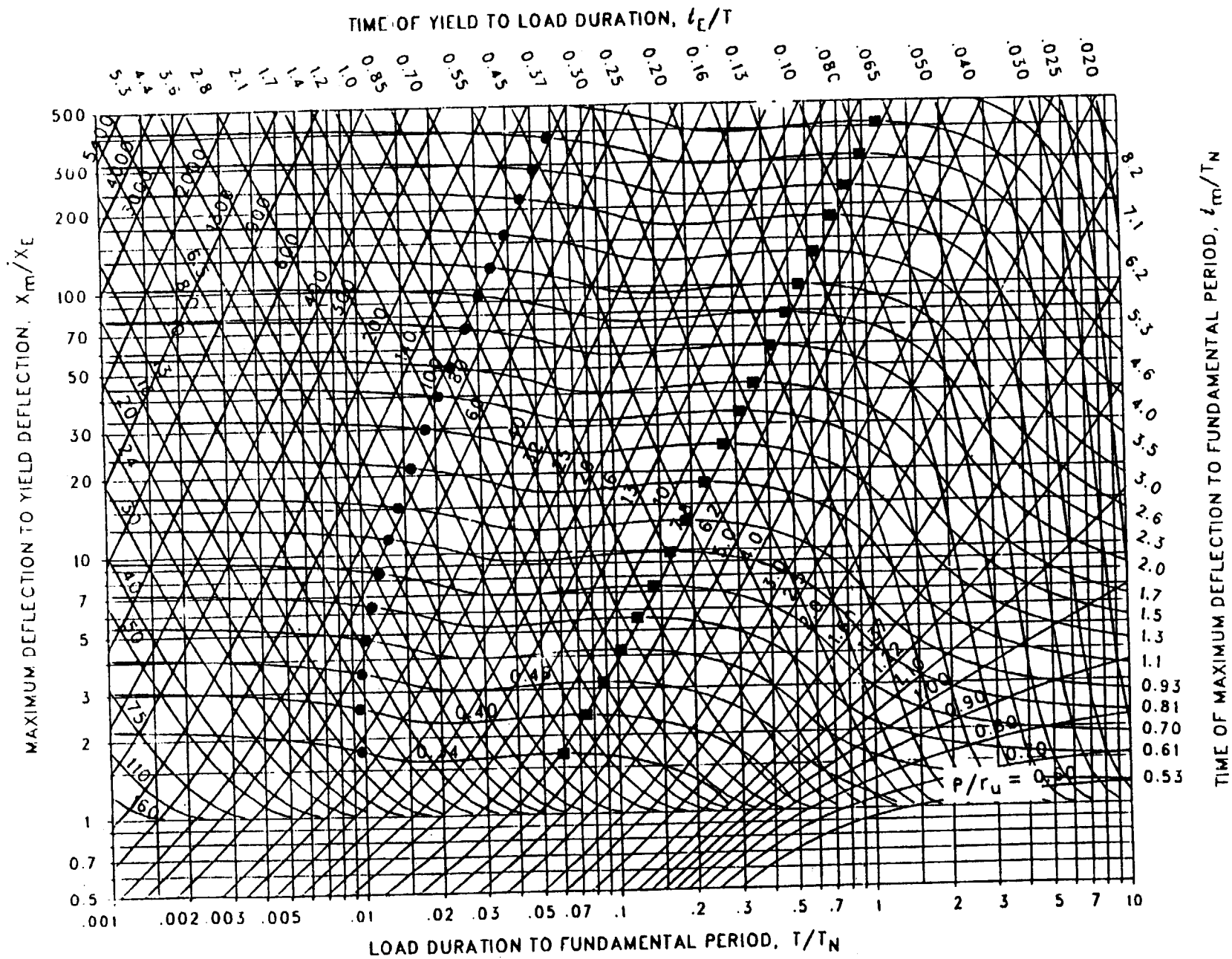


Figure 3-138 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.018$ ,  $C_2 = 30$ )

3-197

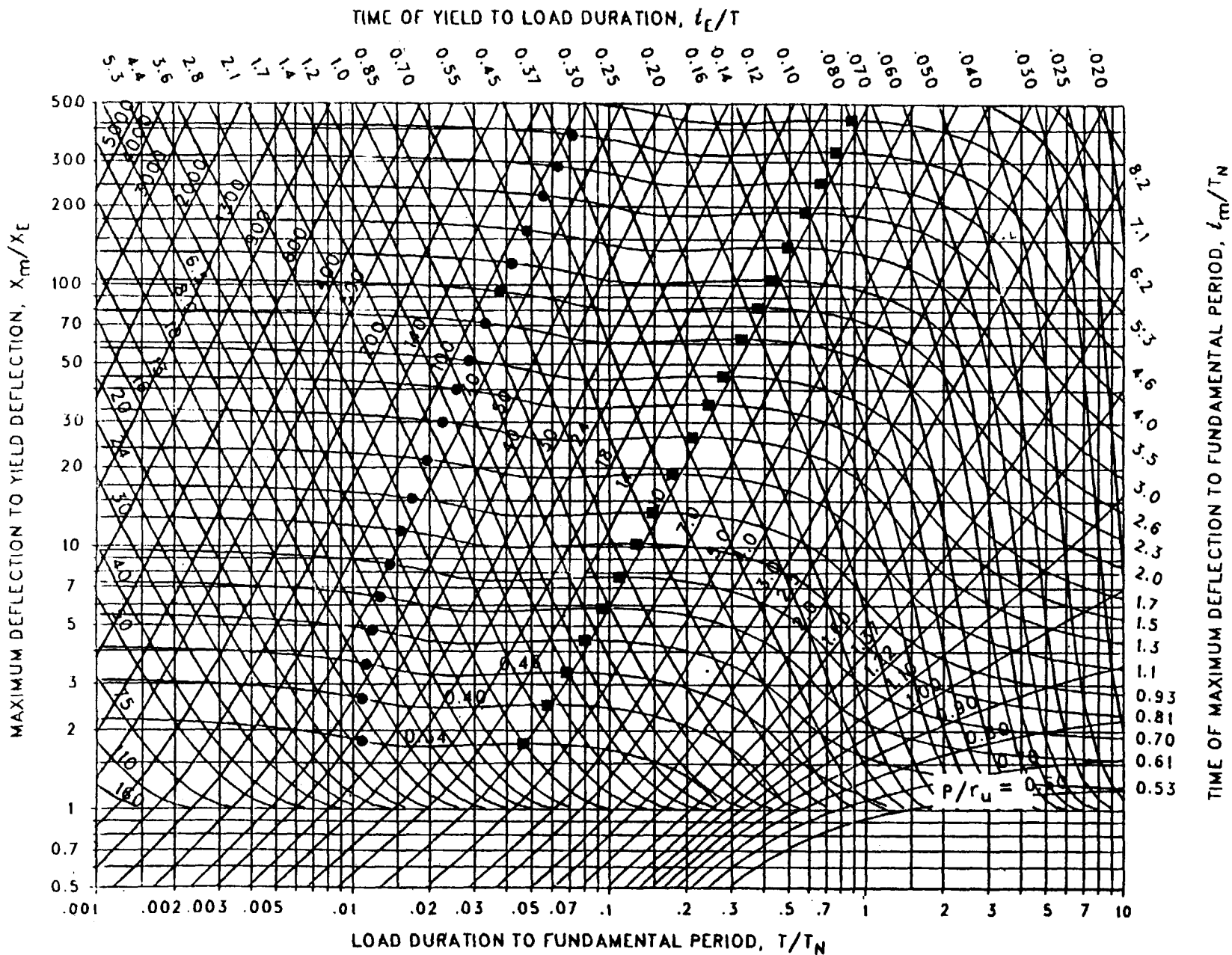


Figure 3-139 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.013$ ,  $C_2 = 30$ )

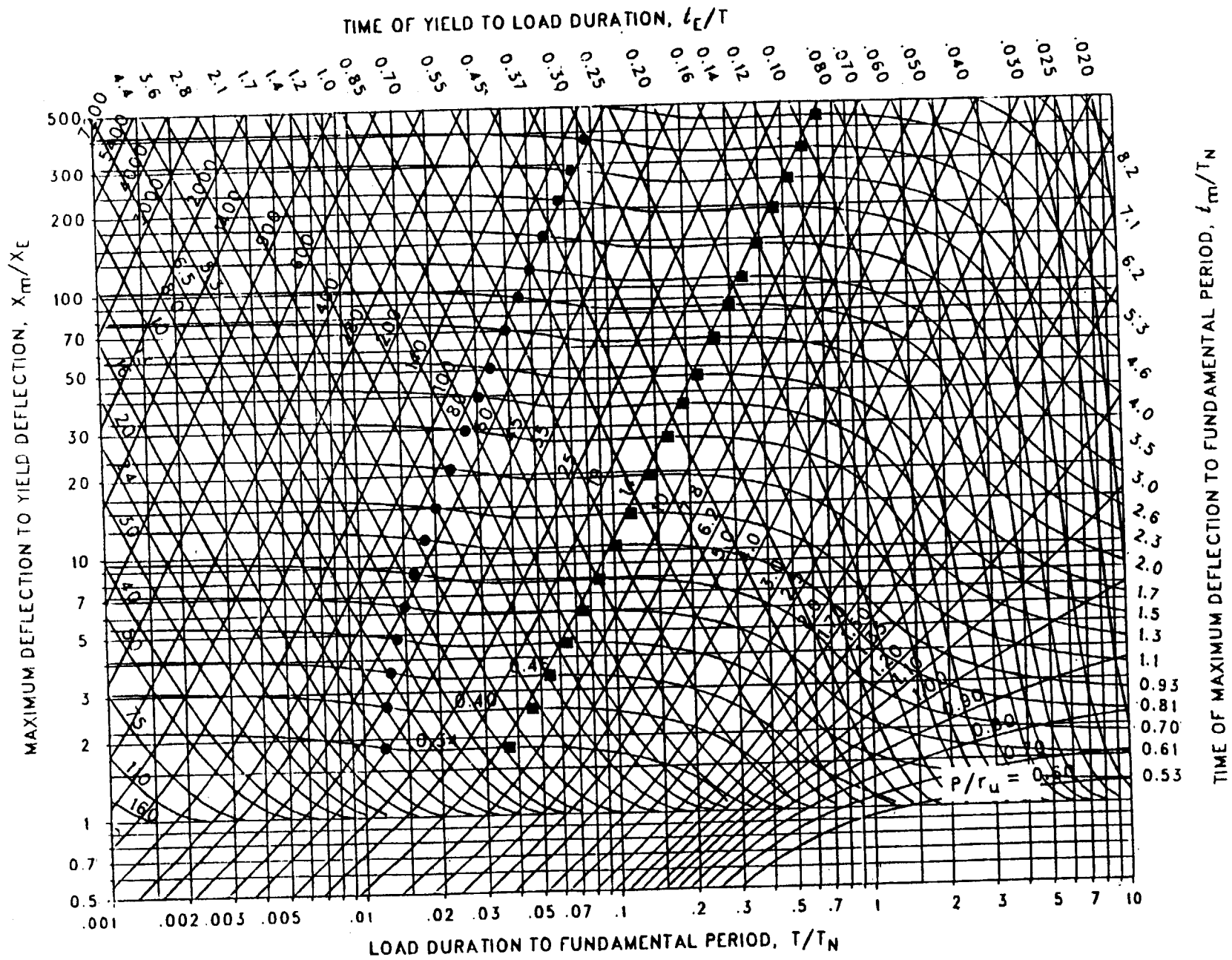


Figure 3-140 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.010$ ,  $C_2 = 30$ )

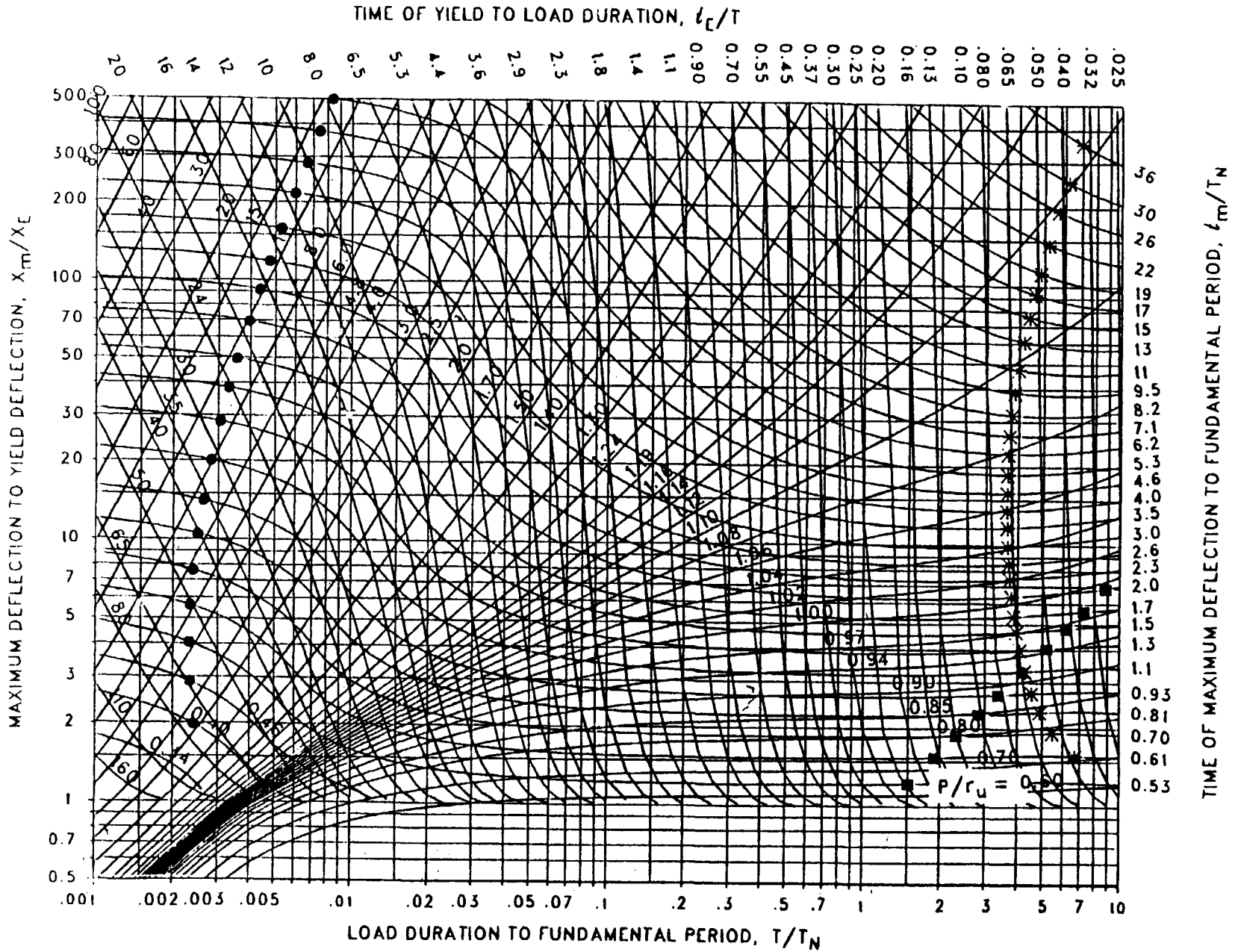


Figure 3-141 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.909$ ,  $C_2 = 100$ .)

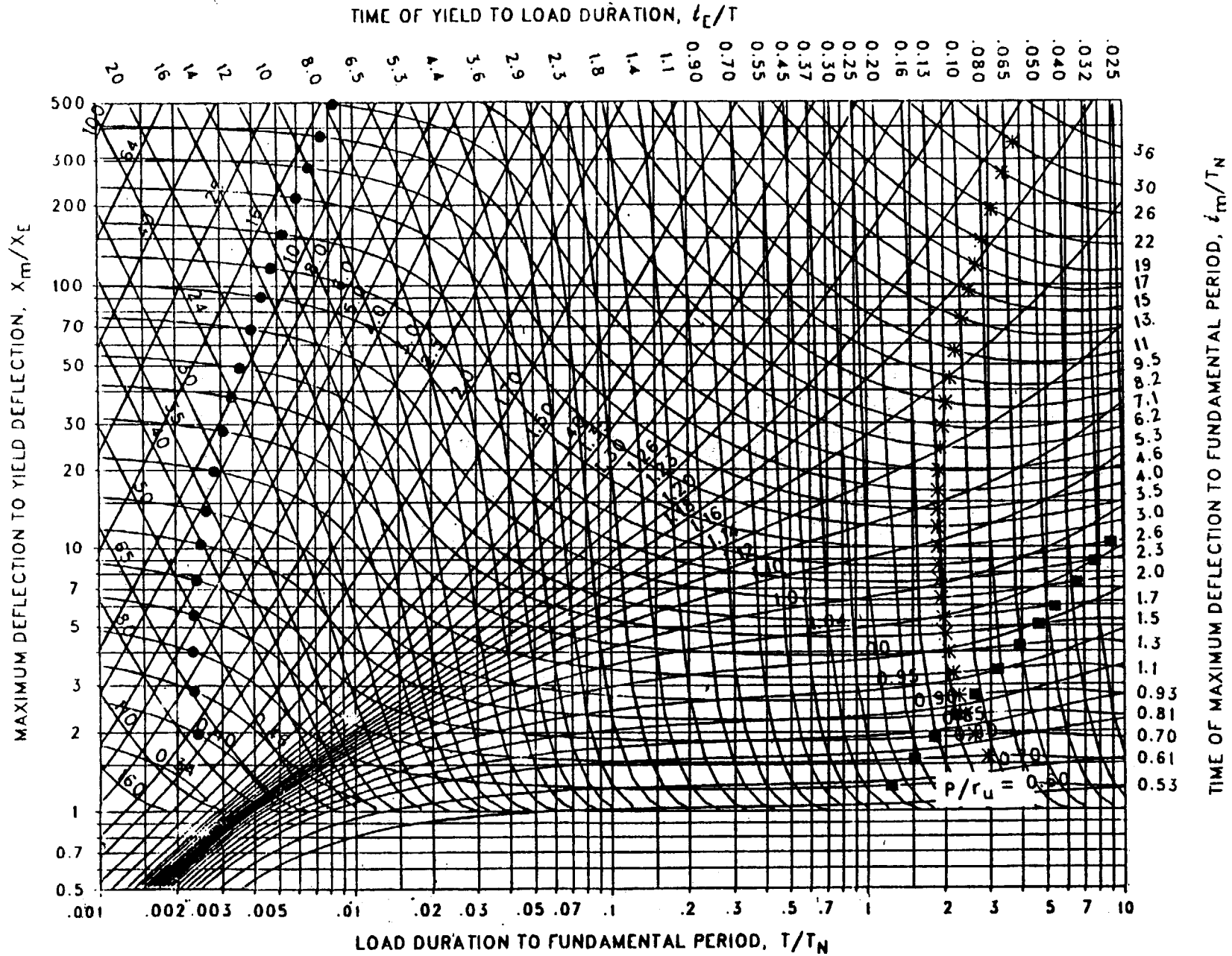


Figure 3-142 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.866$ ,  $C_2 = 100$ .)

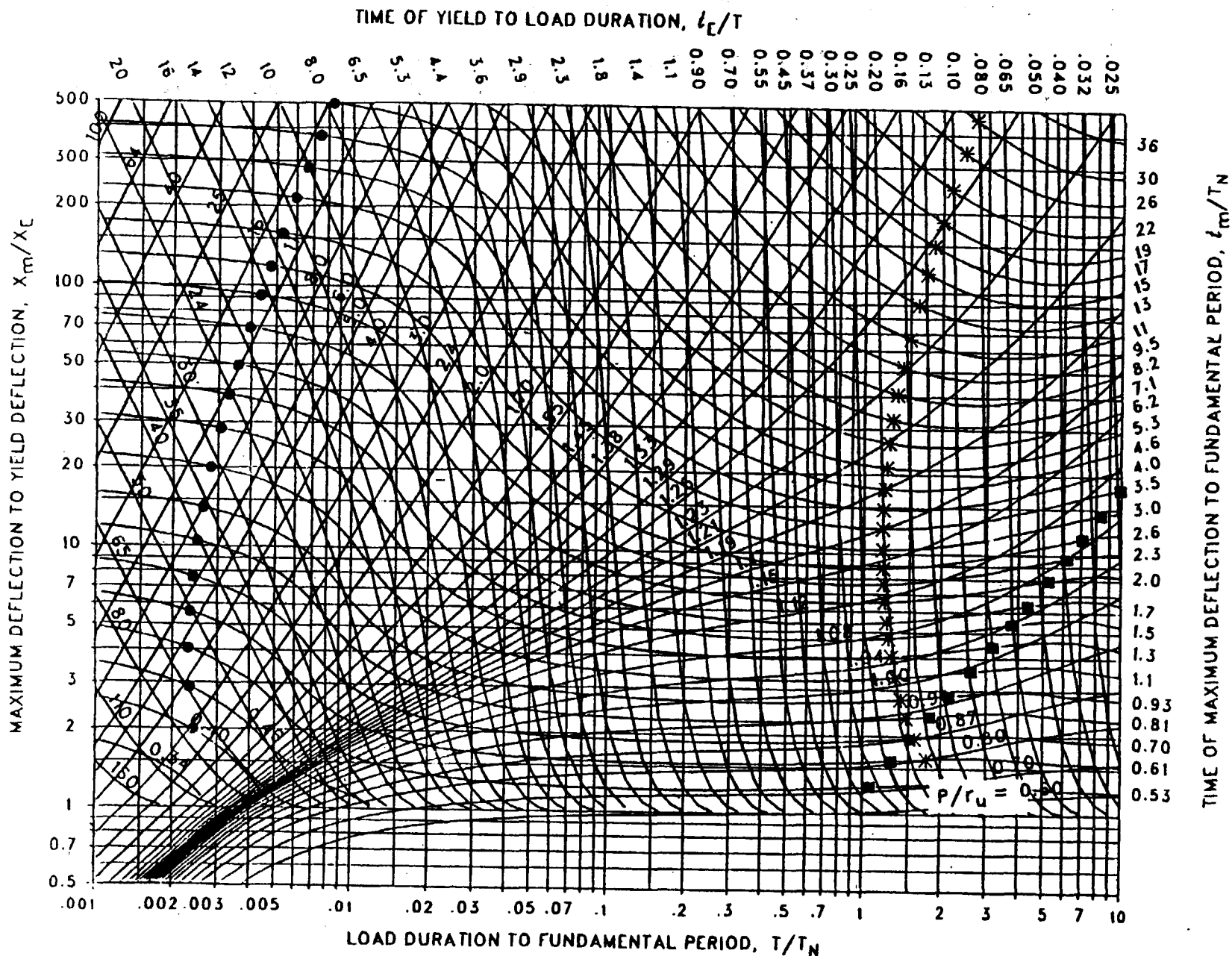


Figure 3-143 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.825$ ,  $C_2 = 100$ .)



3-202

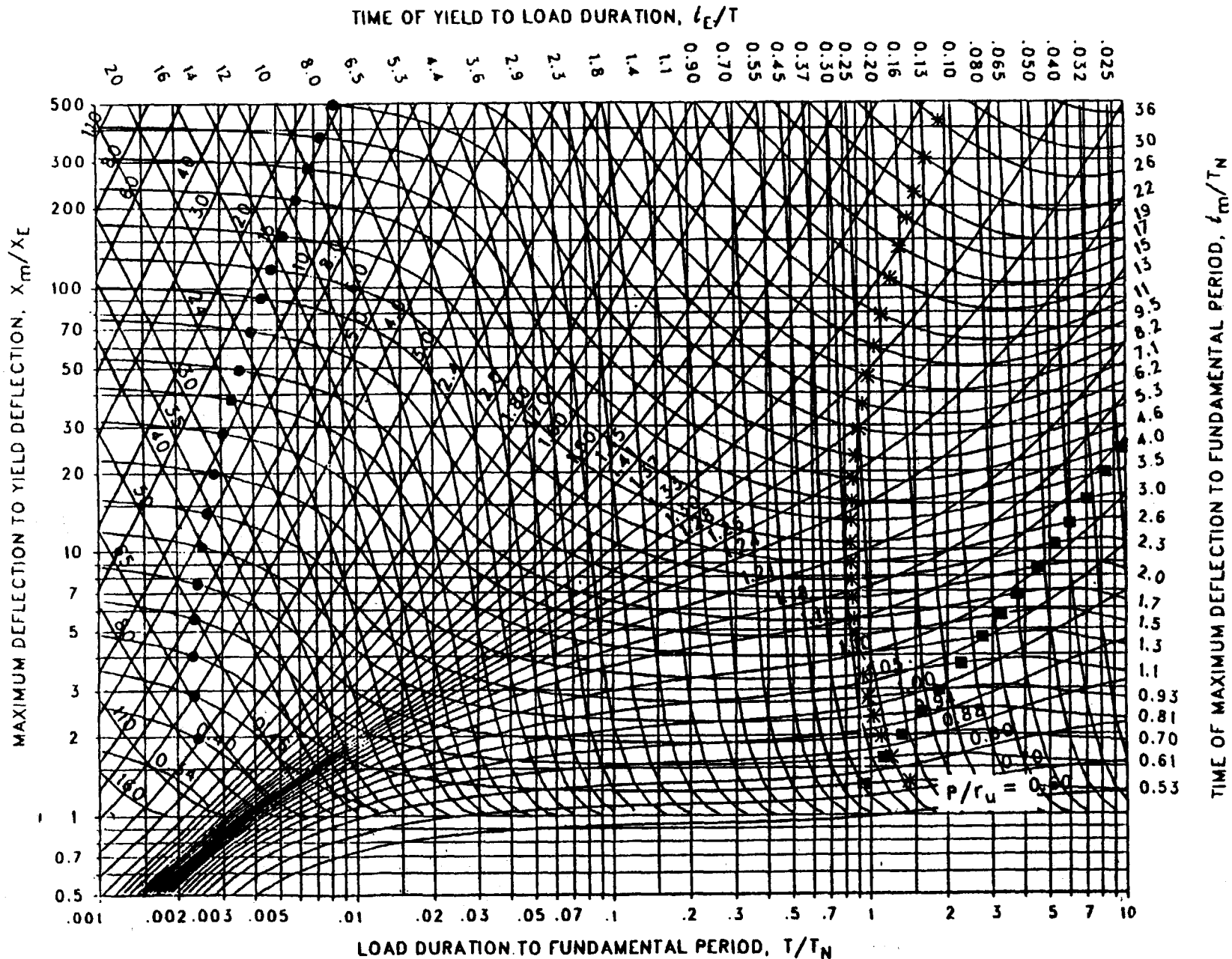


Figure 3-144 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.787$ ,  $C_2 = 100$ .)

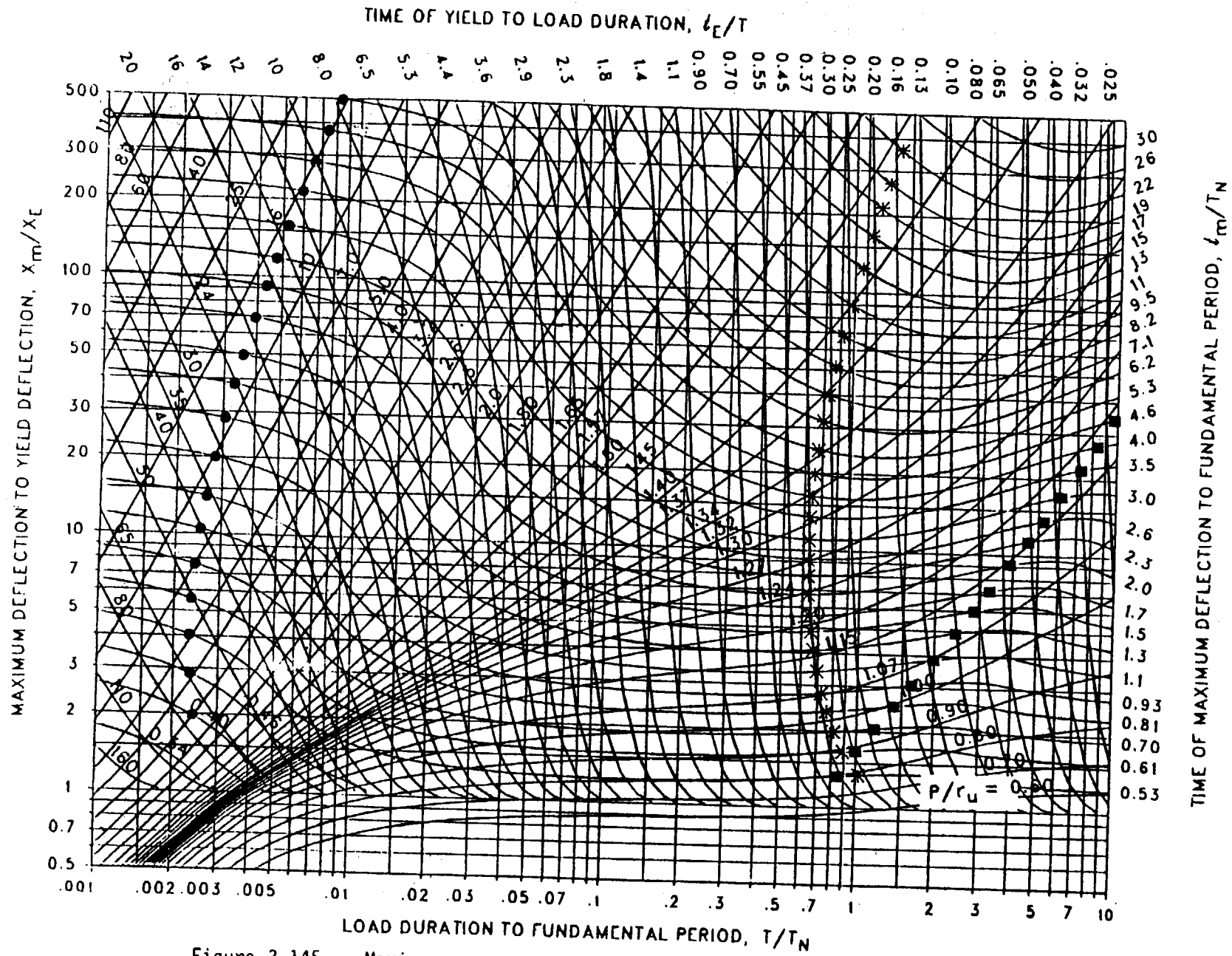


Figure 3-145 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.750$ ,  $C_2 = 100.$ )

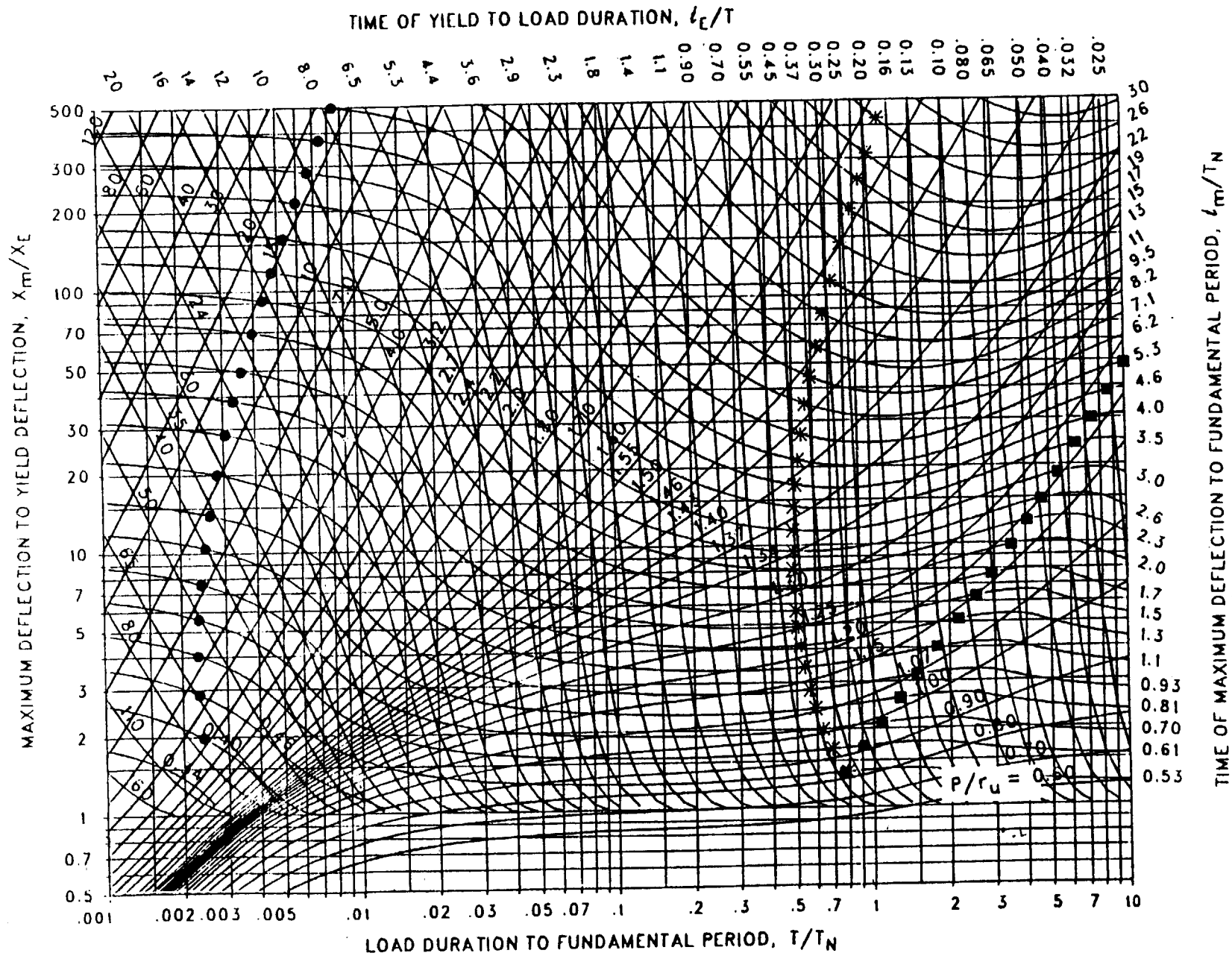


Figure 3-146 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.715$ ,  $C_2 = 100$ .)

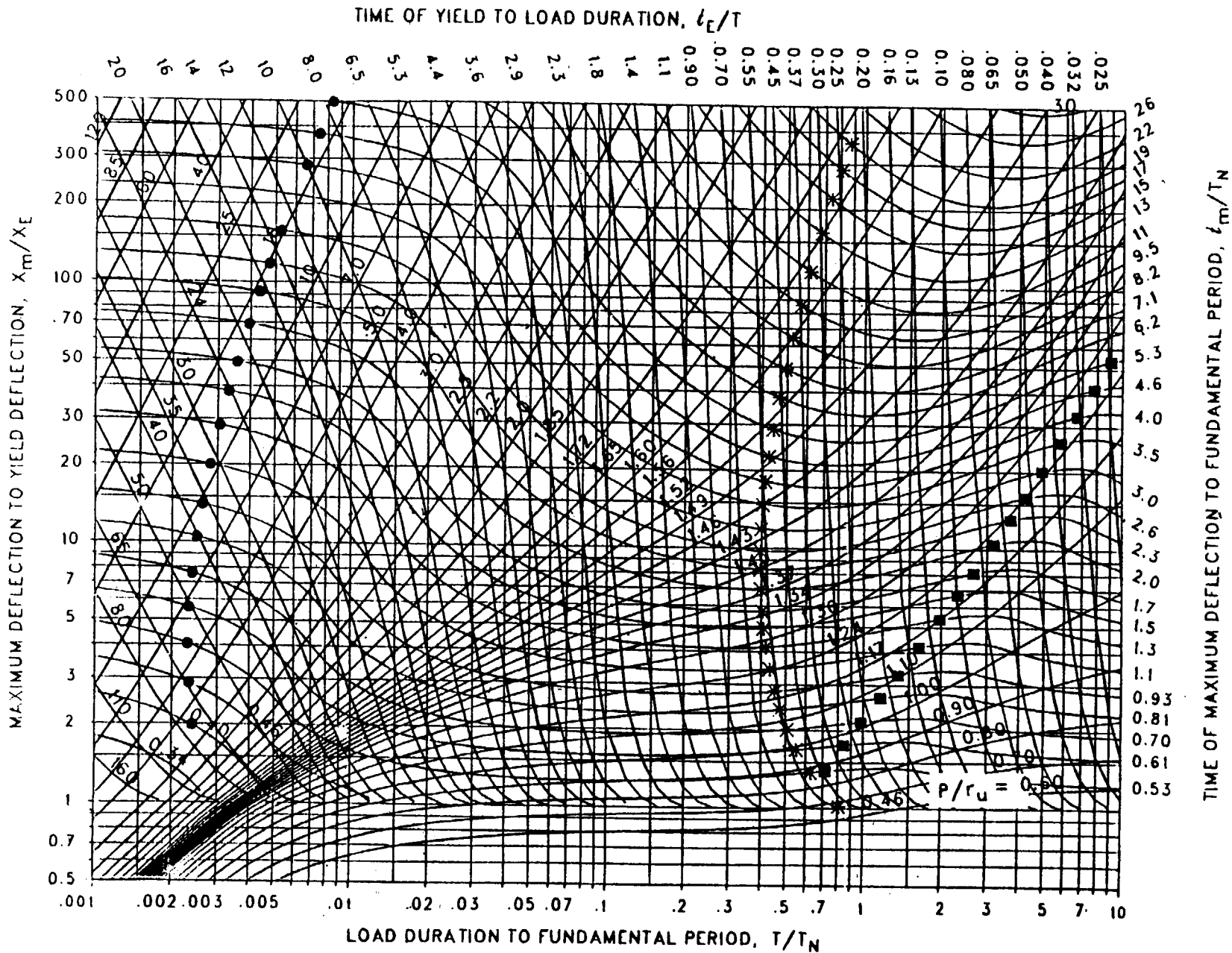


Figure 3-147 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.681$ ,  $C_2 = 100$ .)

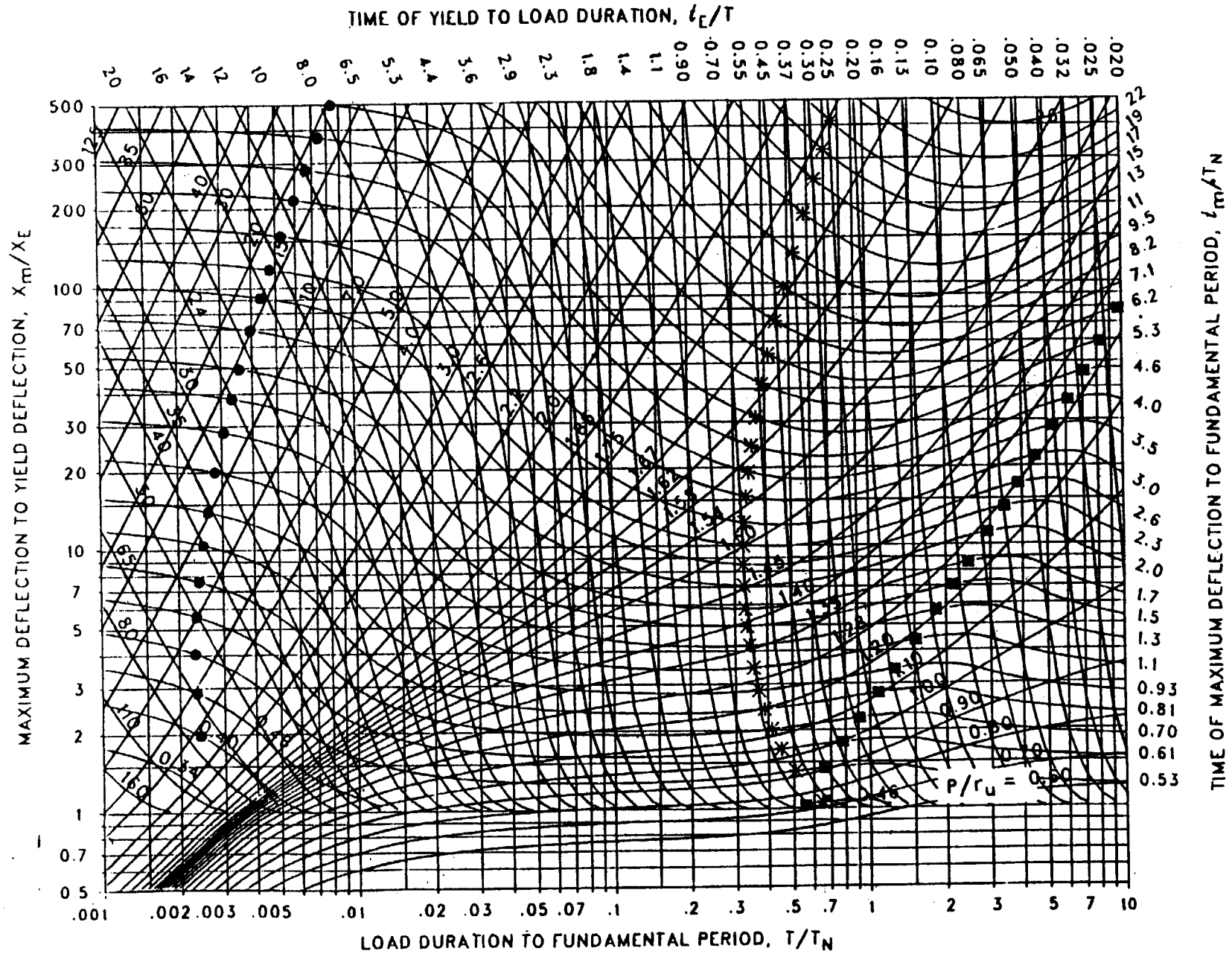


Figure 3-148 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.648$ ,  $C_2 = 100.$ )

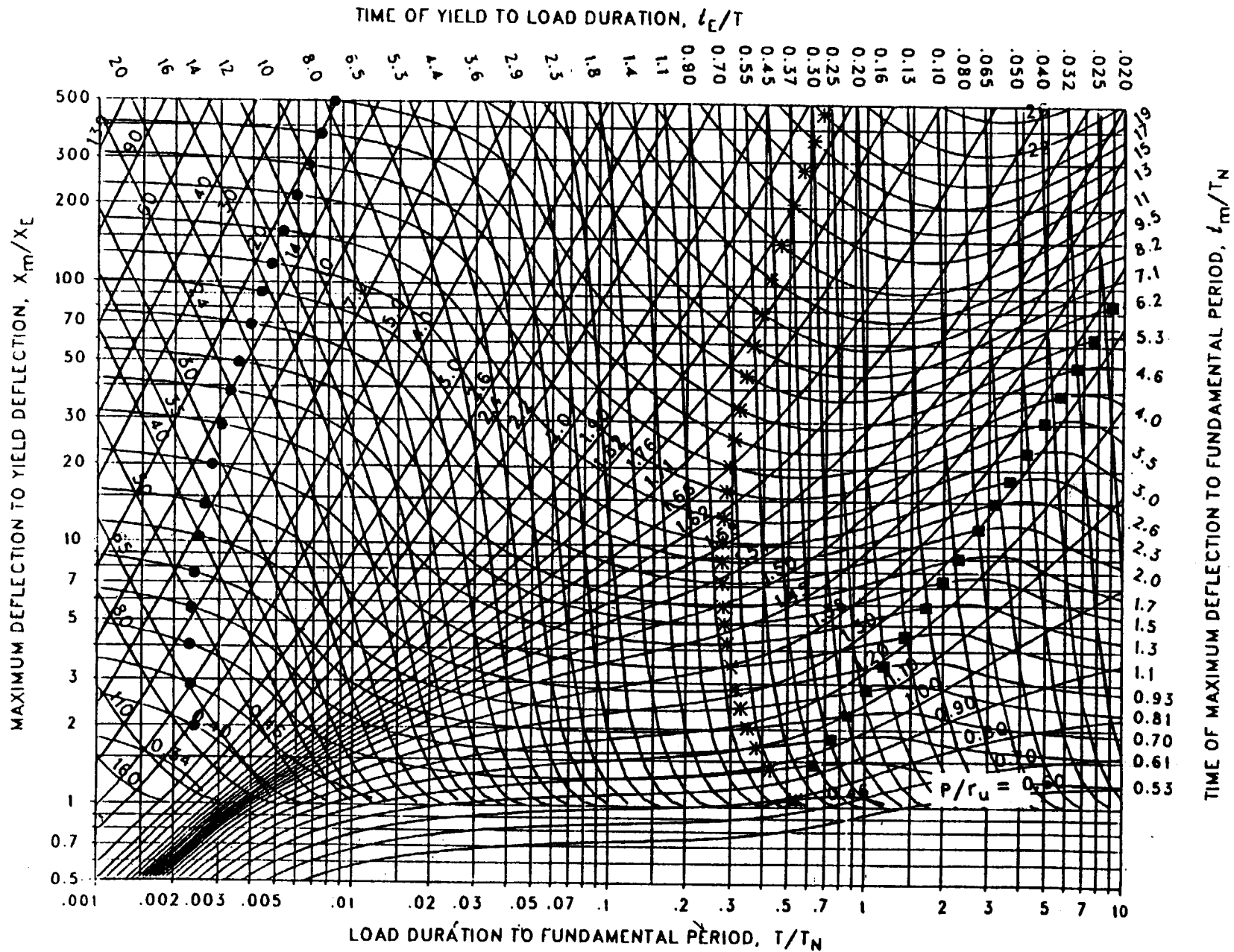


Figure 3-149 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.619$ ,  $C_2 = 100$ .)

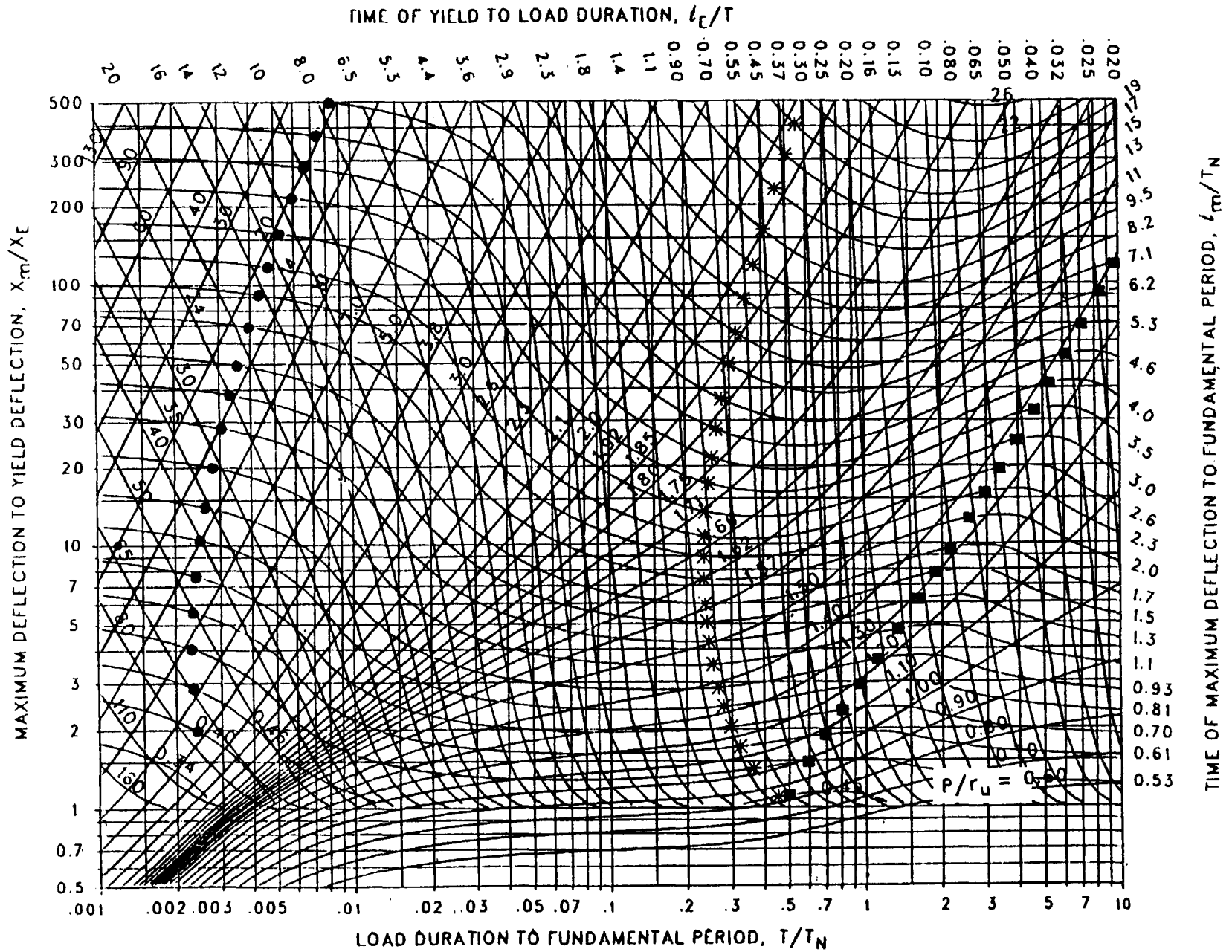


Figure 3-150 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.590$ ,  $C_2 = 100.$ )

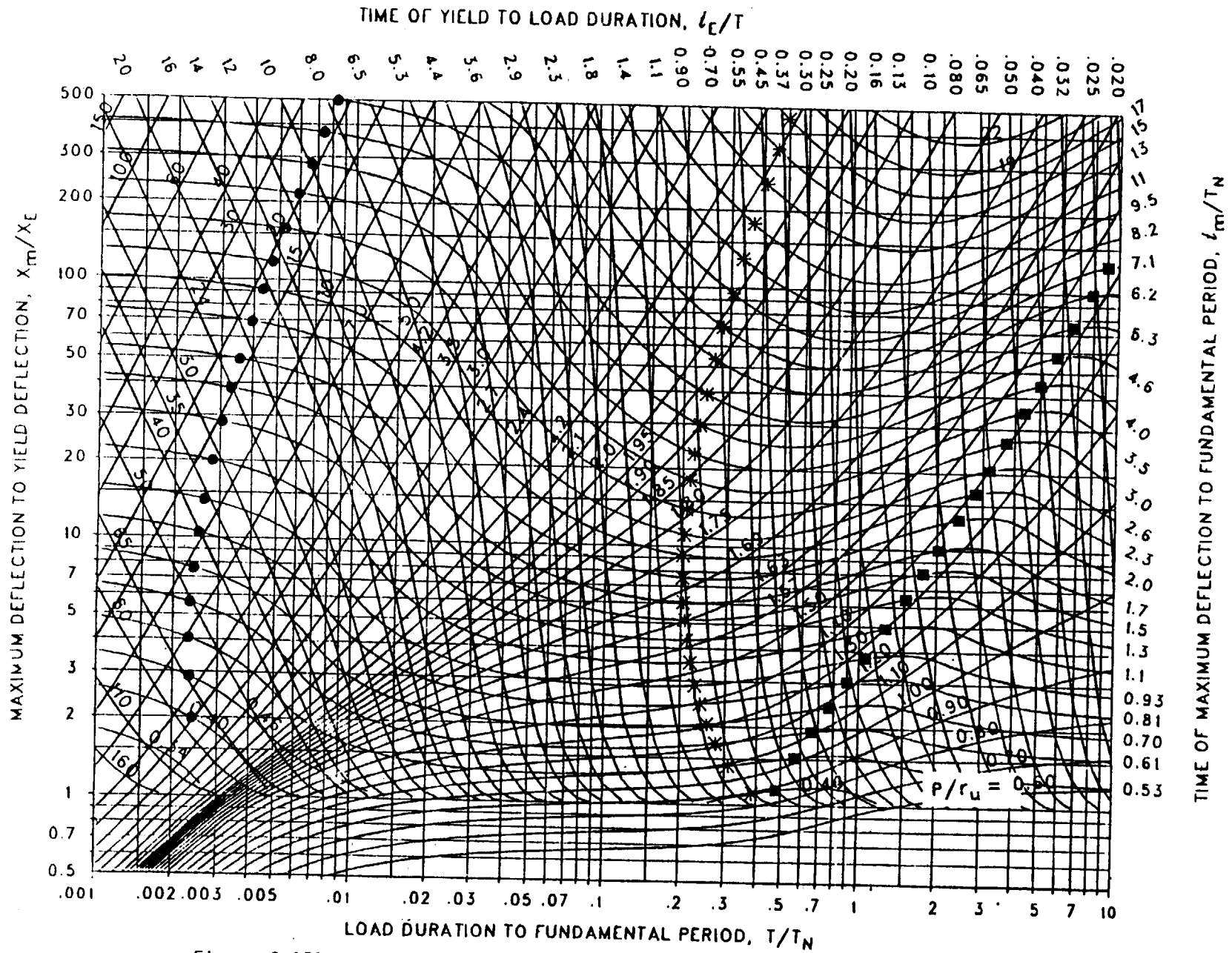


Figure 3-151 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.562$ ,  $C_2 = 100$ .)



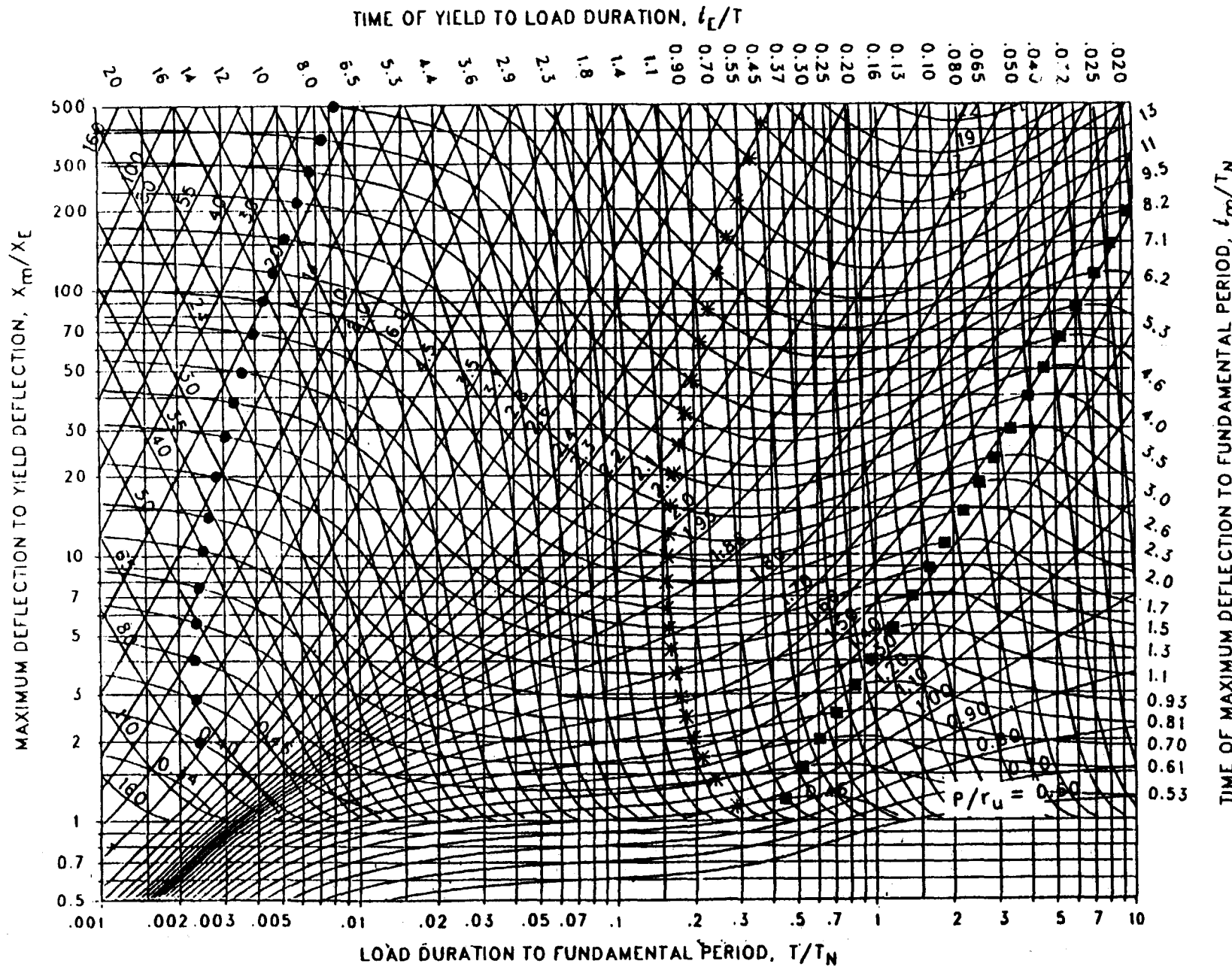


Figure 3-152 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.511$ ,  $C_2 = 100$ .)

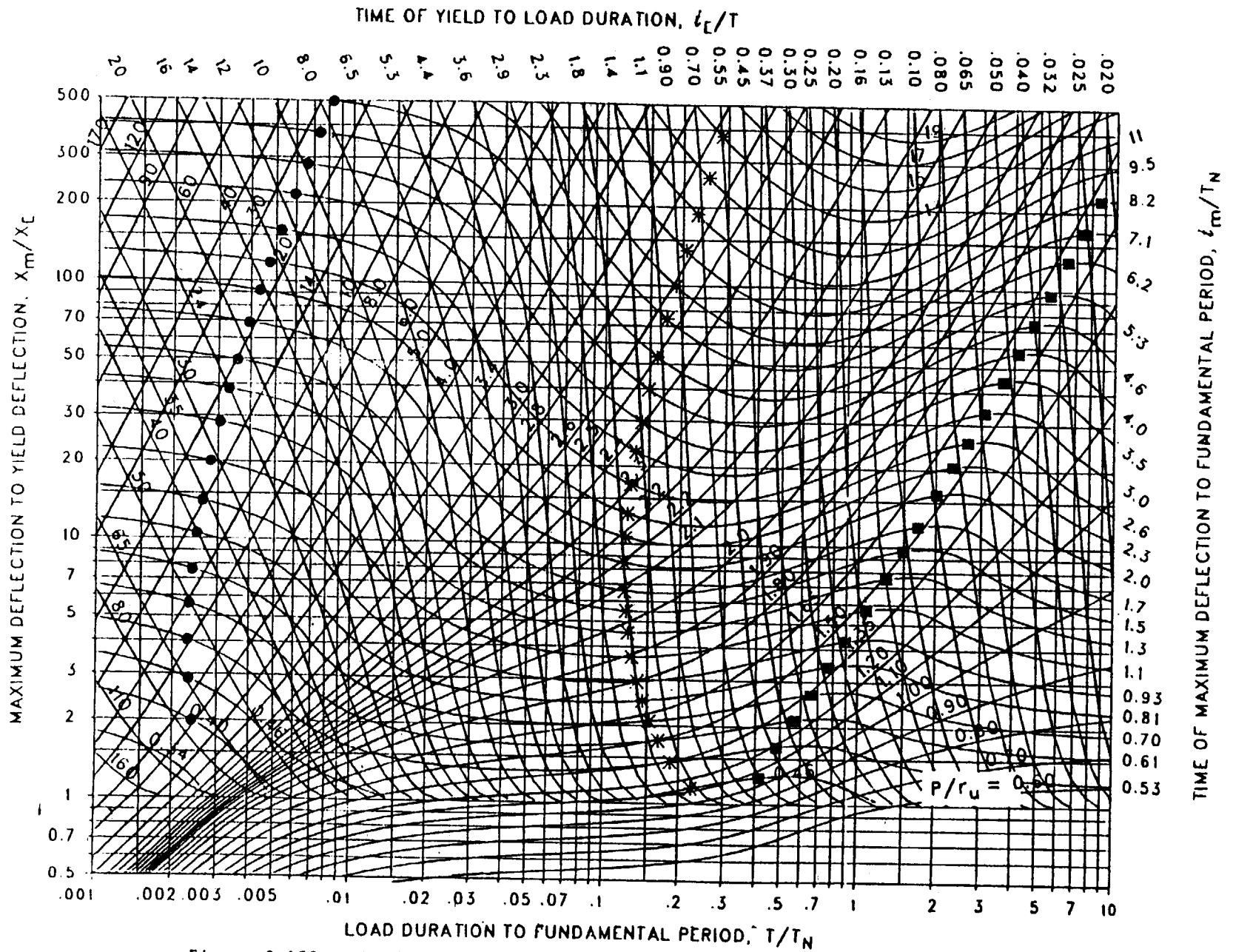


Figure 3-153 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.464$ ,  $C_2 = 100.$ )

3-212

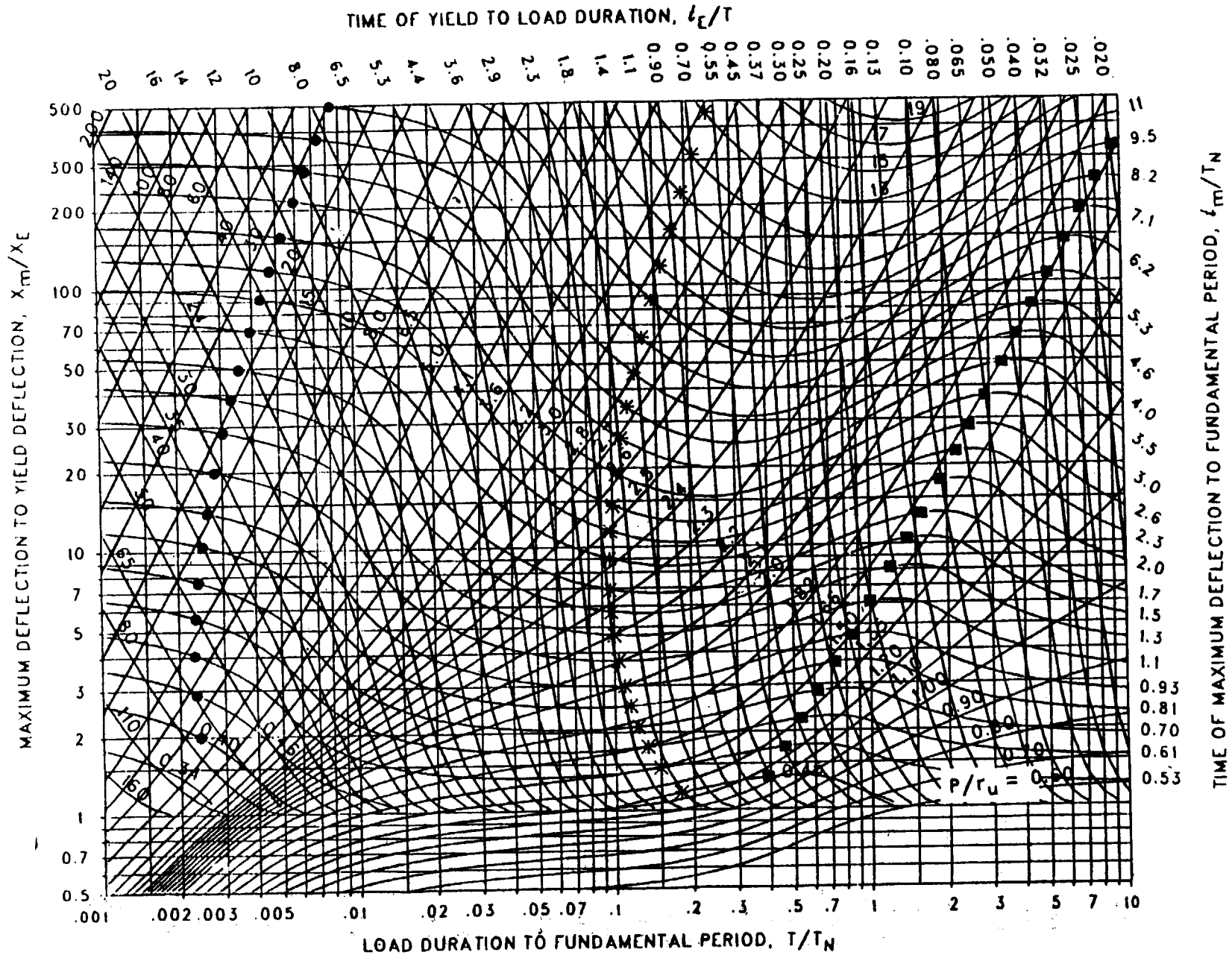


Figure 3-154 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.422$ ,  $C_2 = 100$ .)

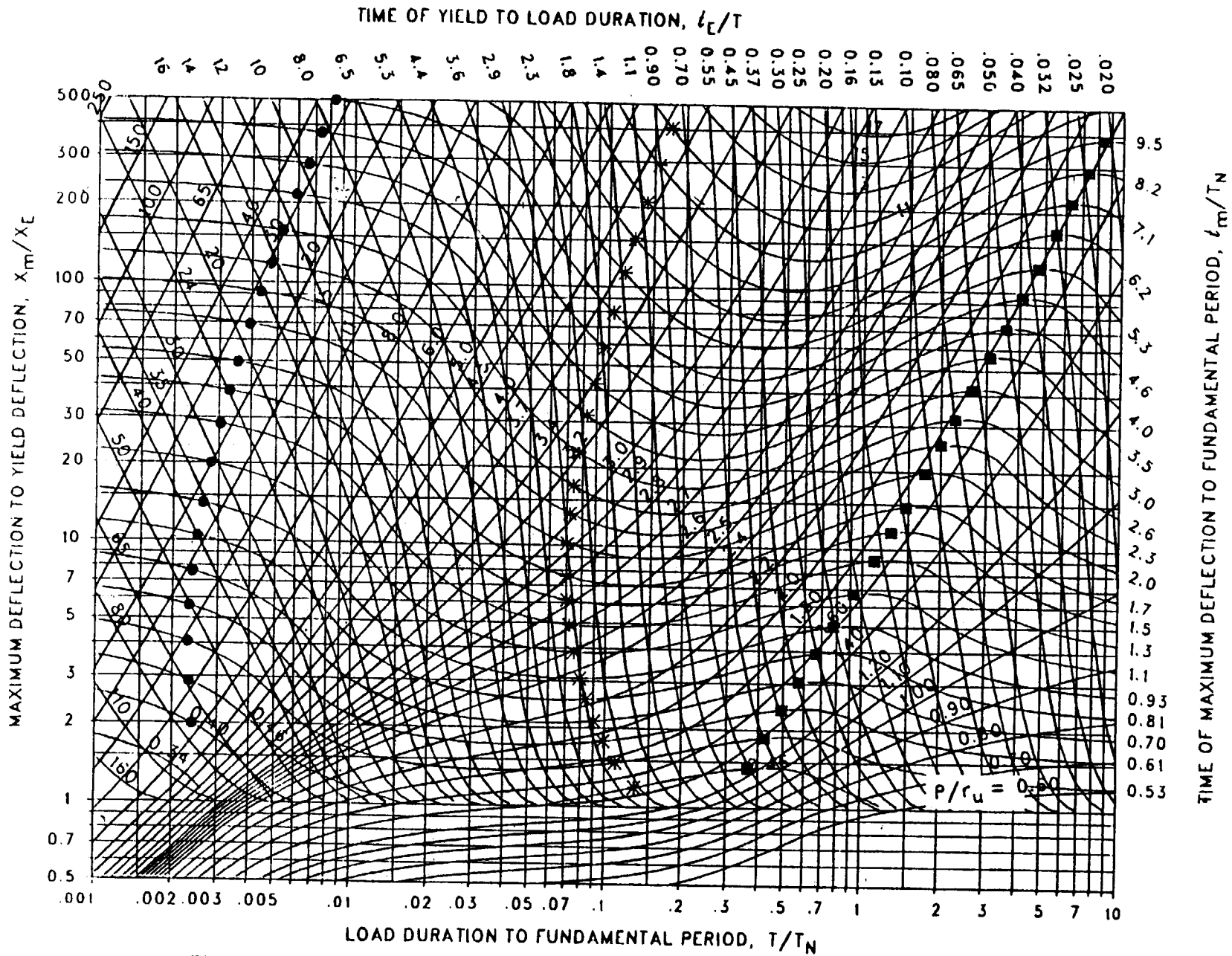


Figure 3-155 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.365$ ,  $C_2 = 100$ .)

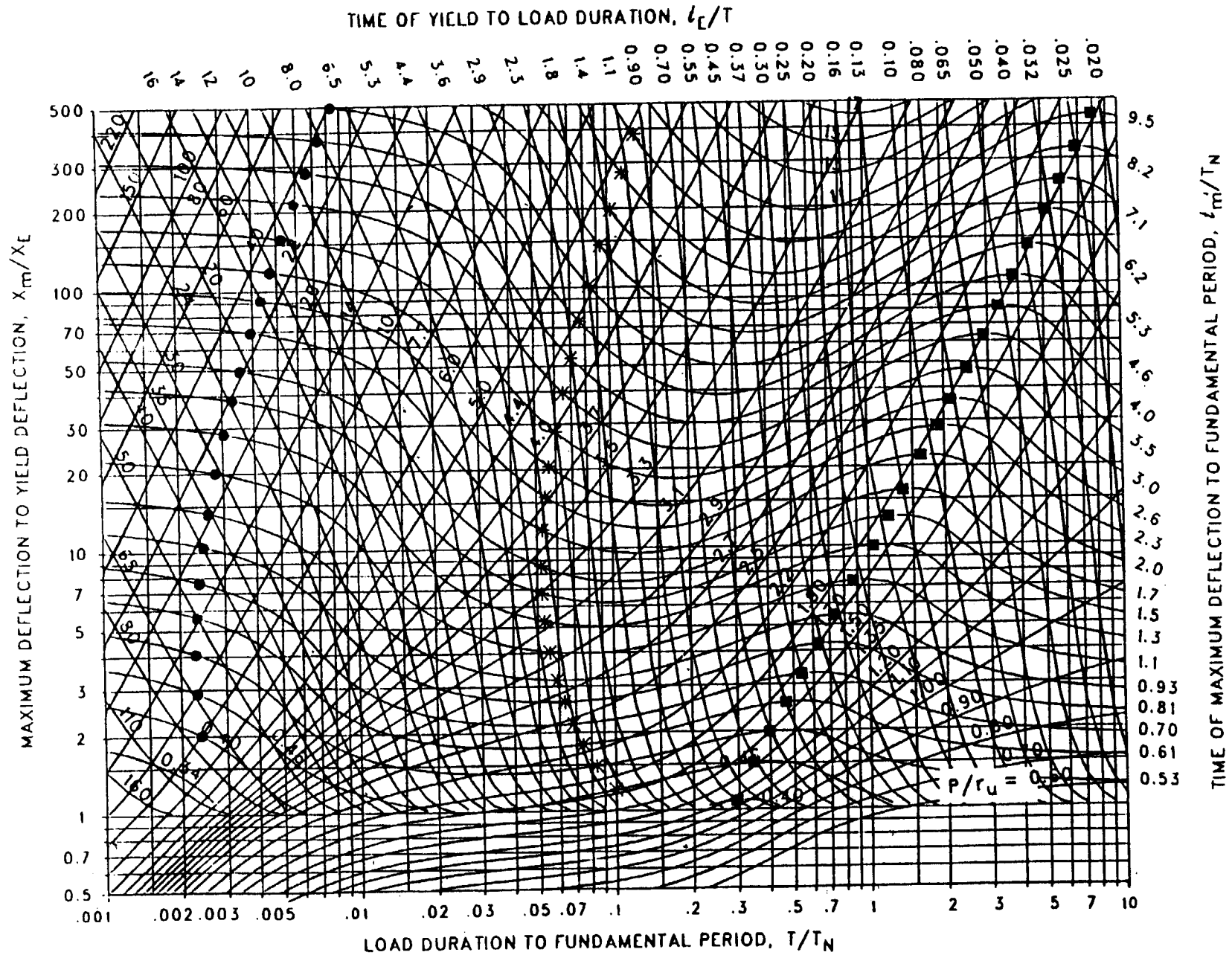


Figure 3-156 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.316$ ,  $C_2 = 100$ .)

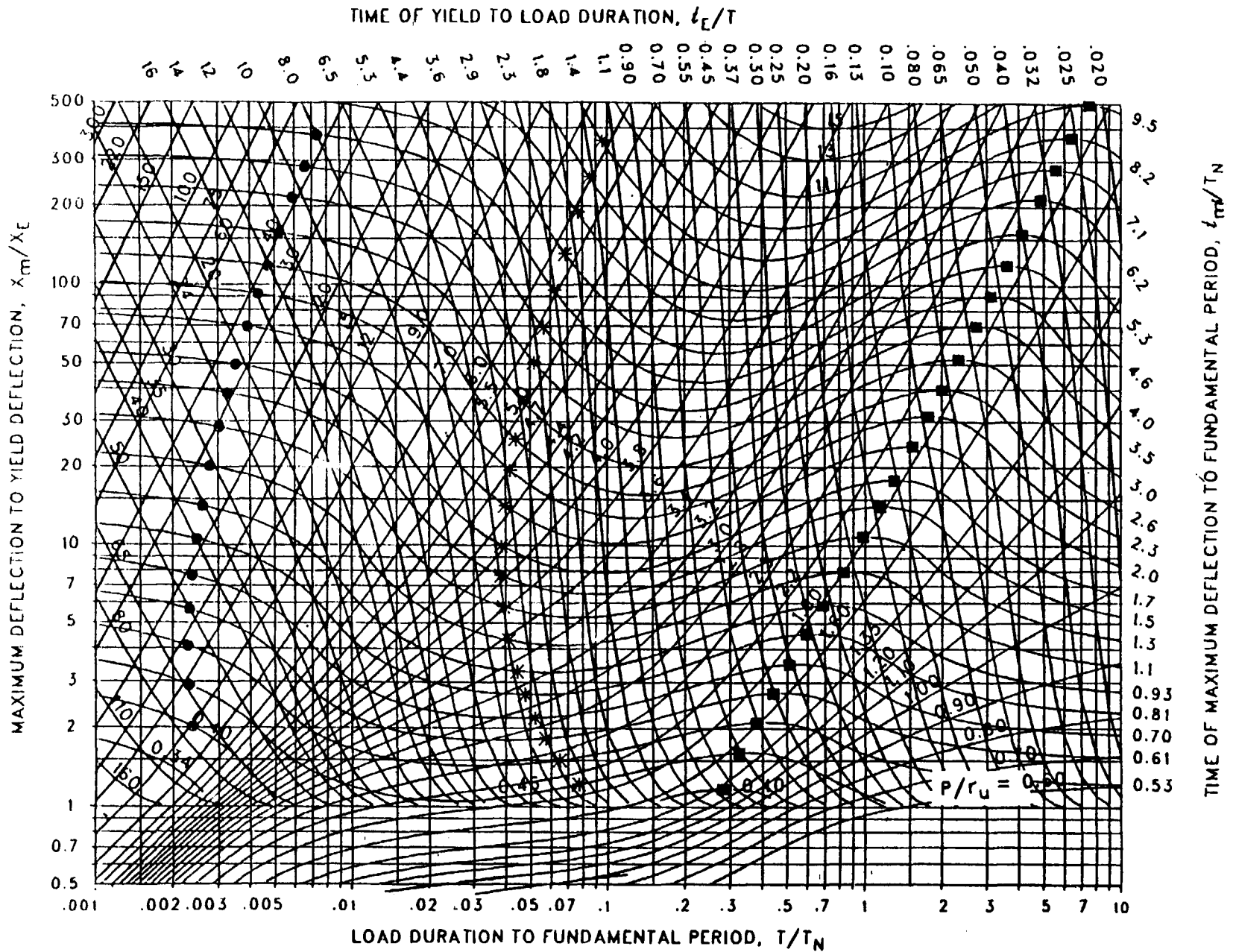


Figure 3-157 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.274$ ,  $C_2 = 100.$ )

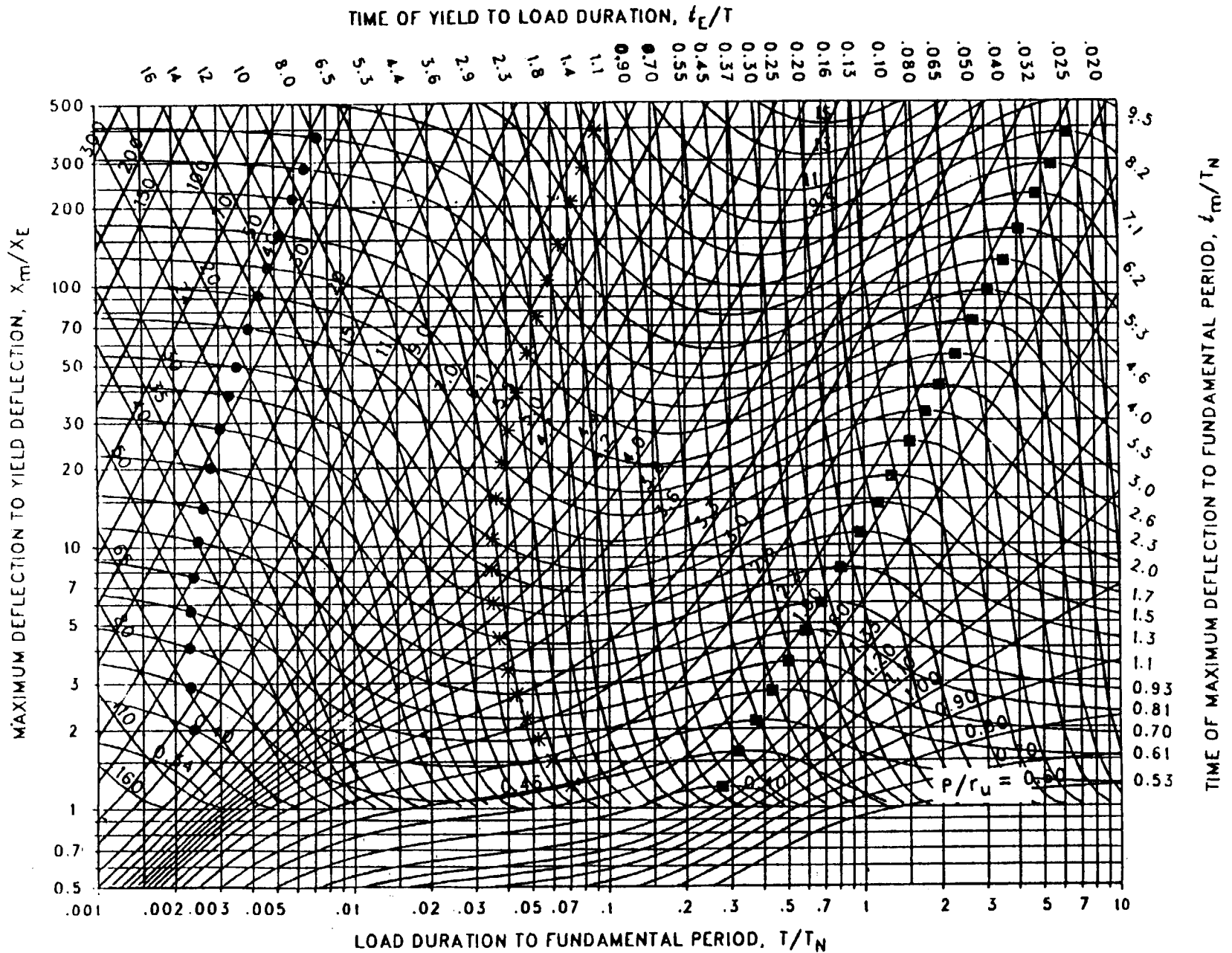


Figure 3-158 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.261$ ,  $C_2 = 100$ .)

3-217

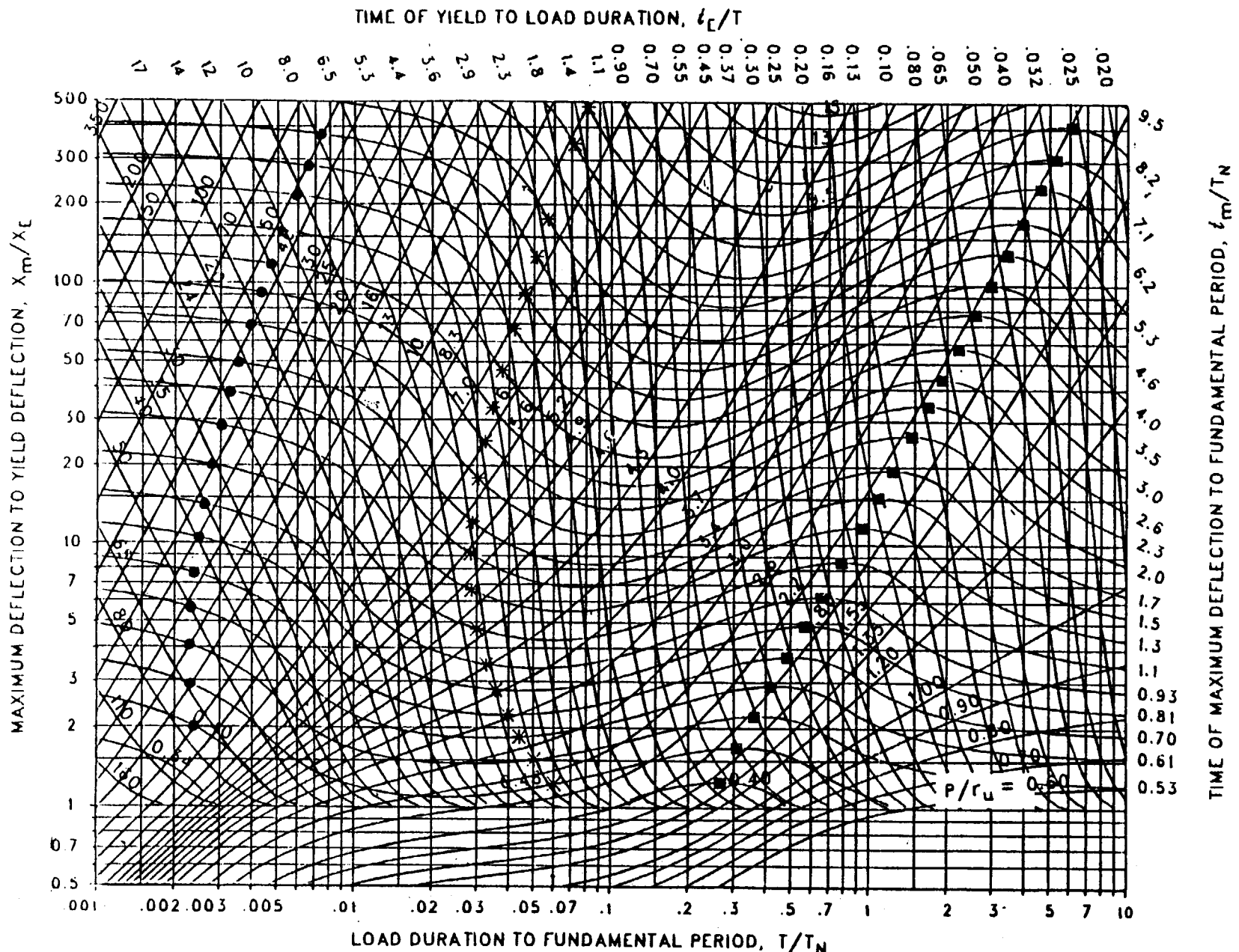


Figure 3-159 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.237$ ,  $C_2 = 100$ .)



3-218

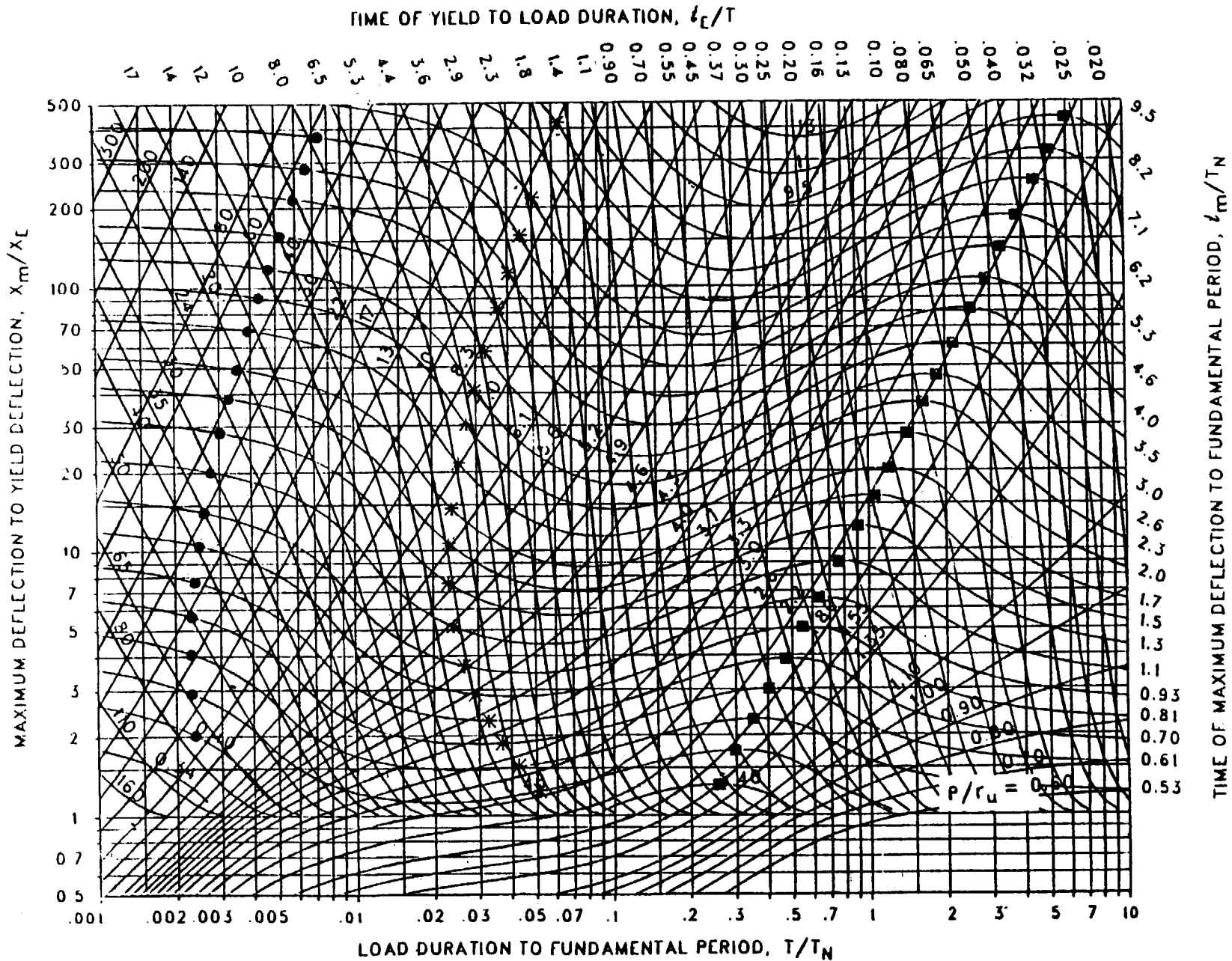


Figure 3-160 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.215$ ,  $C_2 = 100$ .)

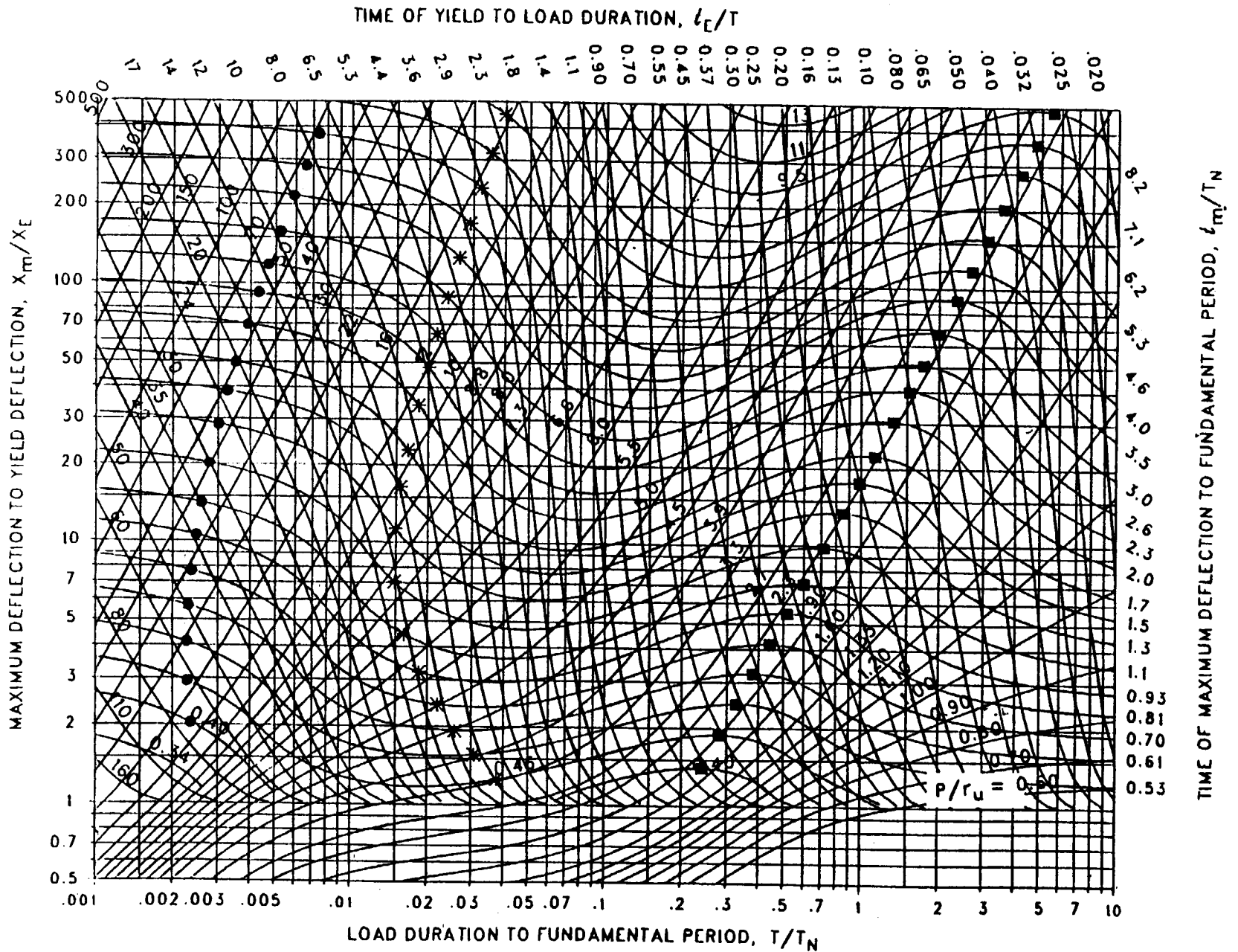


Figure 3-161 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.178$ ,  $C_2 = 100$ .)

3-2220

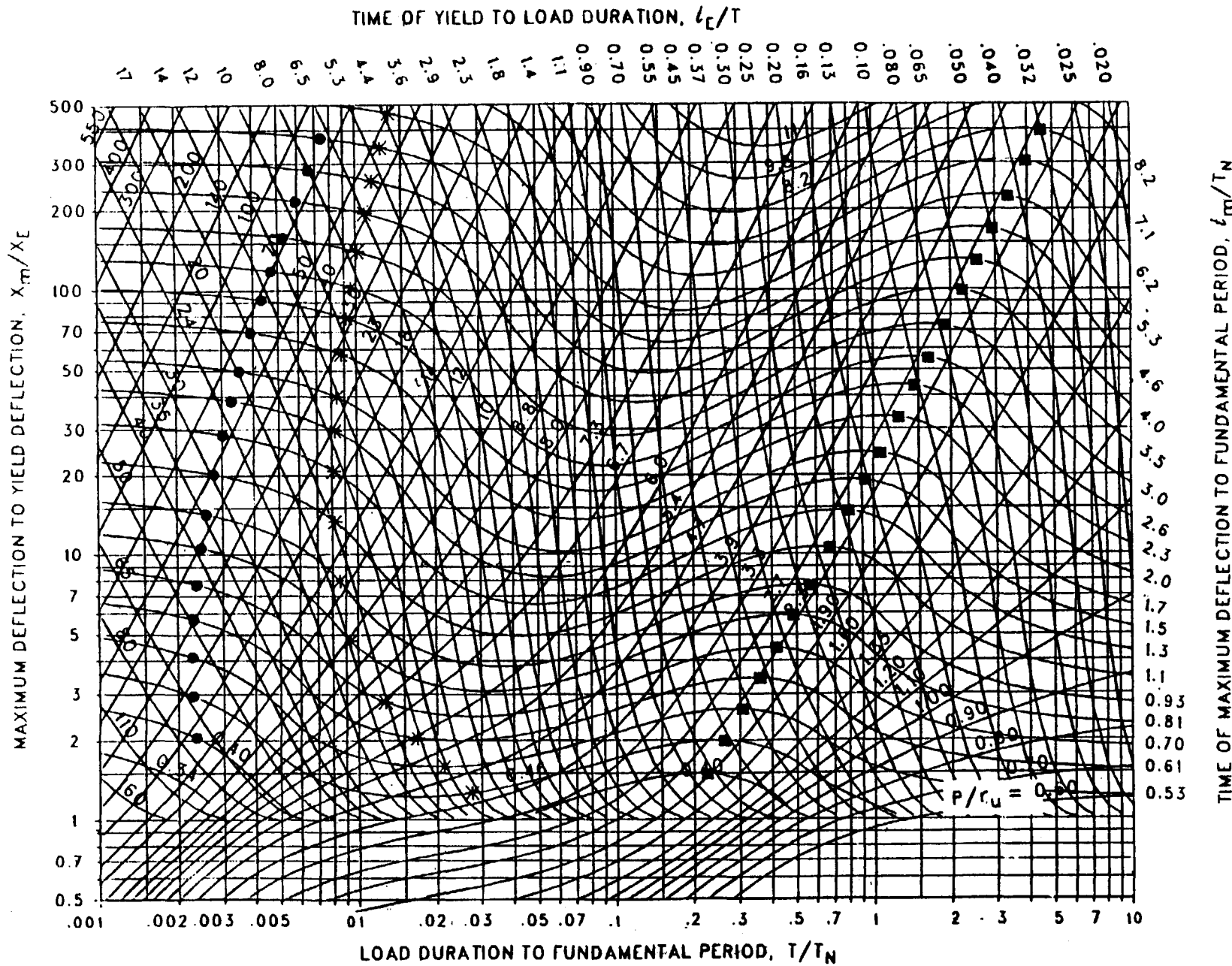


Figure 3-162 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.147$ ,  $C_2 = 100$ .)

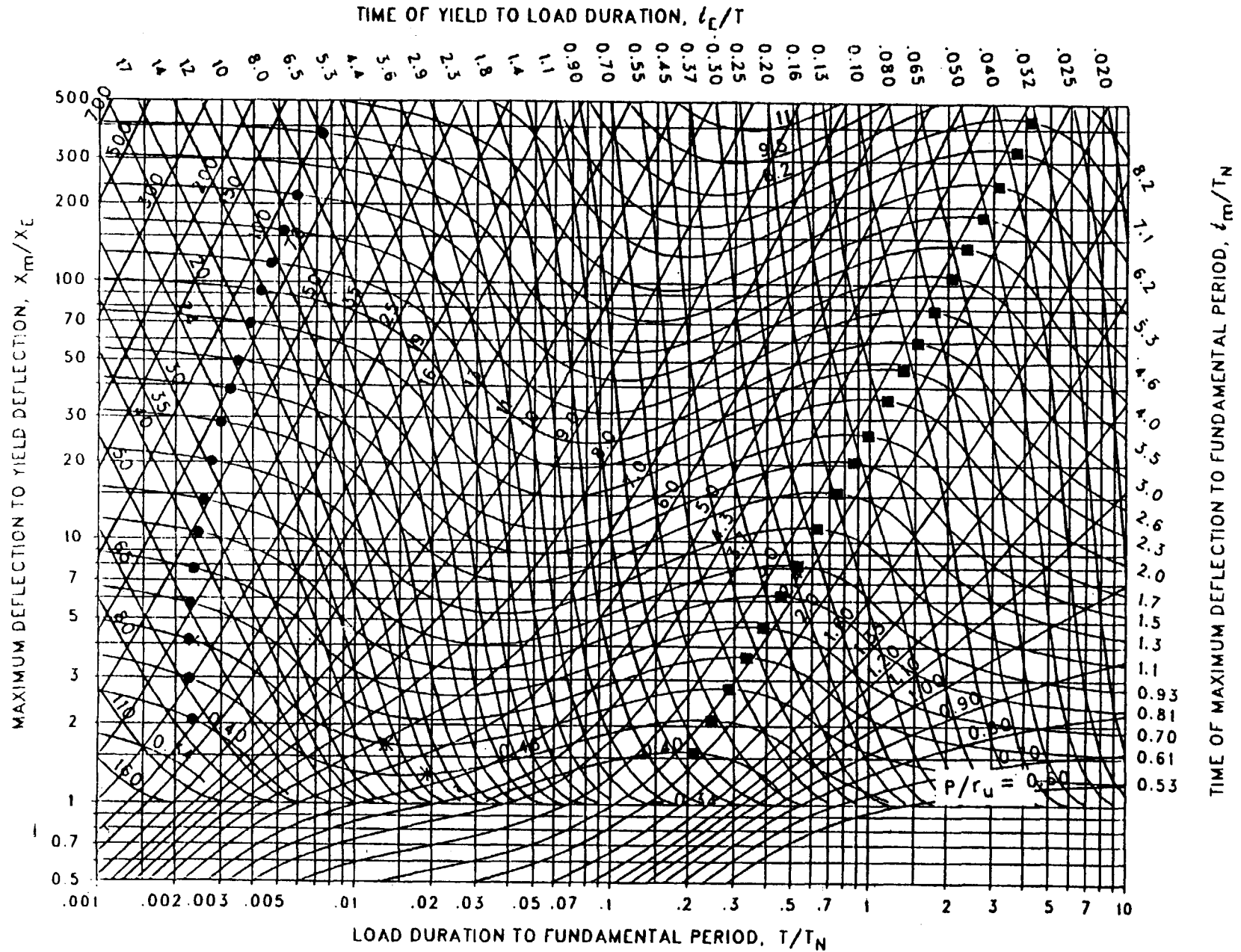


Figure 3-163 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.121$ ,  $C_2 = 100.$ )

3-2222

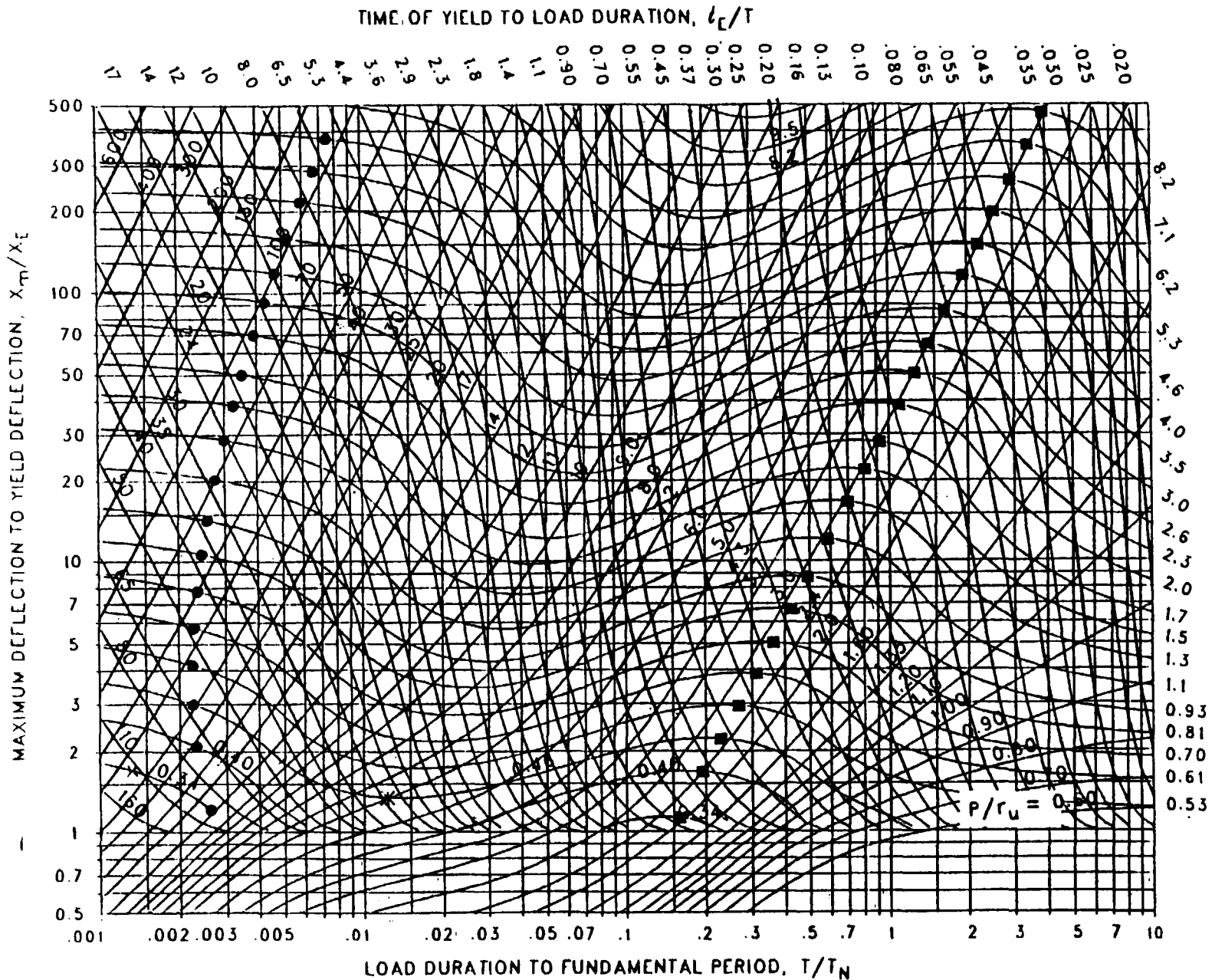


Figure 3-164 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.100$ ,  $C_2 = 100$ .)

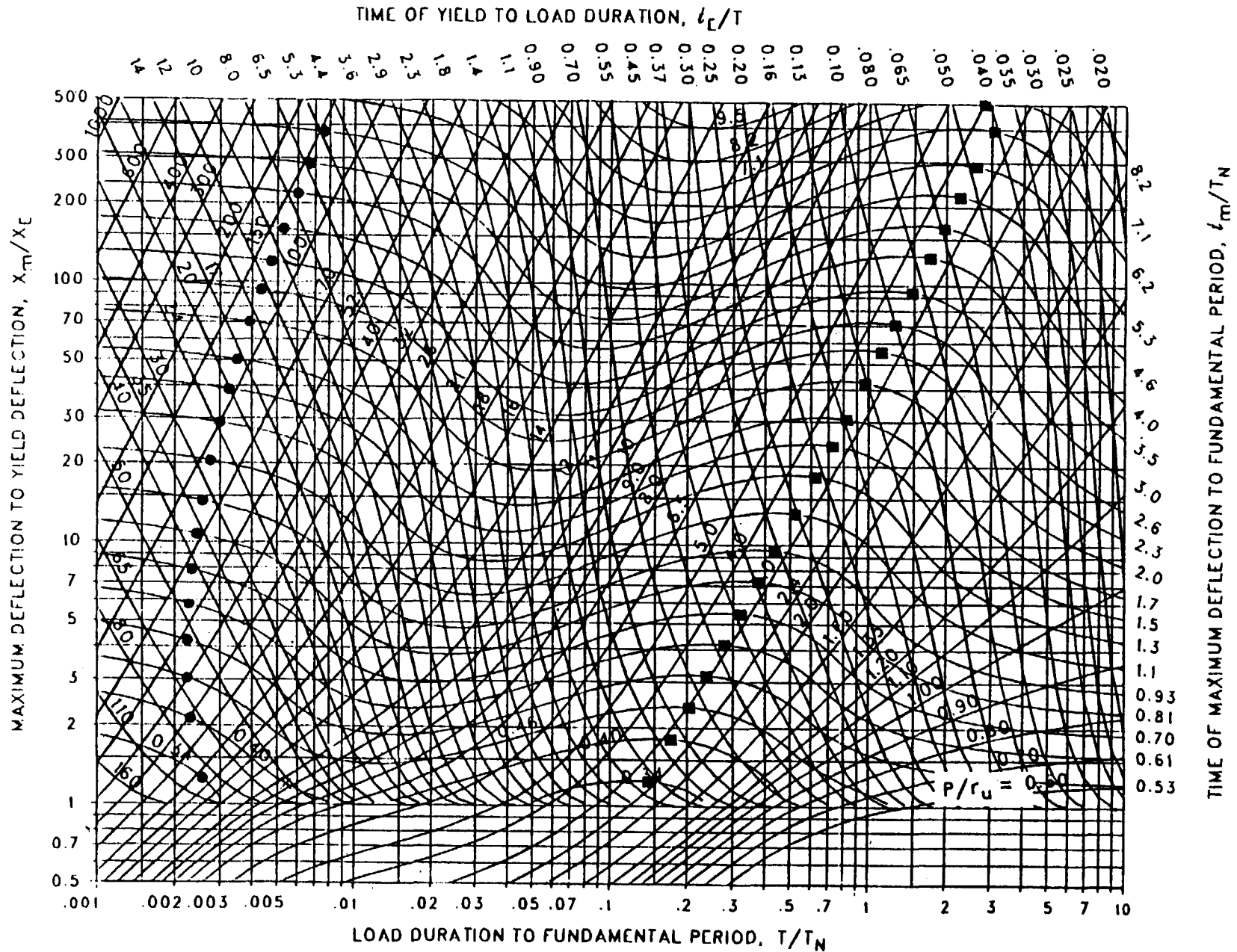


Figure 3-165 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.075$ ,  $C_2 = 100$ .)

3-224

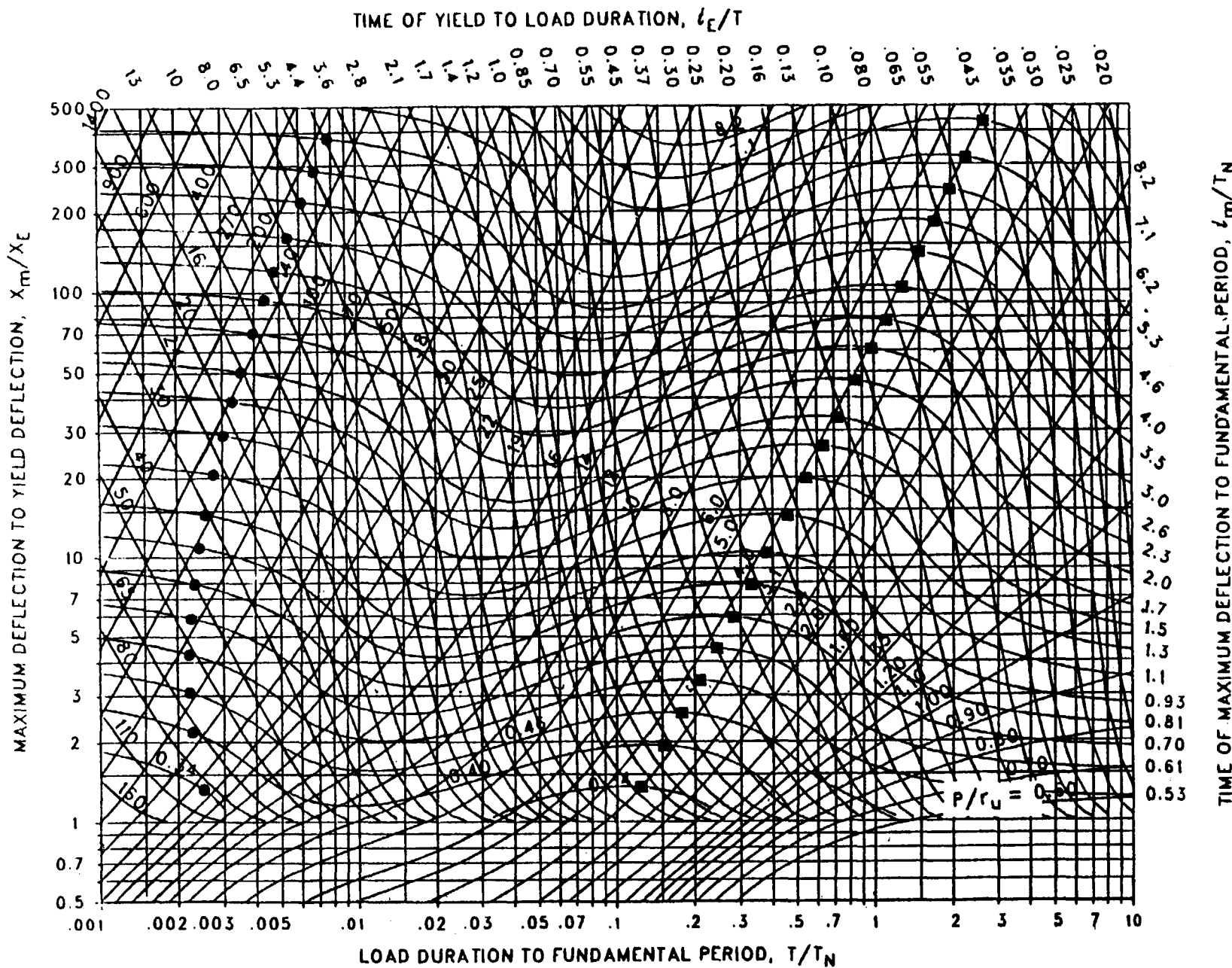


Figure 3-166 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.056$ ,  $C_2 = 100.$ )

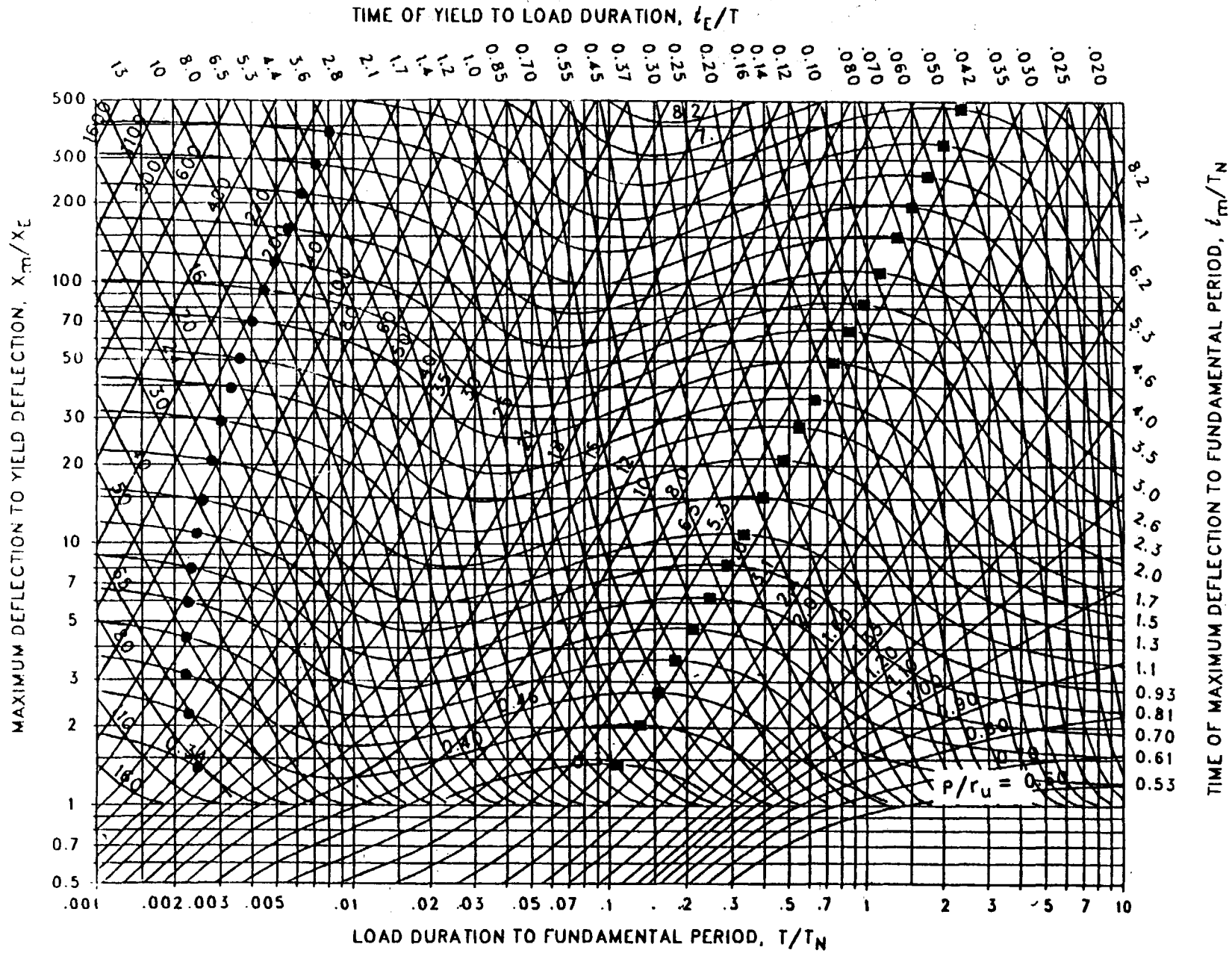


Figure 3-167 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.042$ ,  $C_2 = 100$ .)



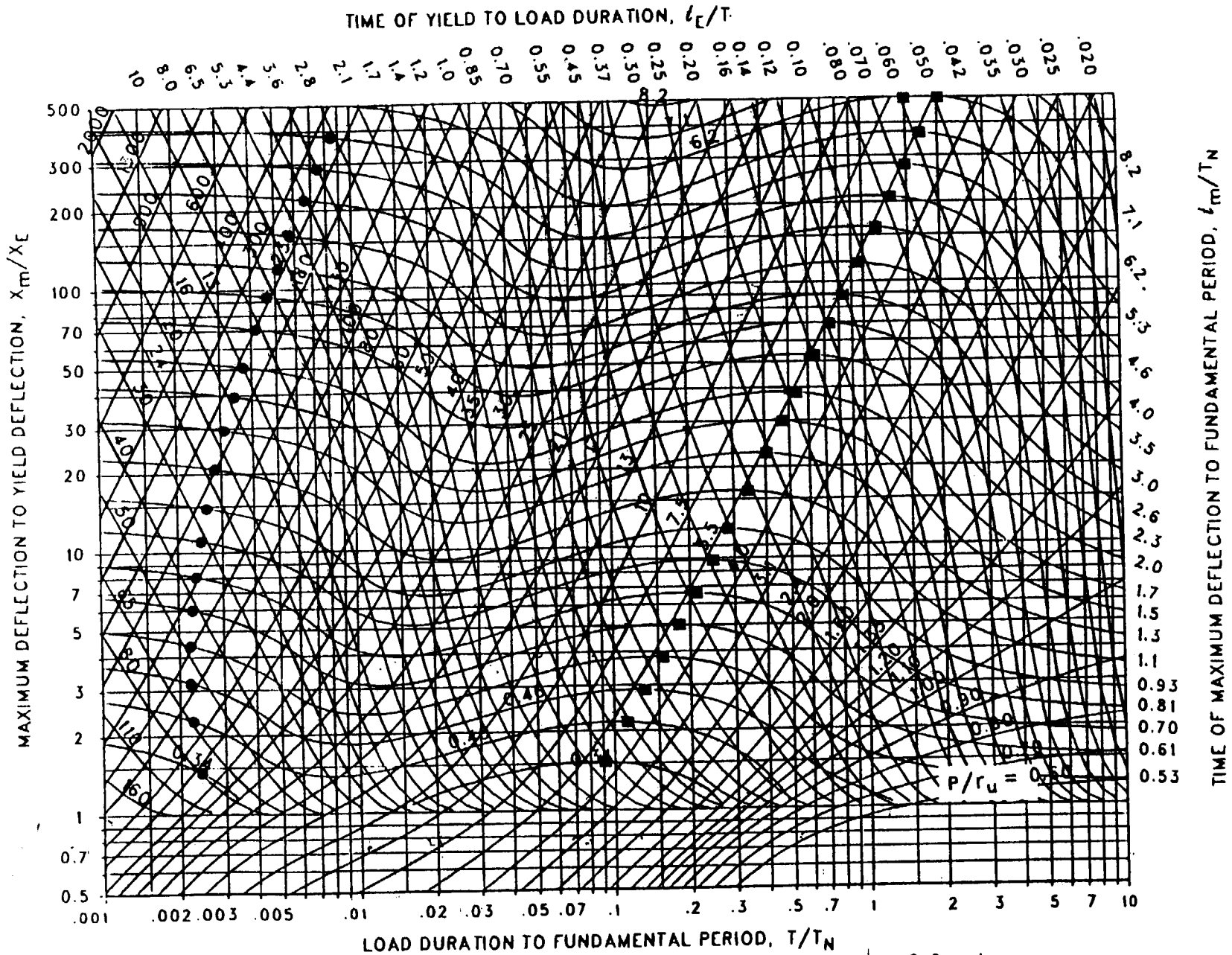


Figure 3-168 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.032$ ,  $C_2 = 100$ .)

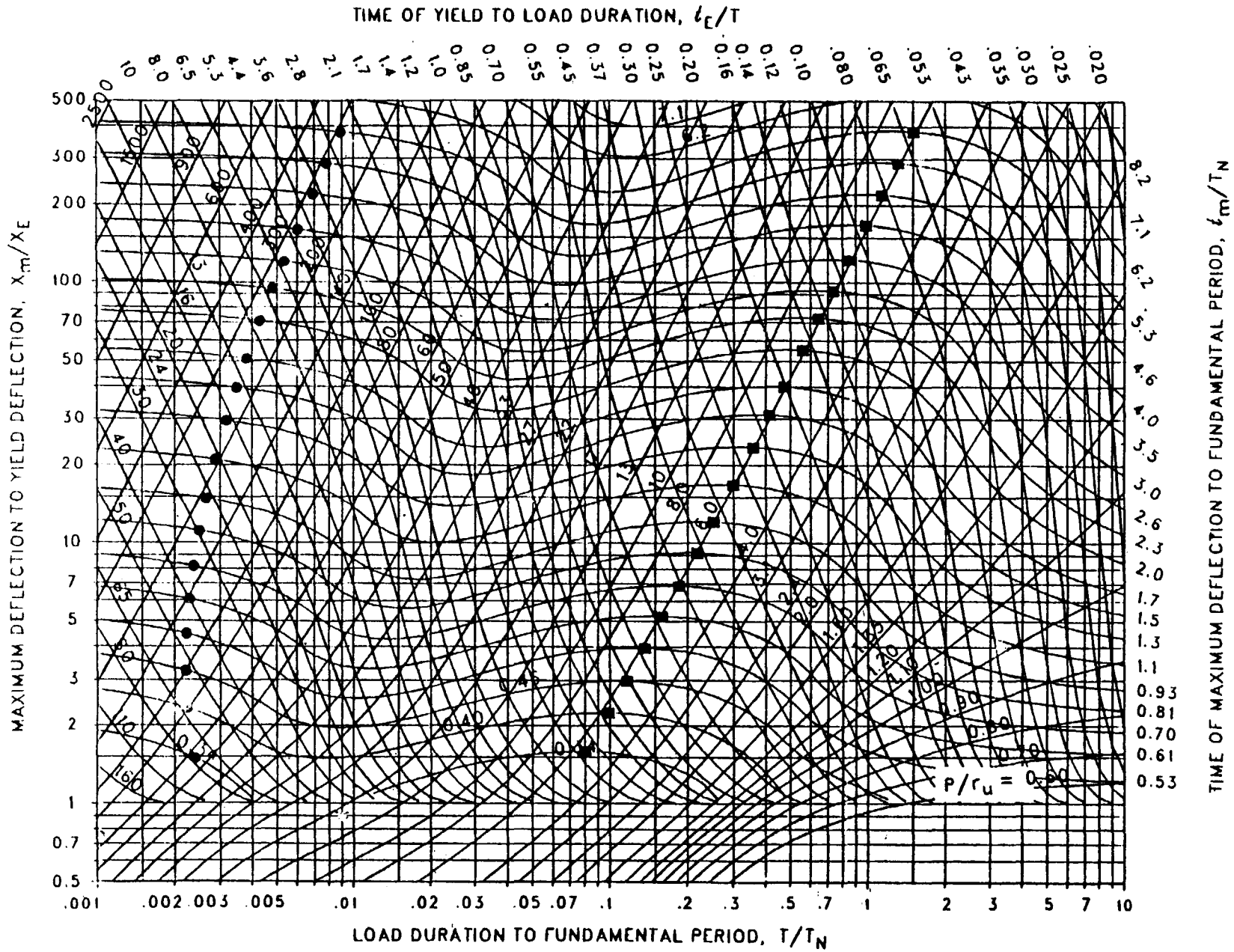


Figure 3-169 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.026$ ,  $C_2 = 100$ .)

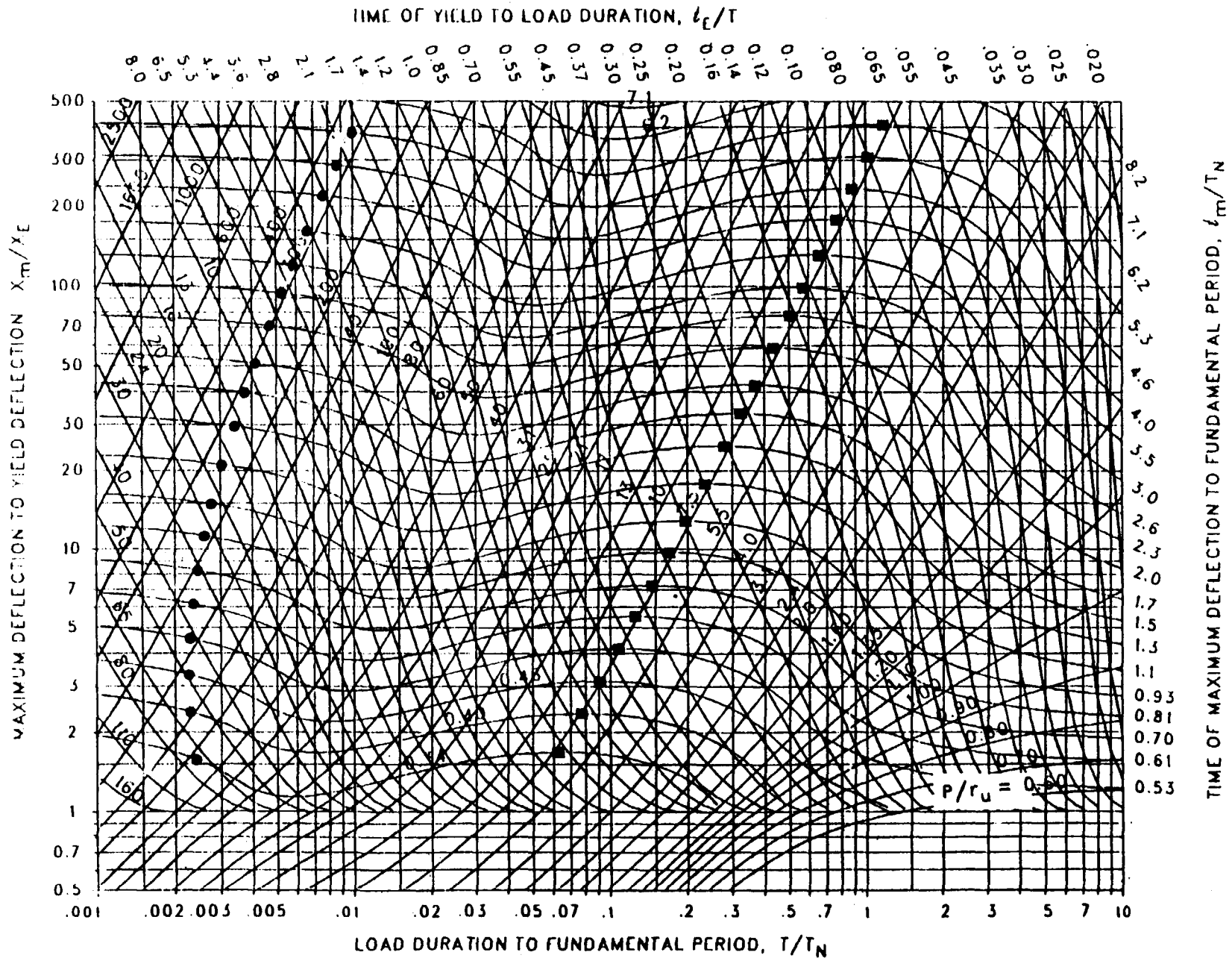


Figure 3-170 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.018$ ,  $C_2 = 100$ .)

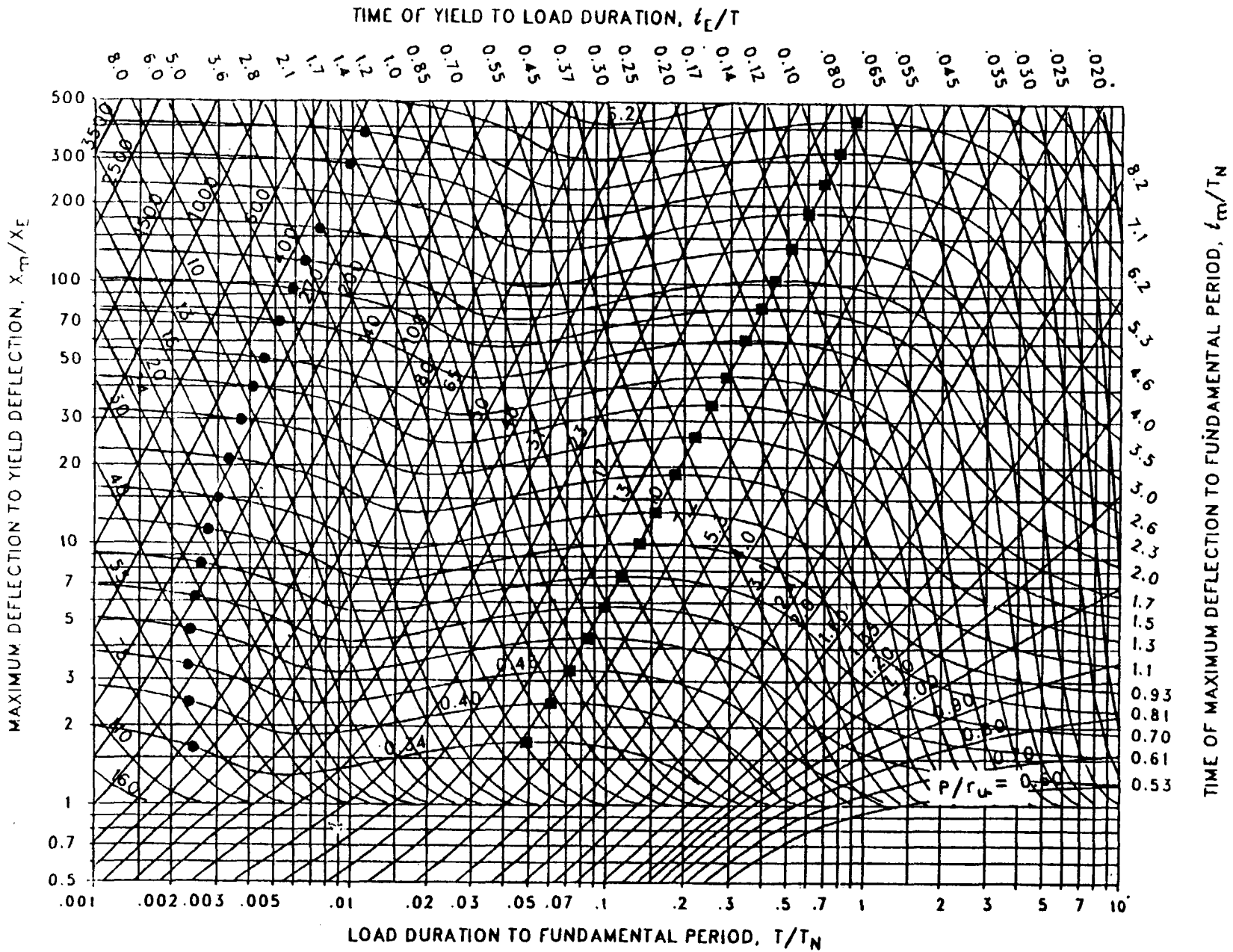


Figure 3-171 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.013$ ,  $C_2 = 100.$ )

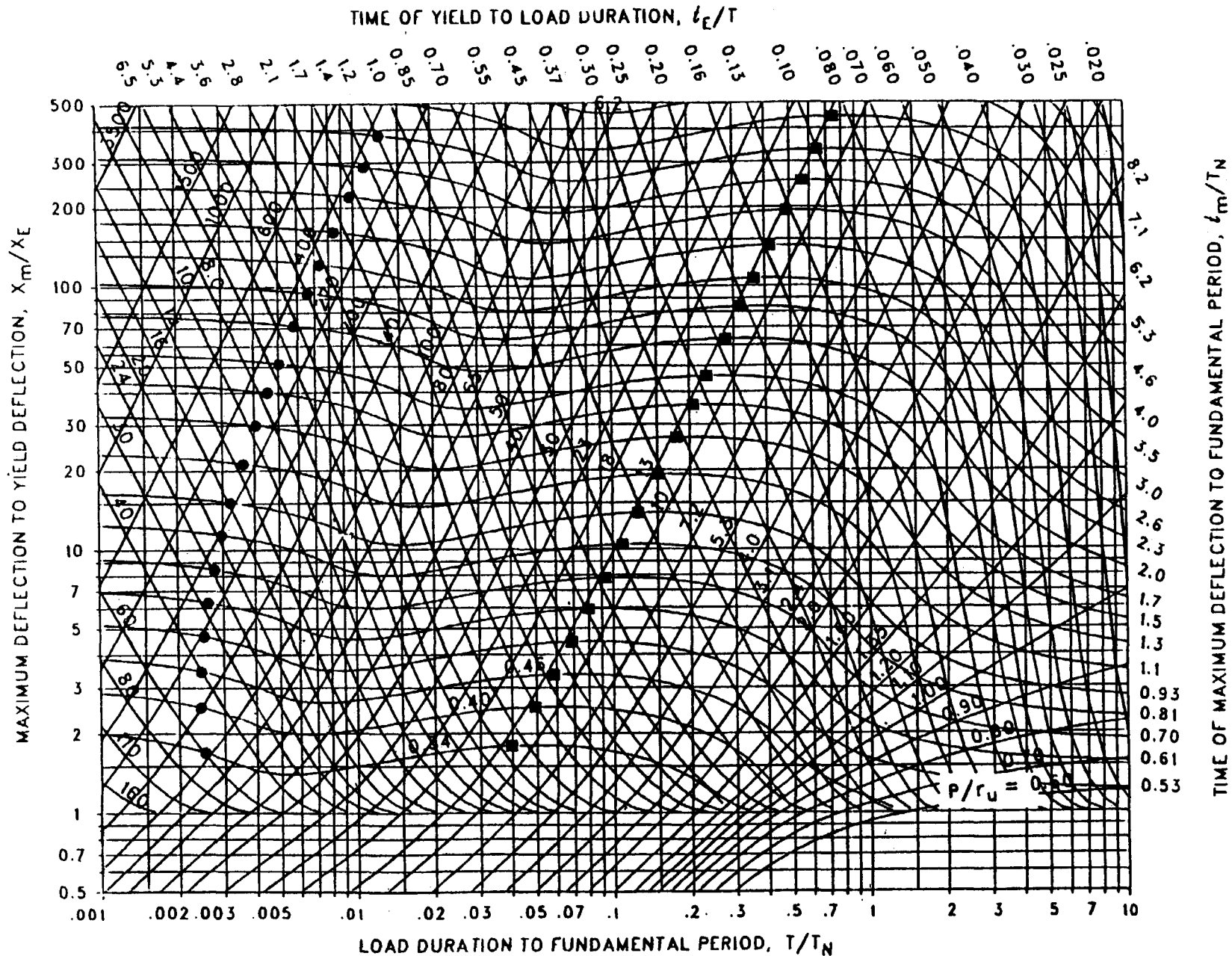


Figure 3-172 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.010$ ,  $C_2 = 100.$ )

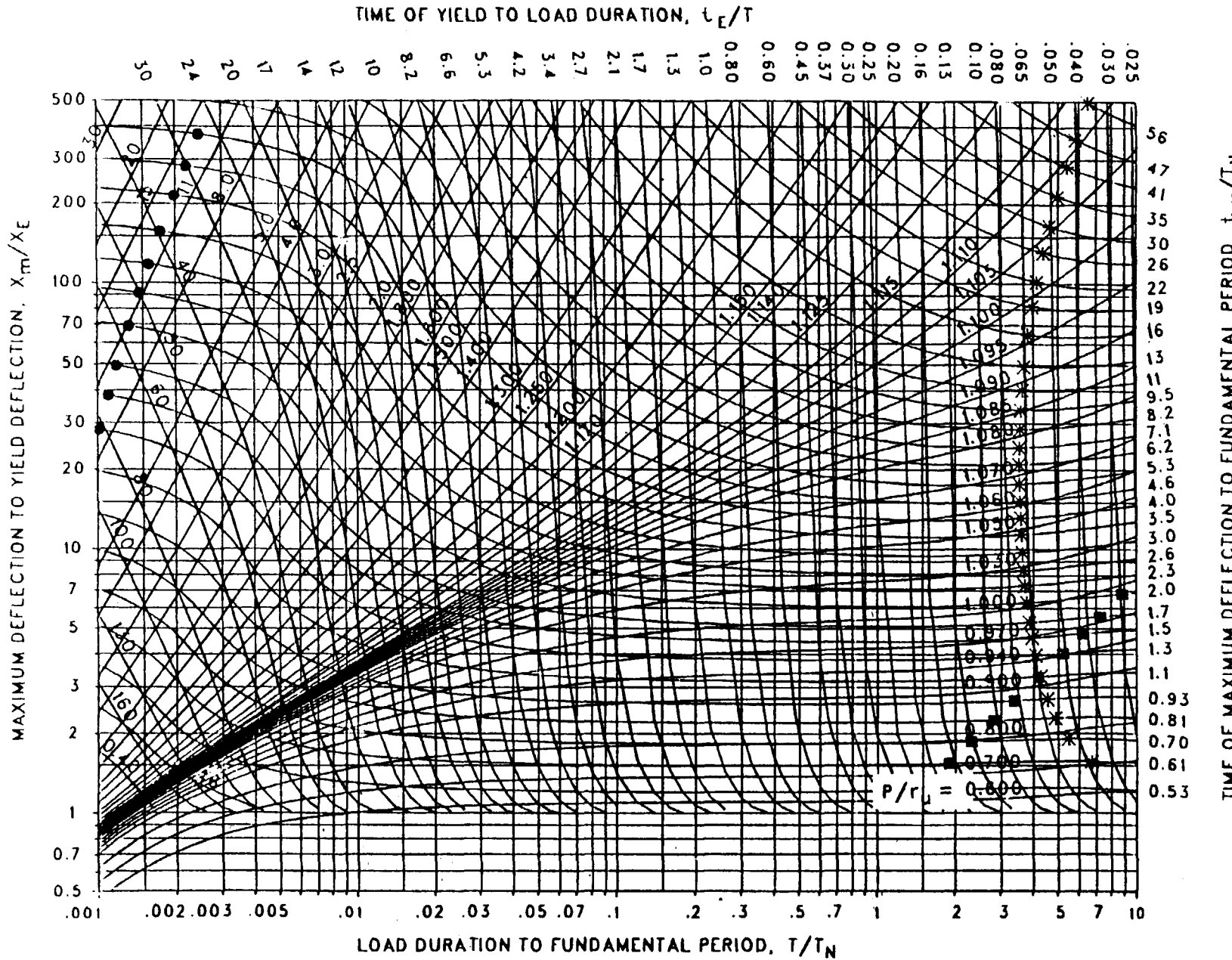


Figure 3-173 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.909$ ,  $C_2 = 300$ .)

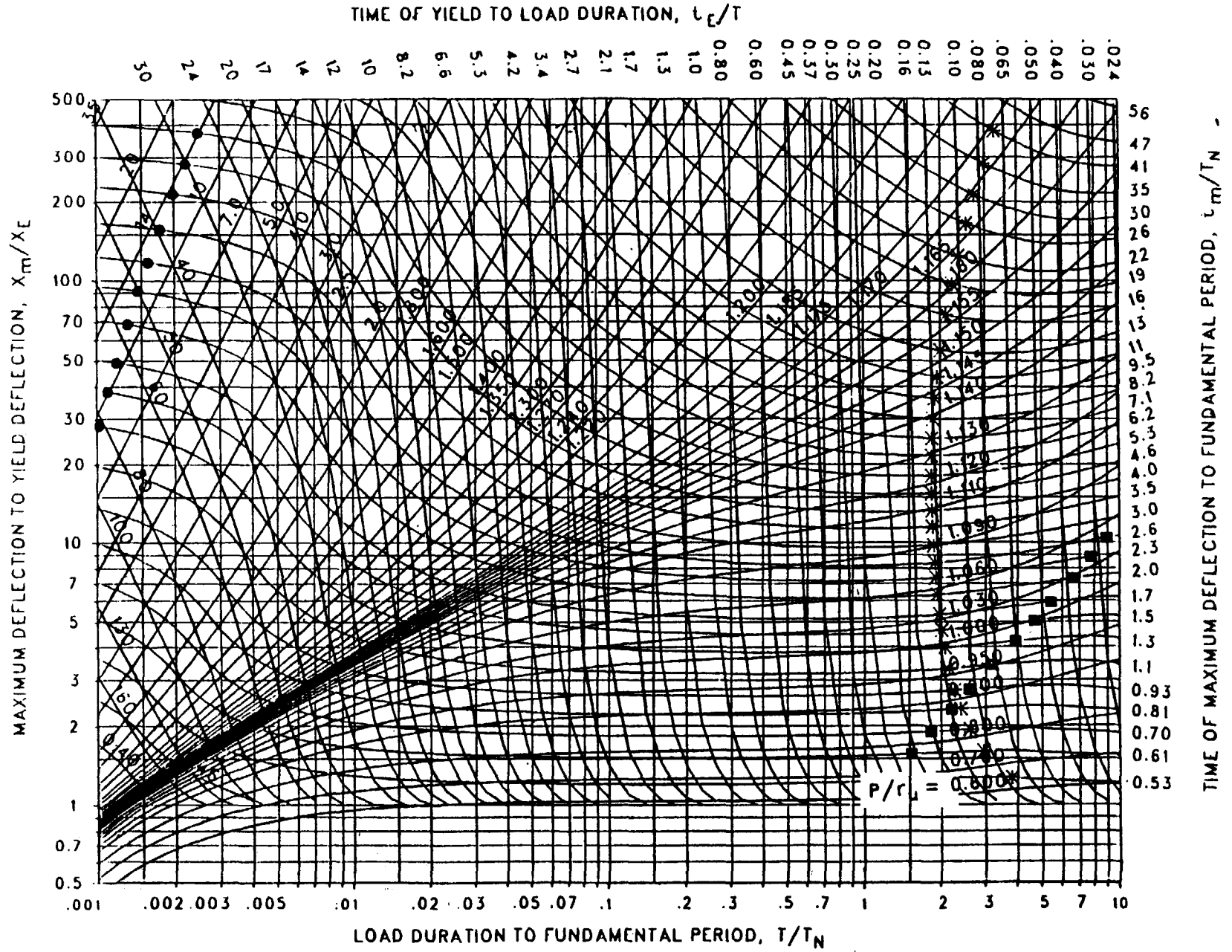


Figure 3-174 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.866$ ,  $C_2 = 300$ .)

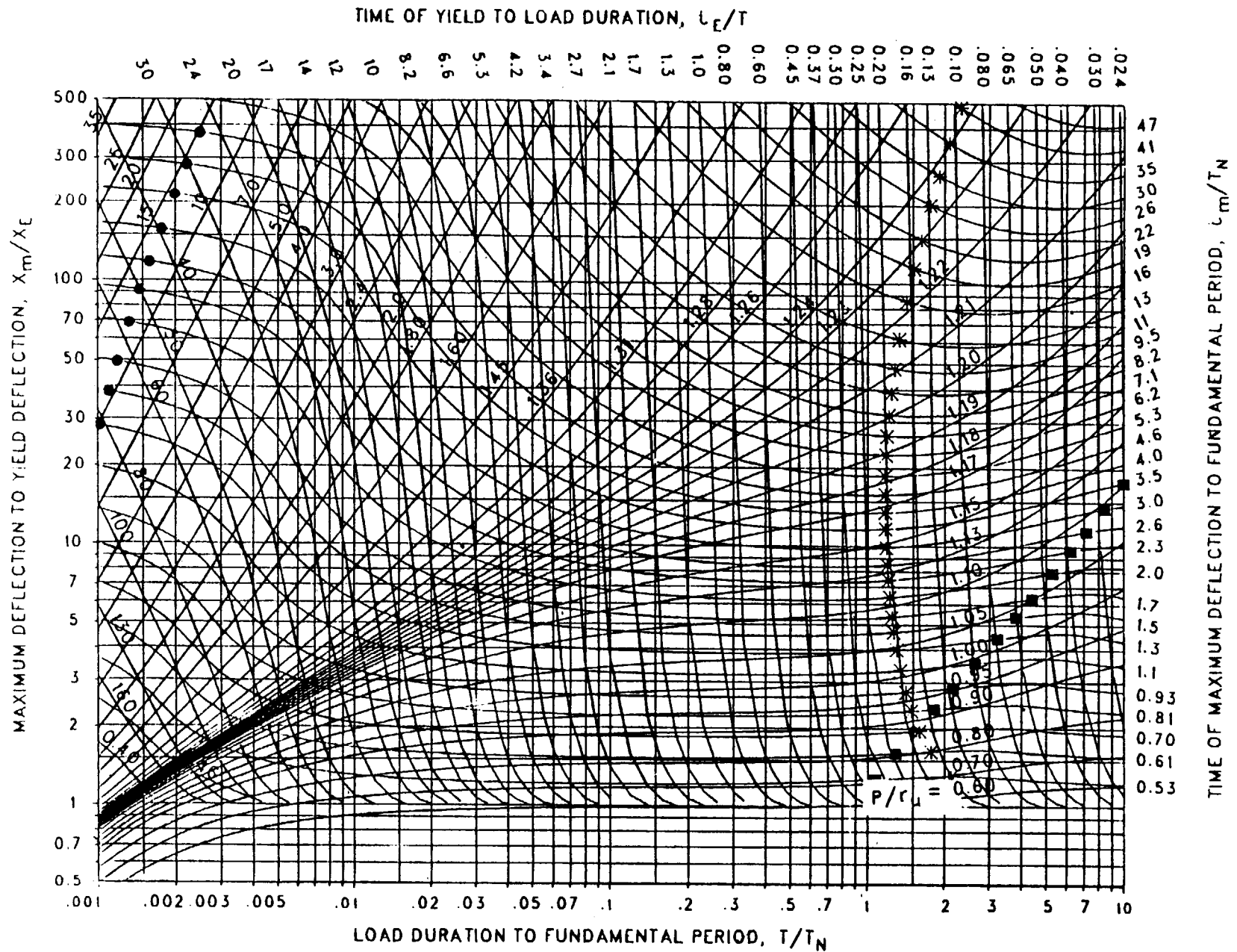


Figure 3-175 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.825$ ,  $C_2 = 300$ .)



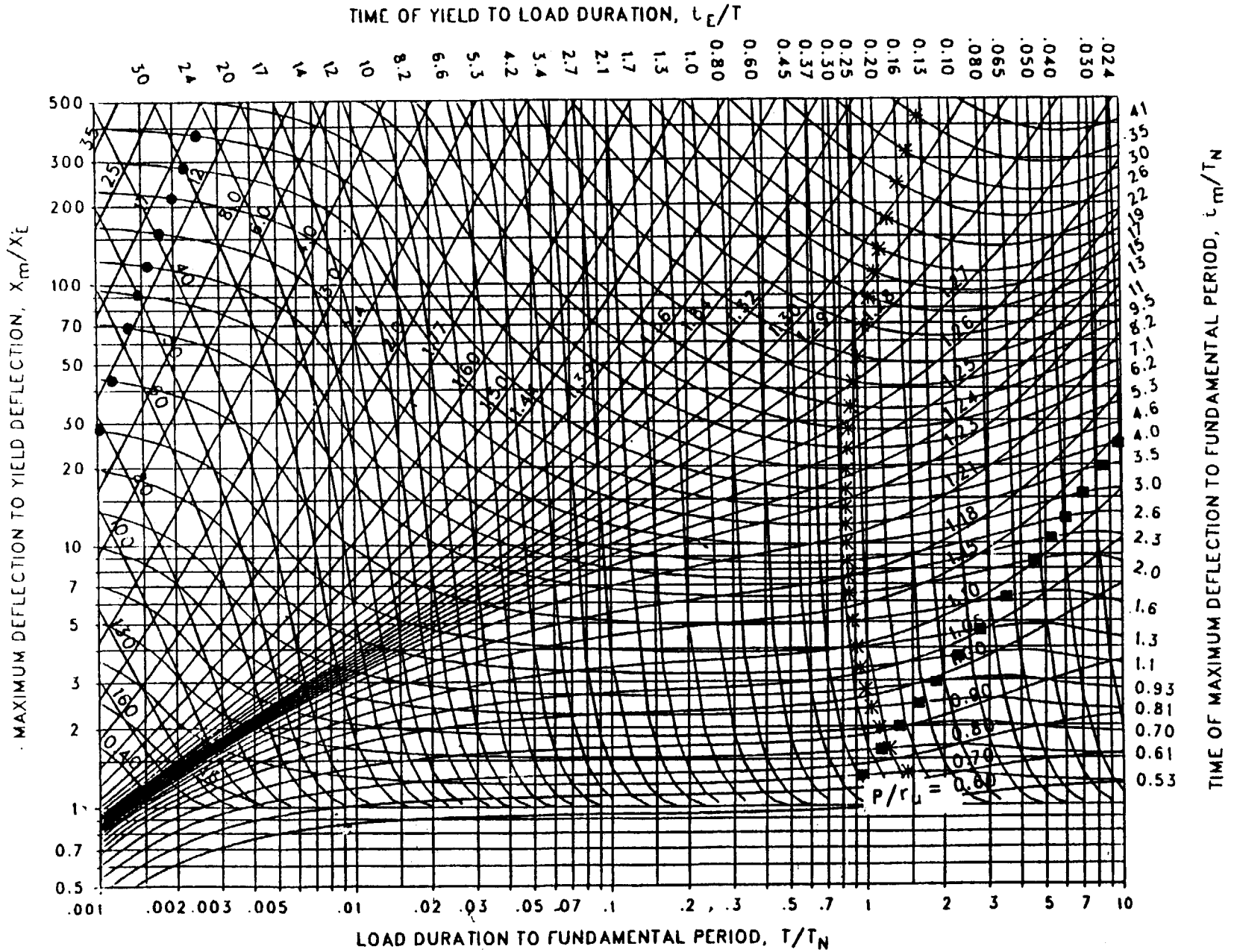


Figure 3-176 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.787$ ,  $C_2 = 300$ .)

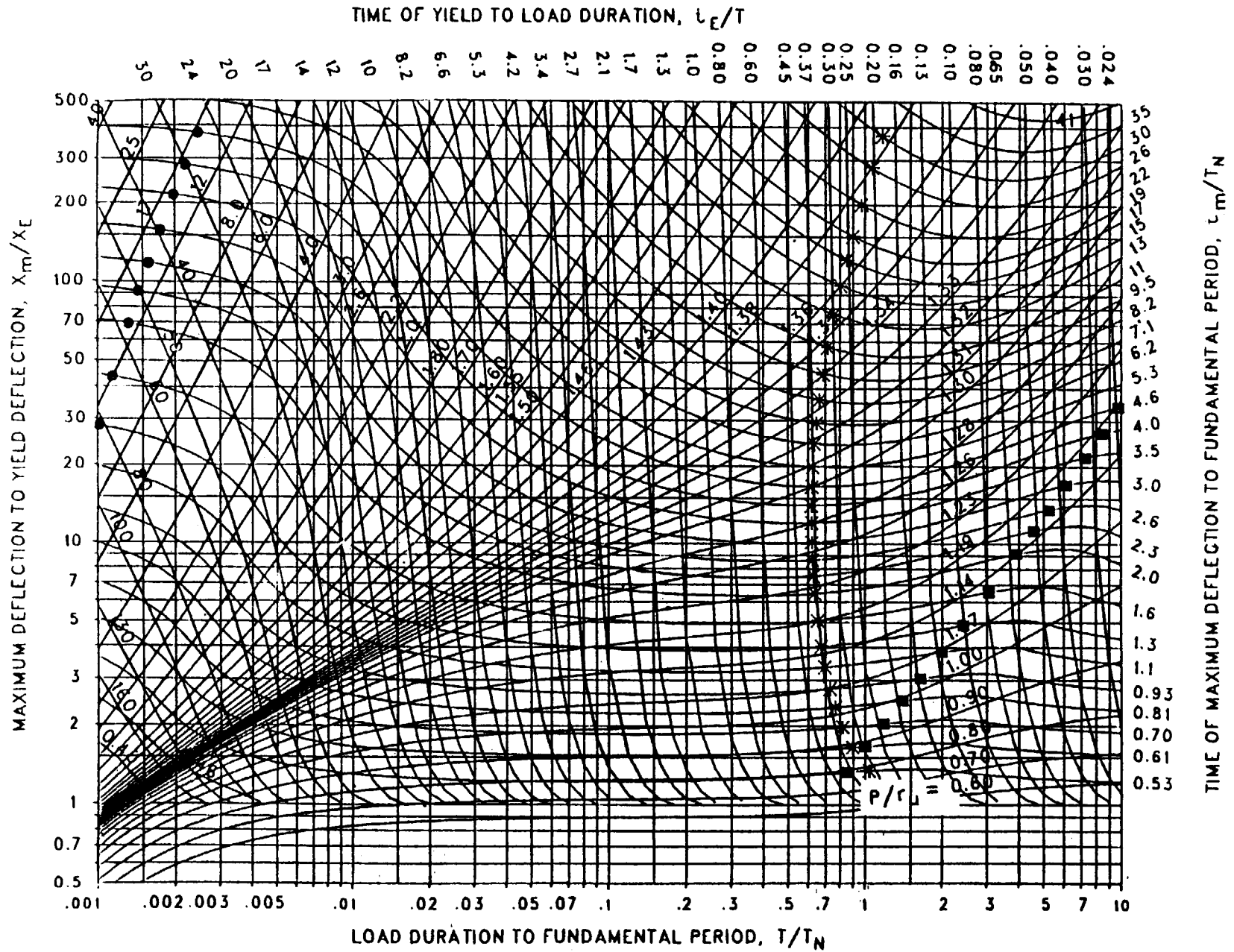


Figure 3-177 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.750$ ,  $C_2 = 300.$ )

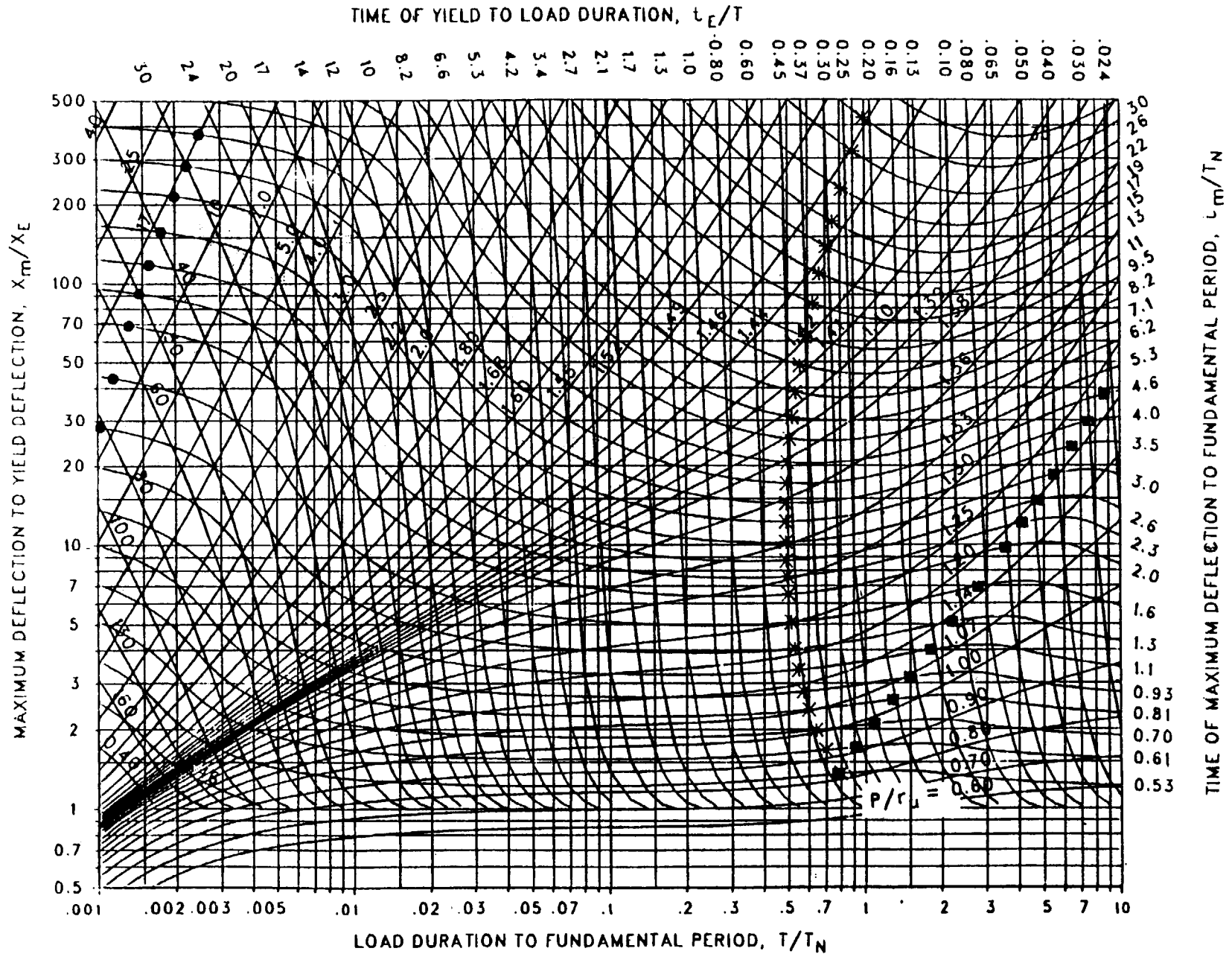


Figure 3-178 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.715$ ,  $C_2 = 300$ .)

3-237

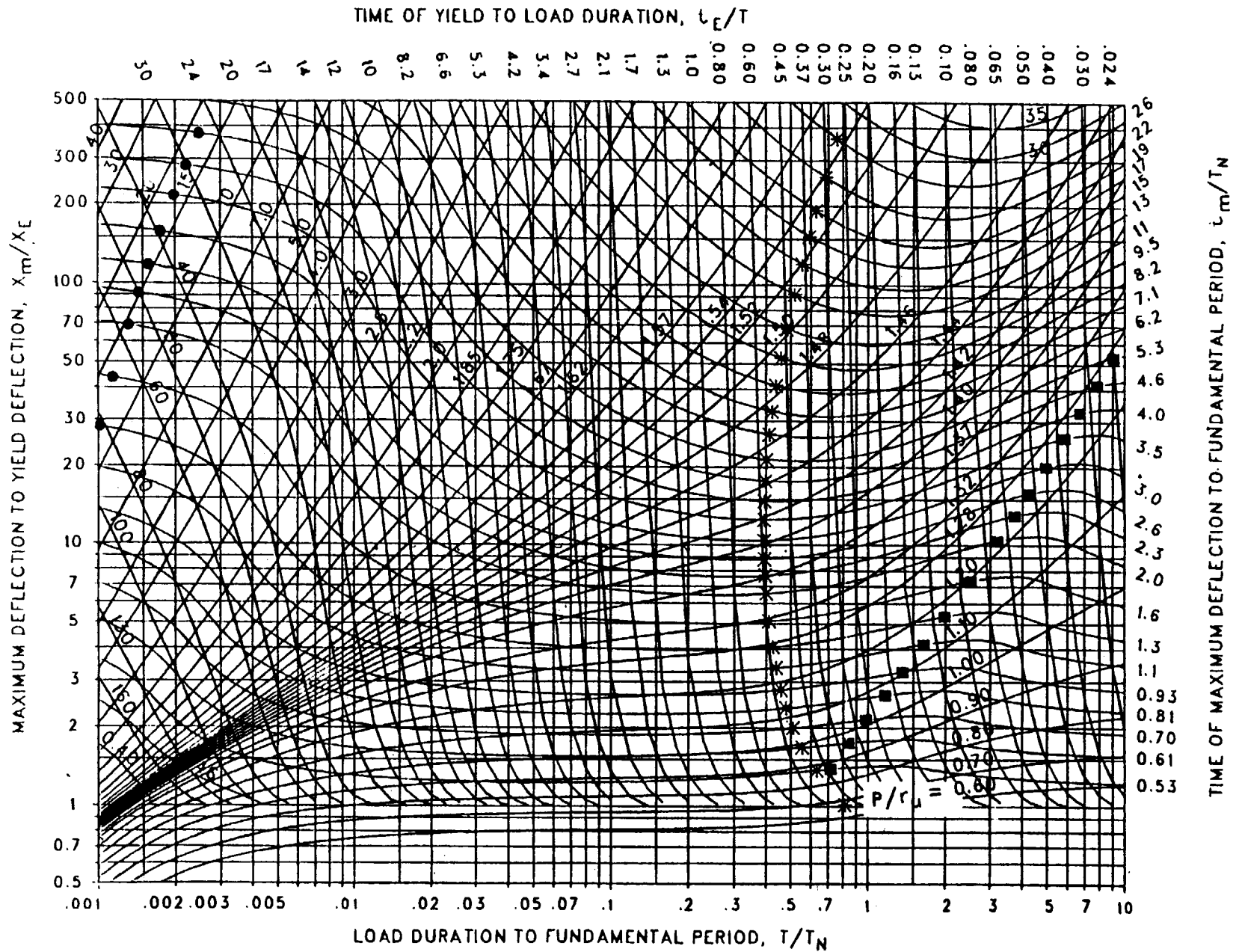


Figure 3-179 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.681$ ,  $C_2 = 300$ .)

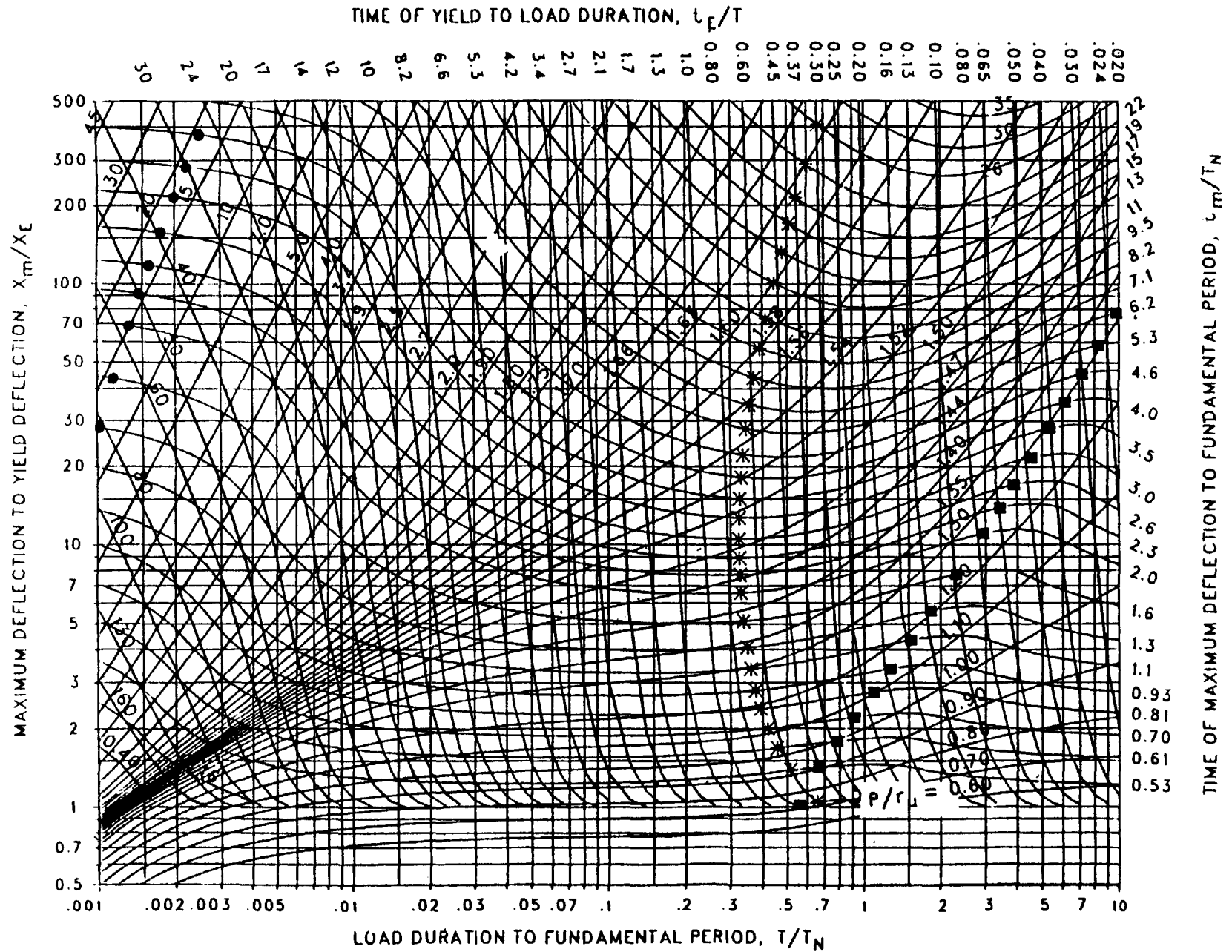


Figure 3-180 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.648$ ,  $C_2 = 300$ .)

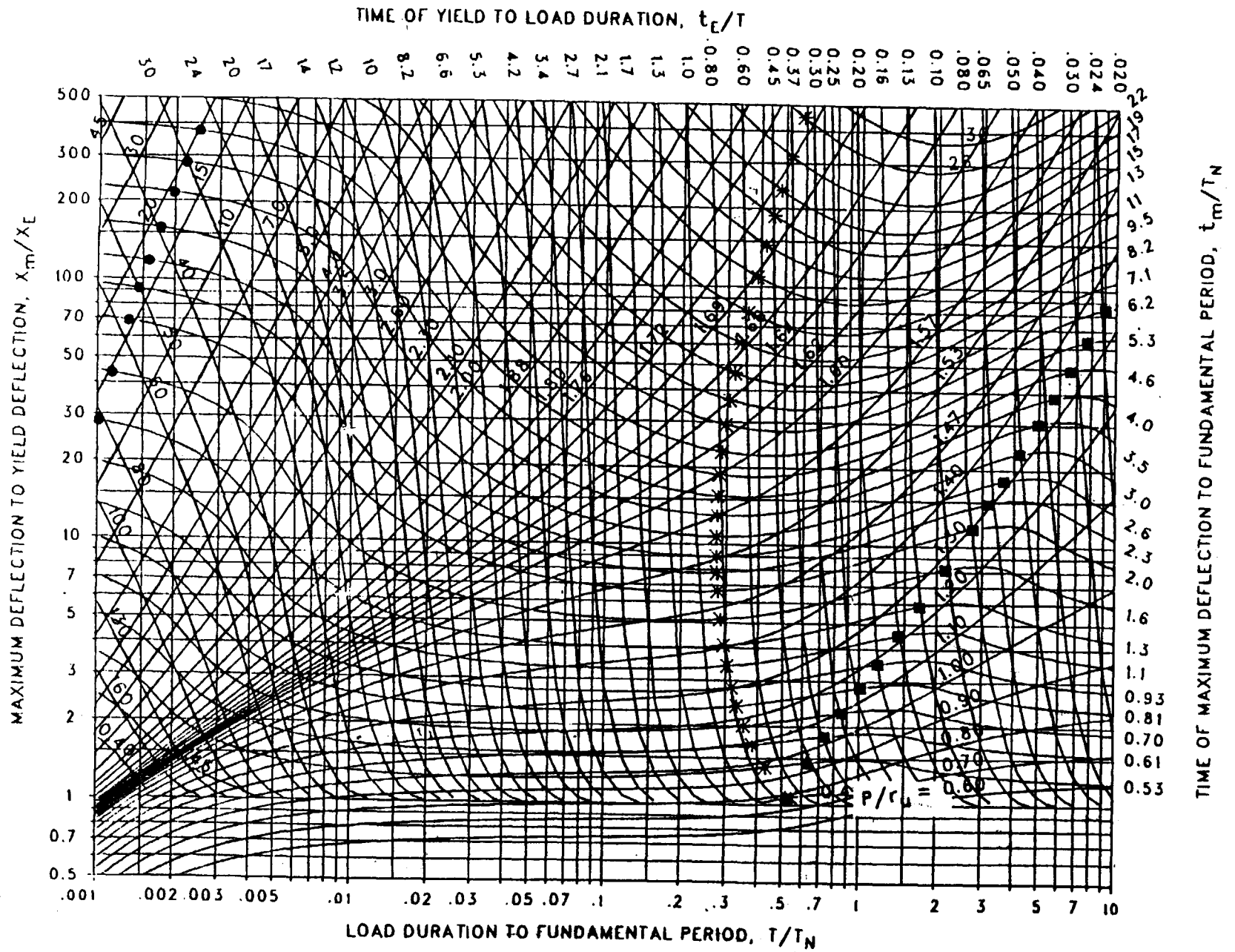


Figure 3-181 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.619$ ,  $C_2 = 300$ .)

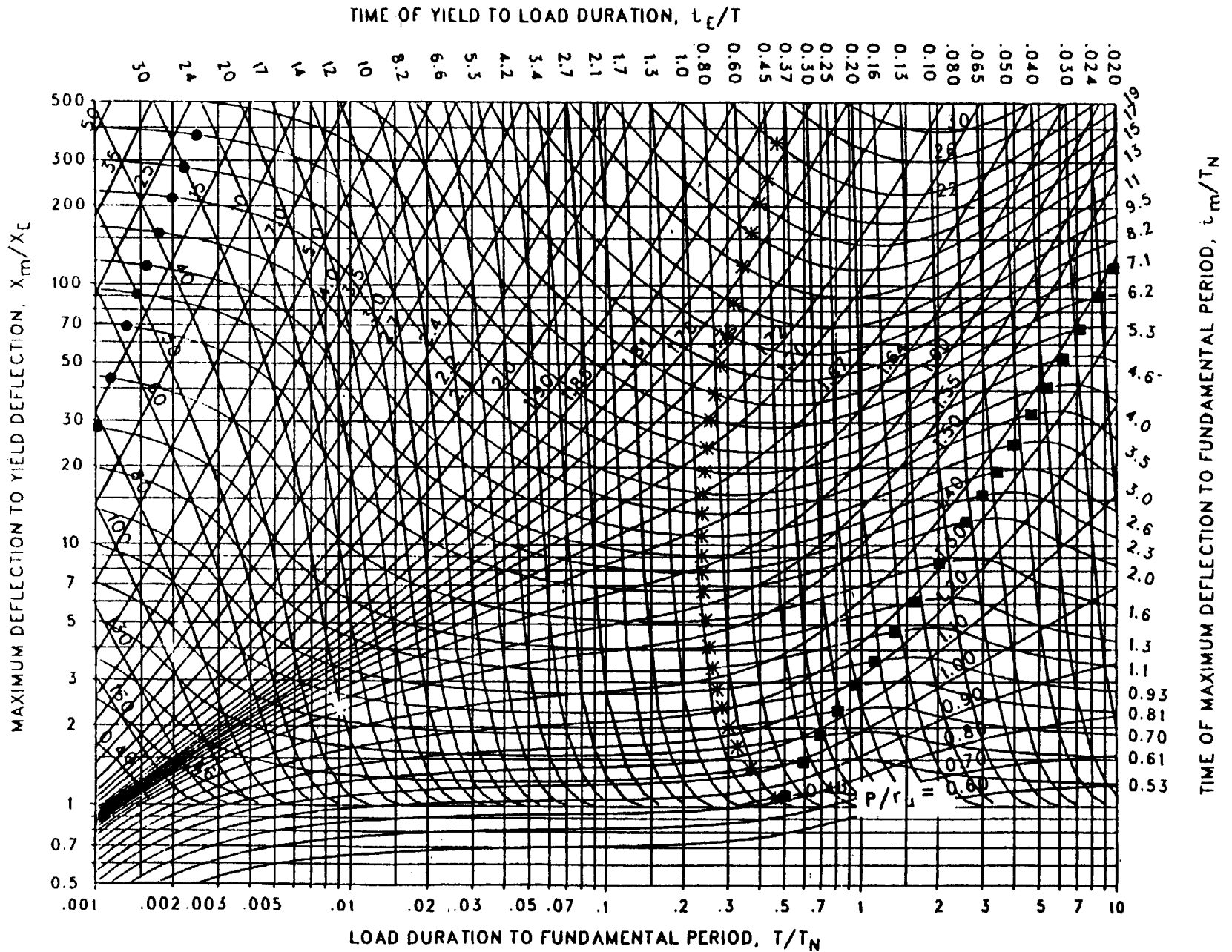


Figure 3-182 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.590$ ,  $C_2 = 300$ .)

3-241

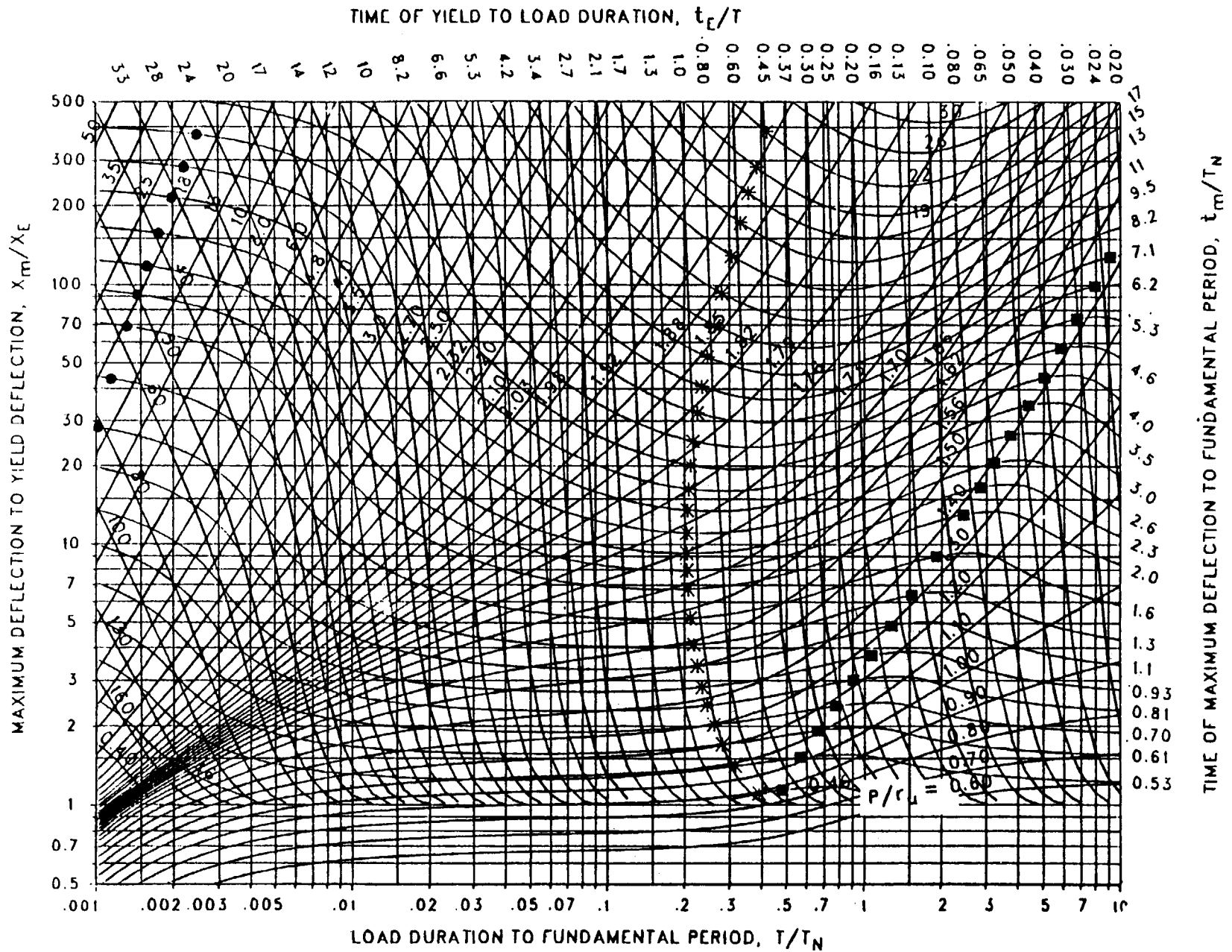


Figure 3-183 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.562$ ,  $C_2 = 300$ .)



3-242

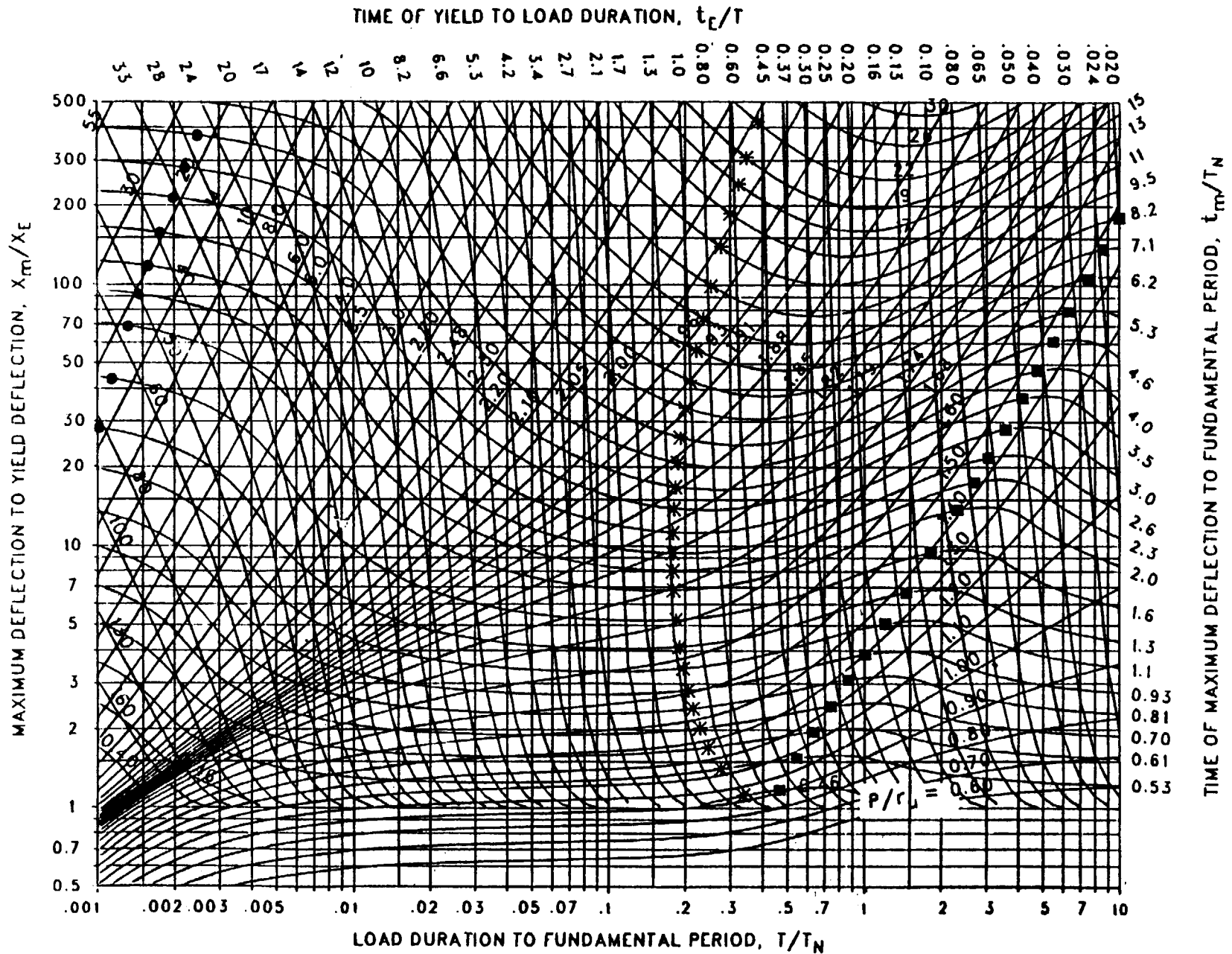


Figure 3-184 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.536$ ,  $C_2 = 300$ .)

3-243

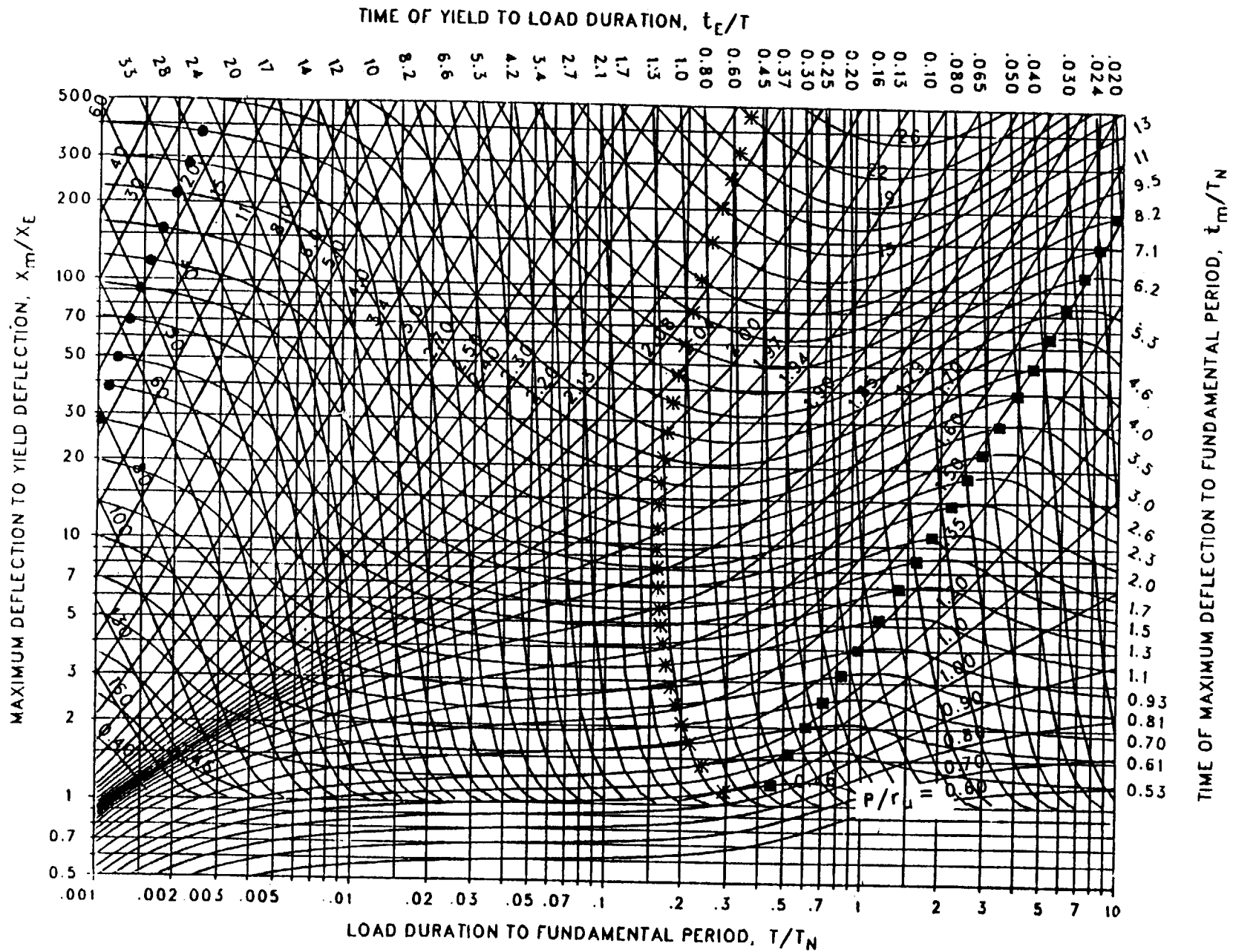


Figure 3-185 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.511$ ,  $C_2 = 300$ .)

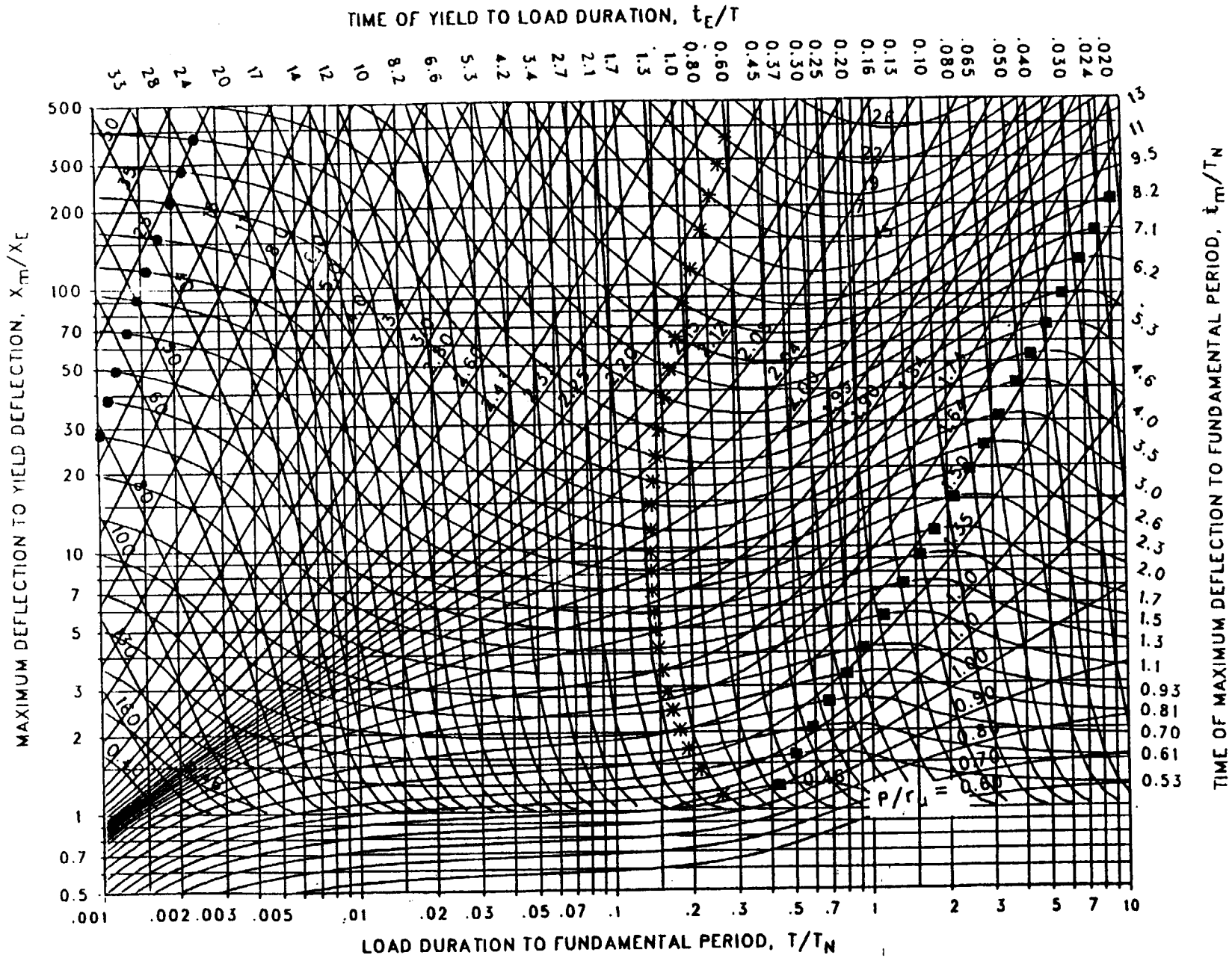


Figure 3-186 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.487$ ,  $C_2 = 300$ .)

3-245

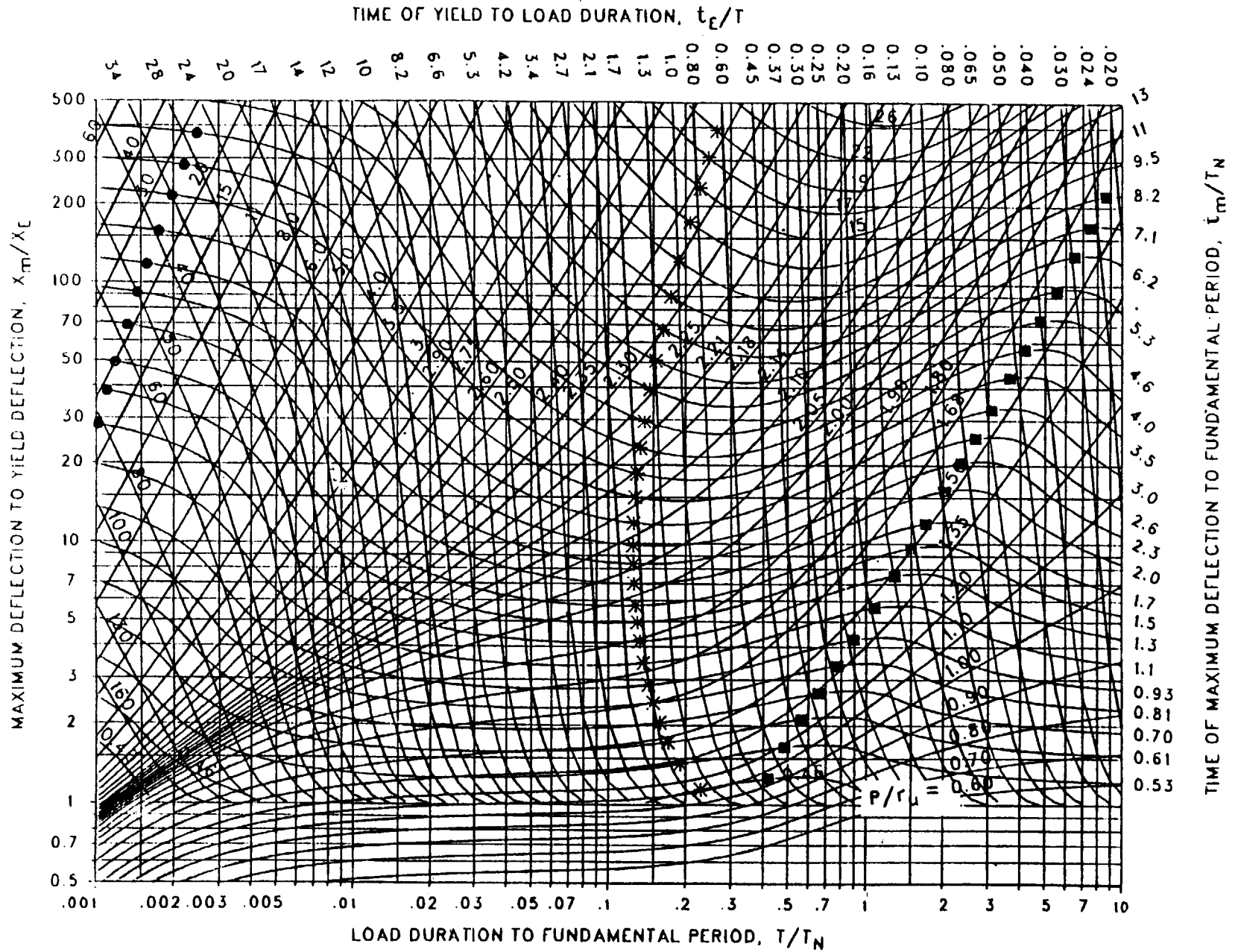


Figure 3-187 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.464$ ,  $C_2 = 300$ .)

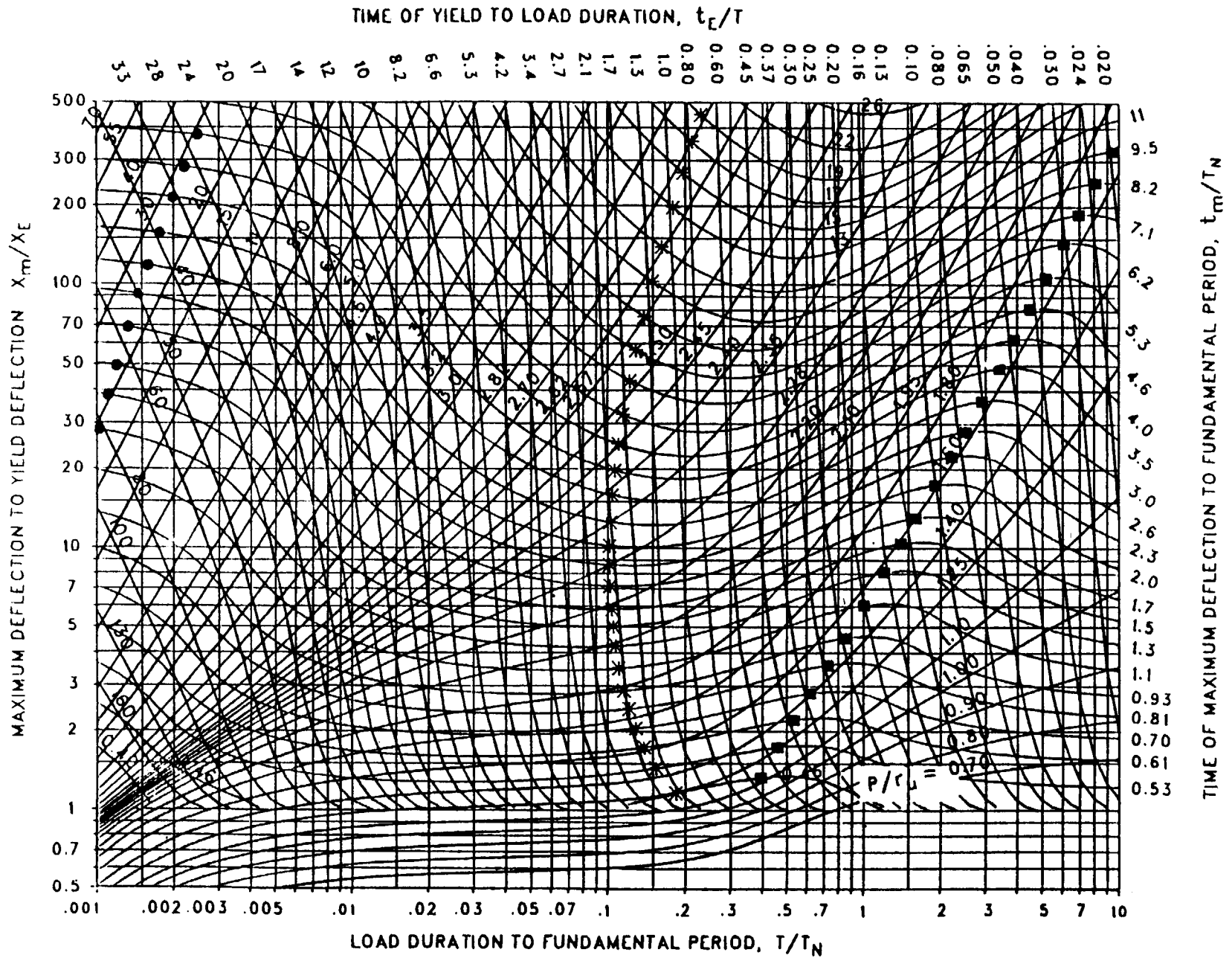


Figure 3-188 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.422$ ,  $C_2 = 300$ .)

3-247

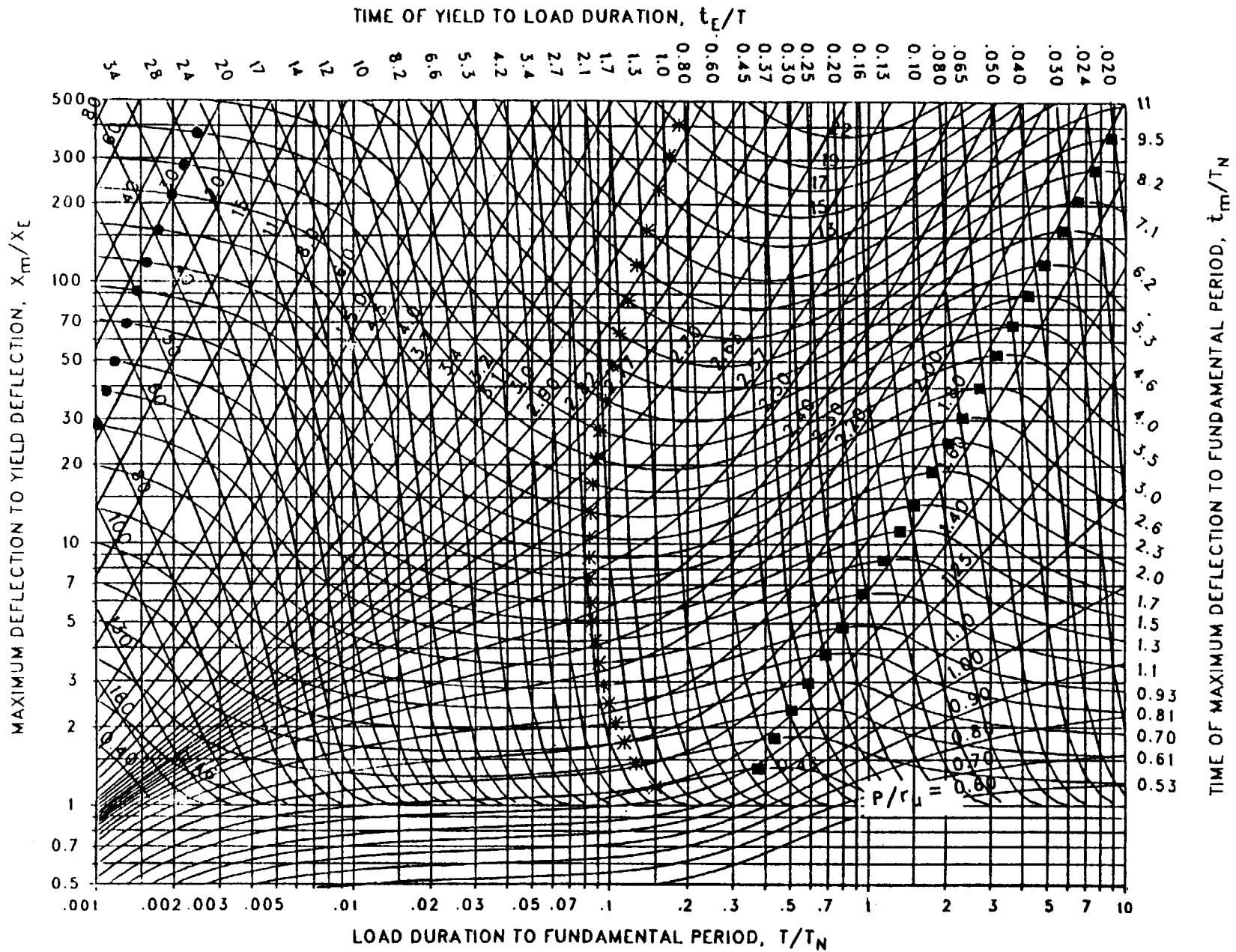


Figure 3-189 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.383$ ,  $C_2 = 300$ .)

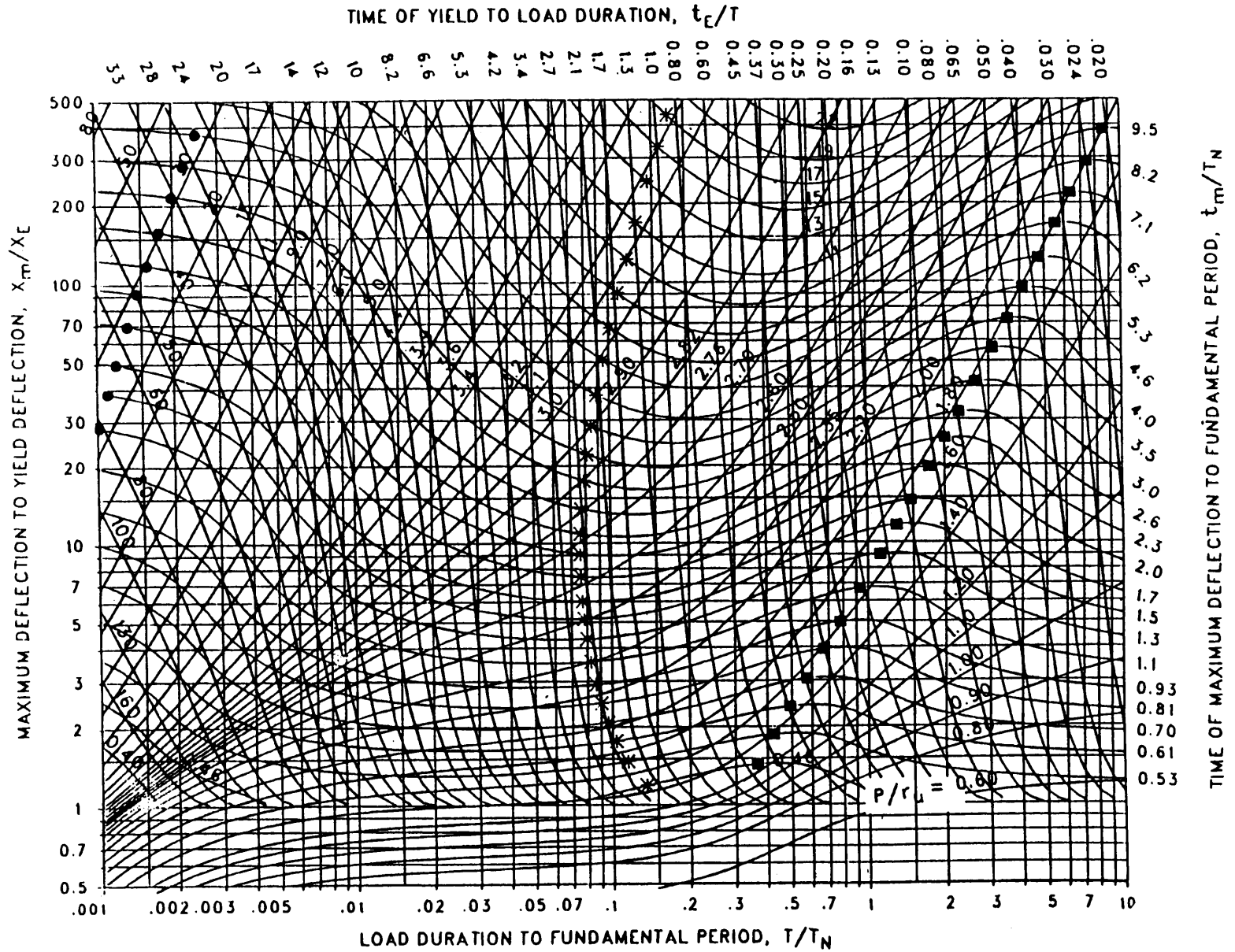


Figure 3-190 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.365$ ,  $C_2 = 300$ .)

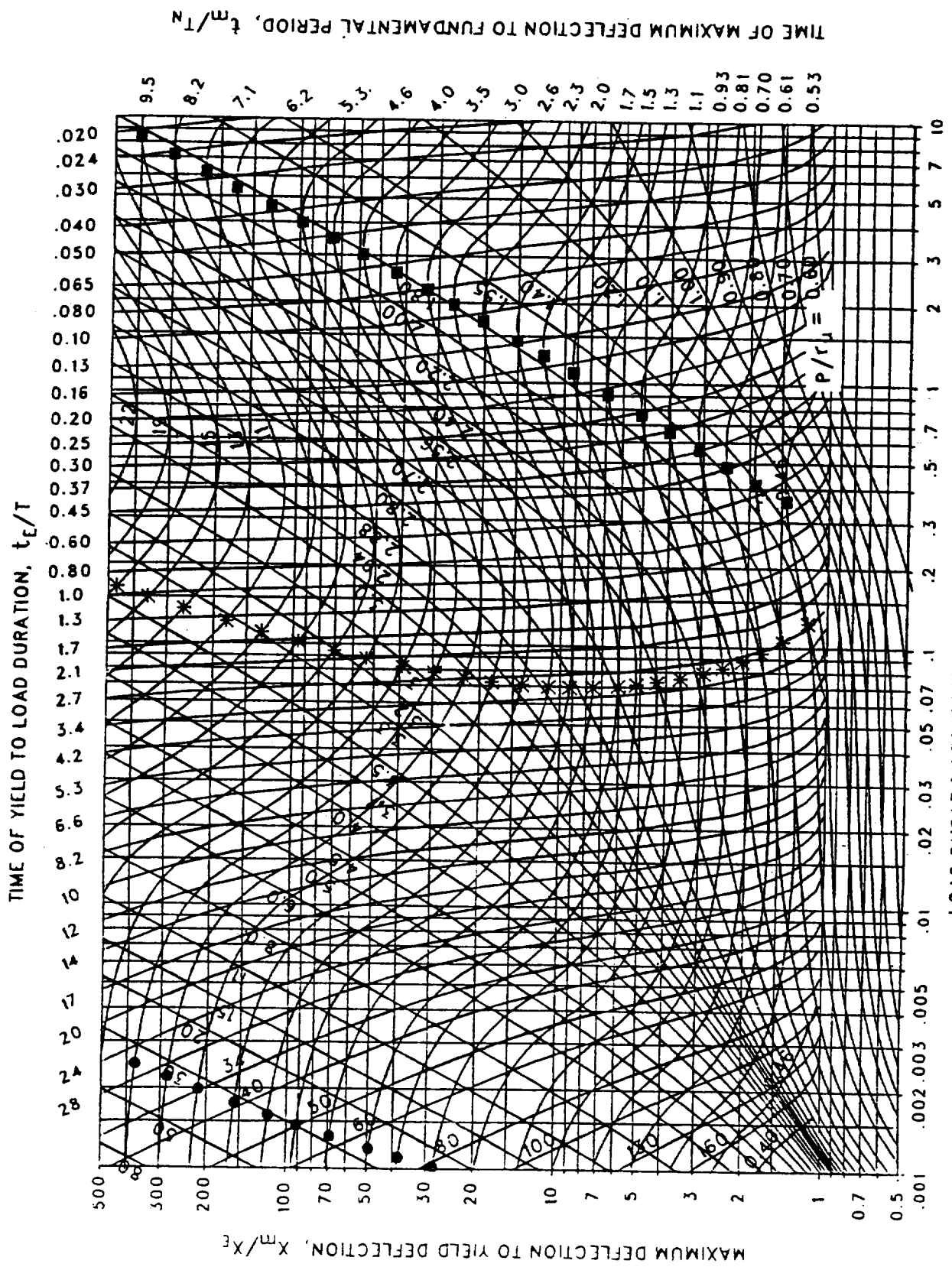


Figure 3-191 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.348$ ,  $C_2 = 300$ .)



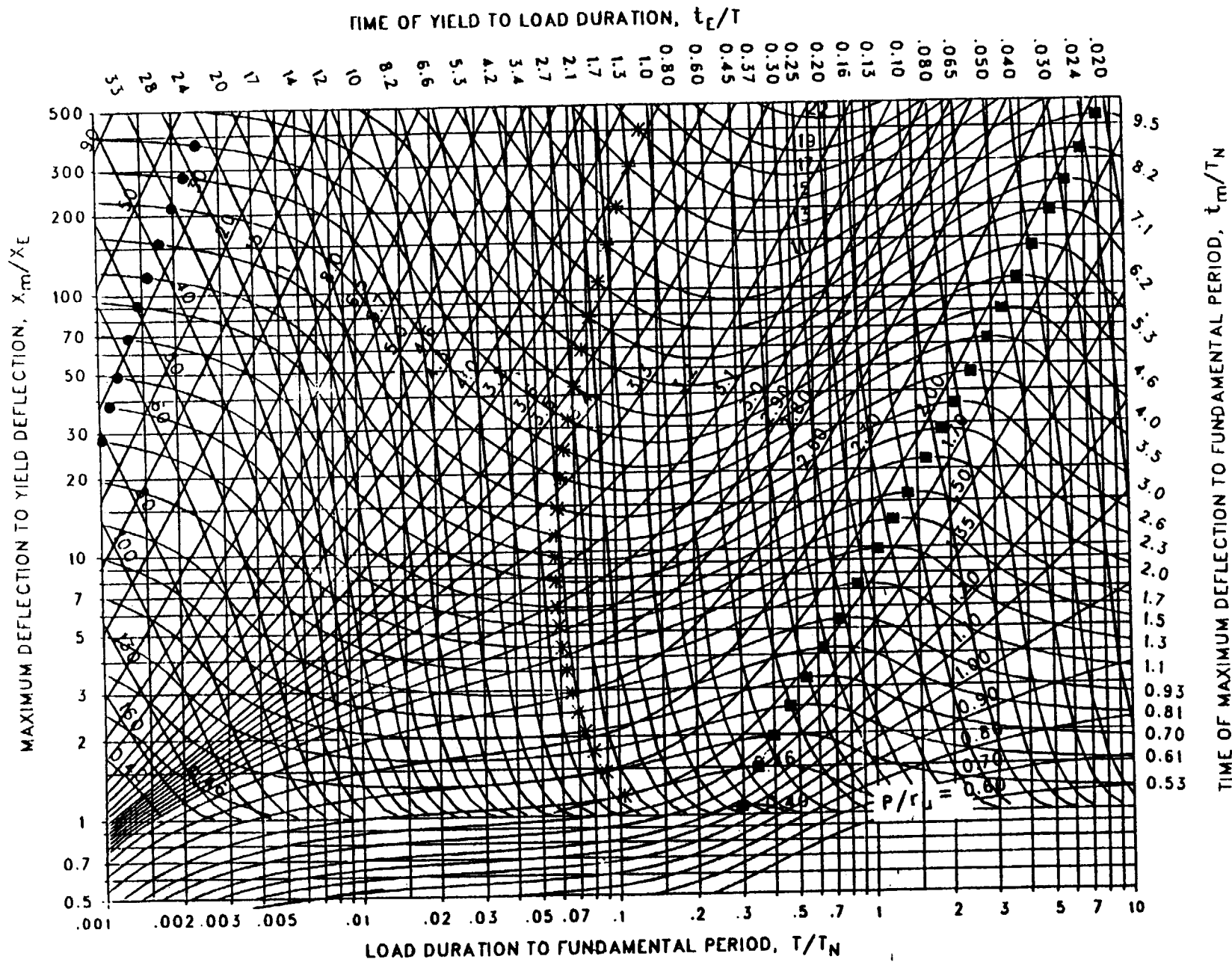


Figure 3-192 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.316$ ,  $C_2 = 300$ .)

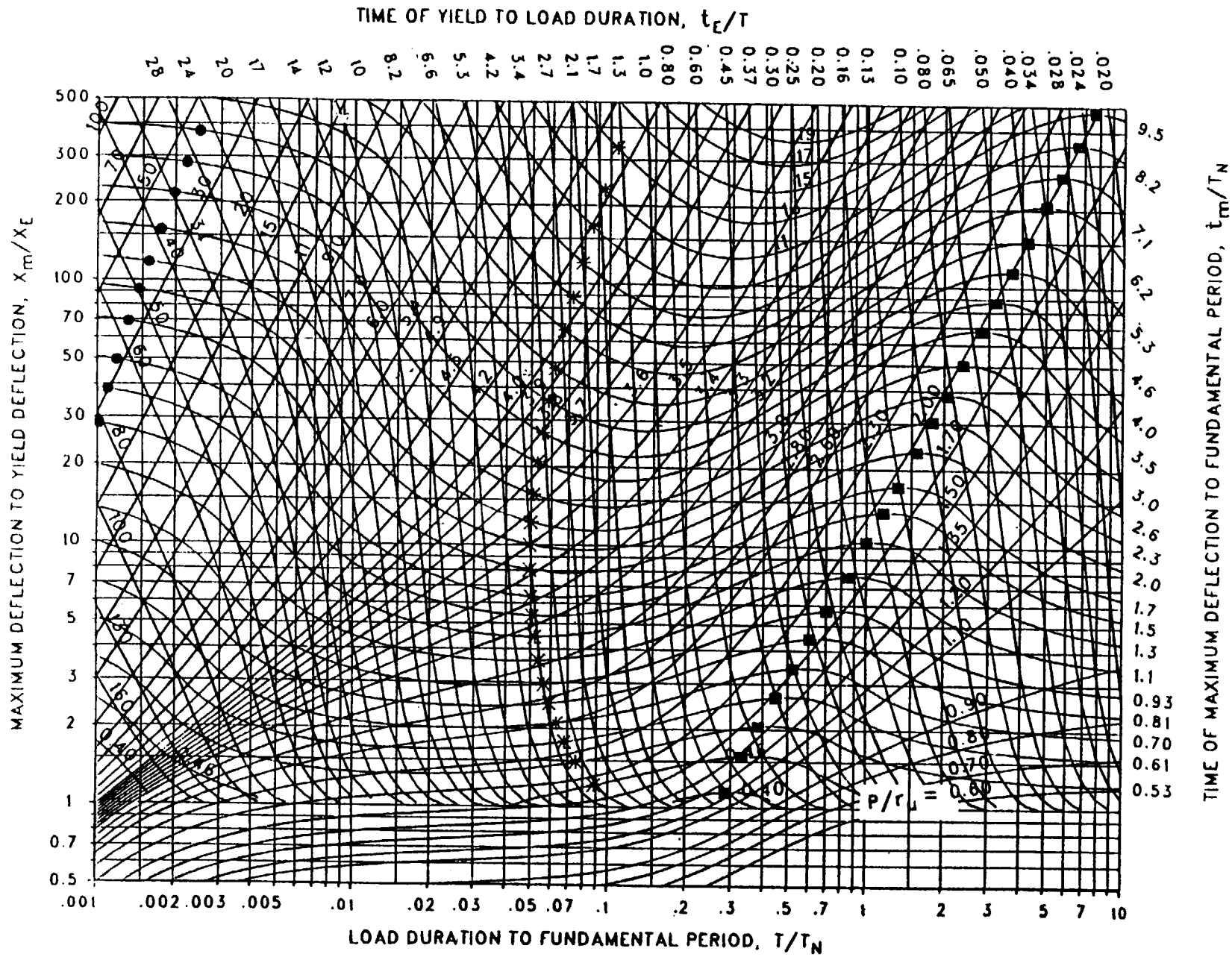


Figure 3-193 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.287$ ,  $C_2 = 300$ .)

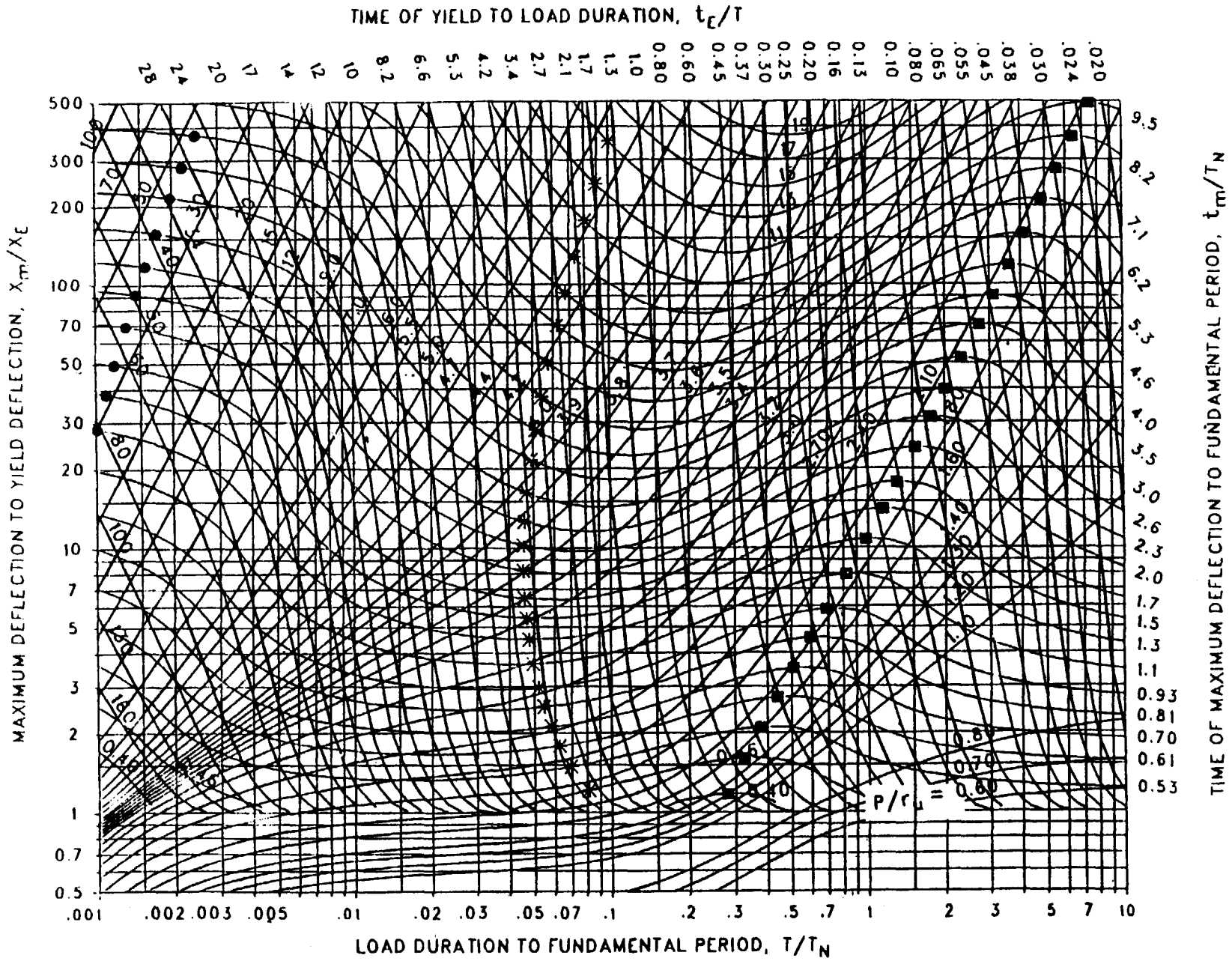


Figure 3-194 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.274$ ,  $C_2 = 300$ .)

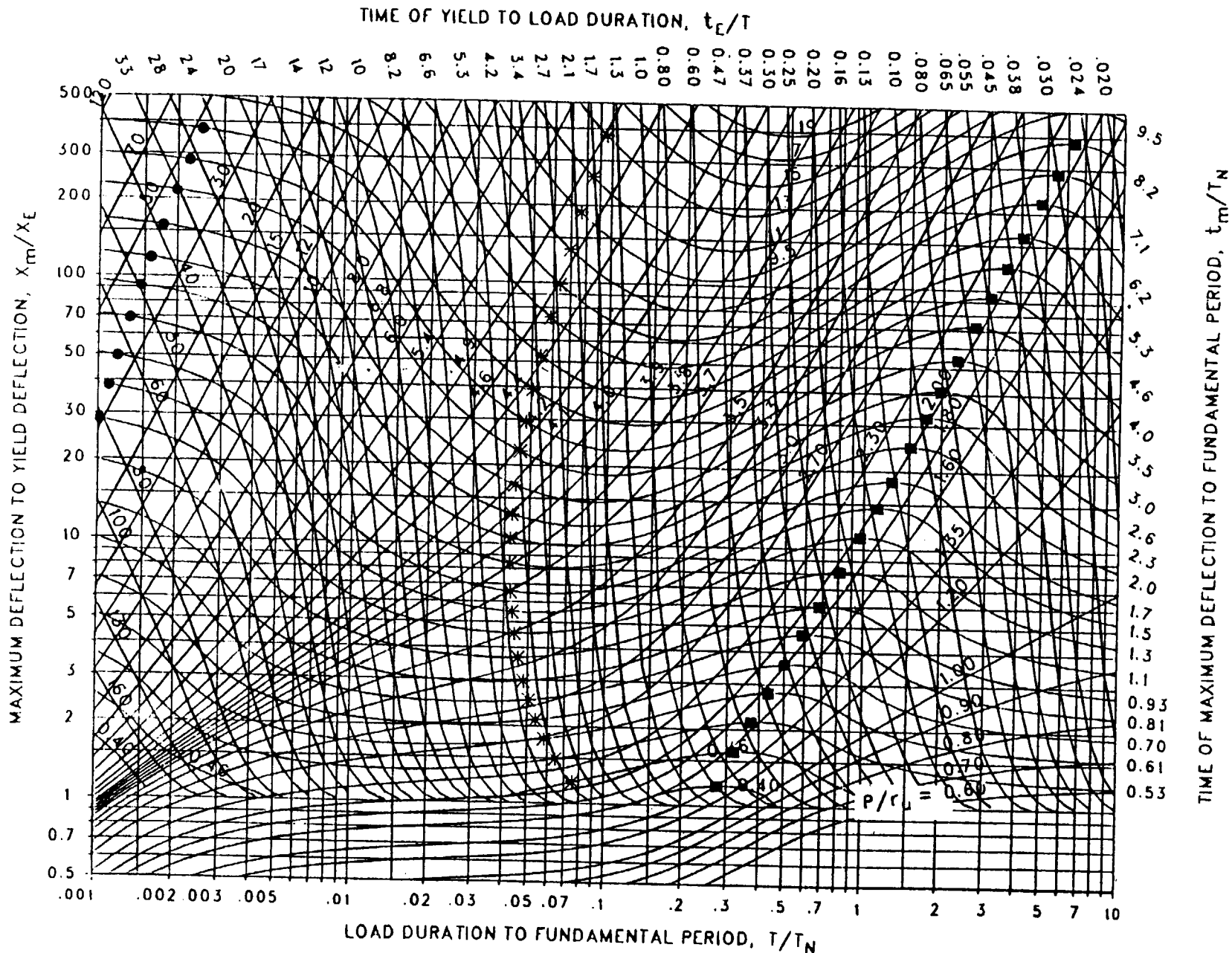


Figure 3-195 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.261$ ,  $C_2 = 300$ .)

3-254

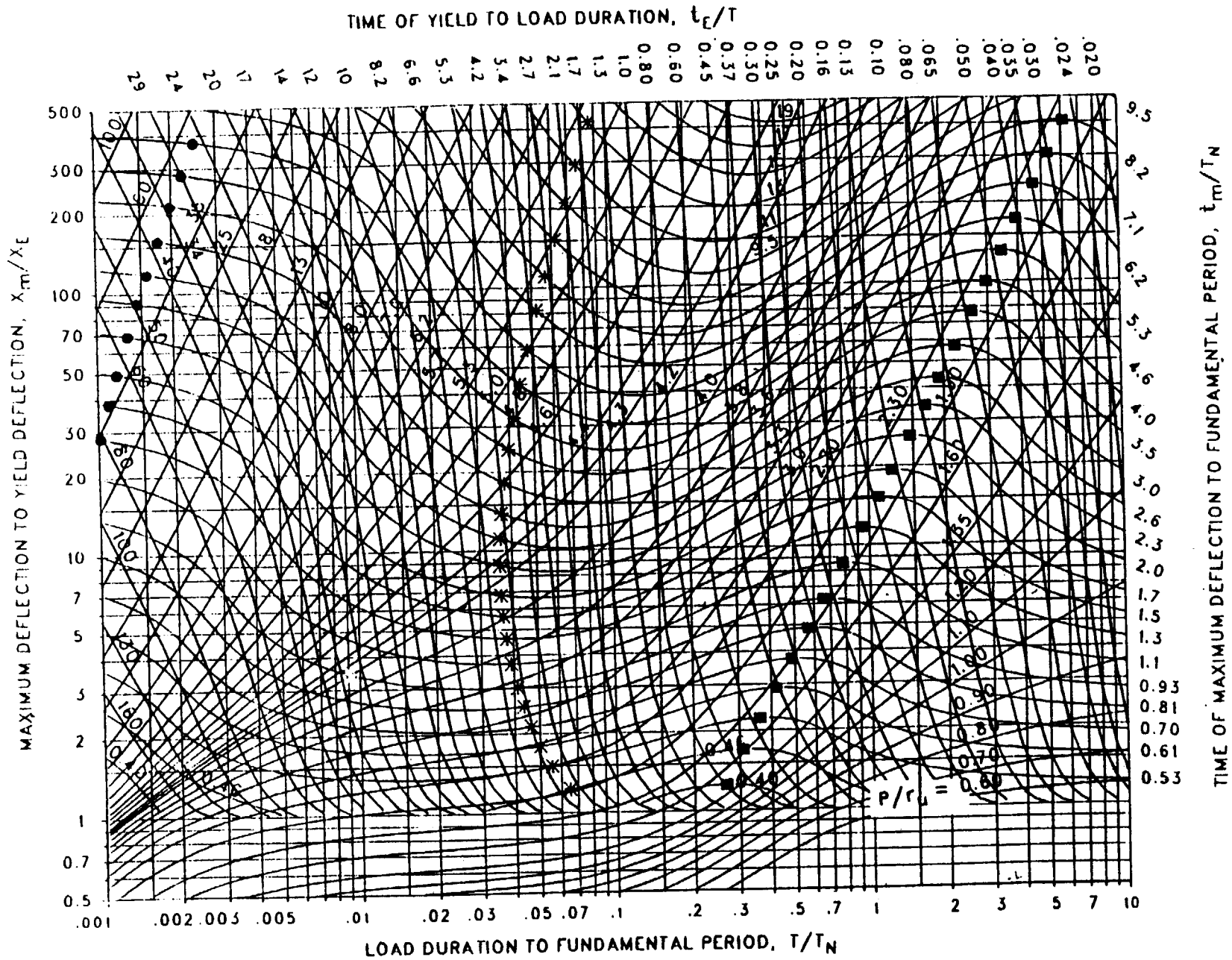


Figure 3-196 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.237$ ,  $C_2 = 300$ .)

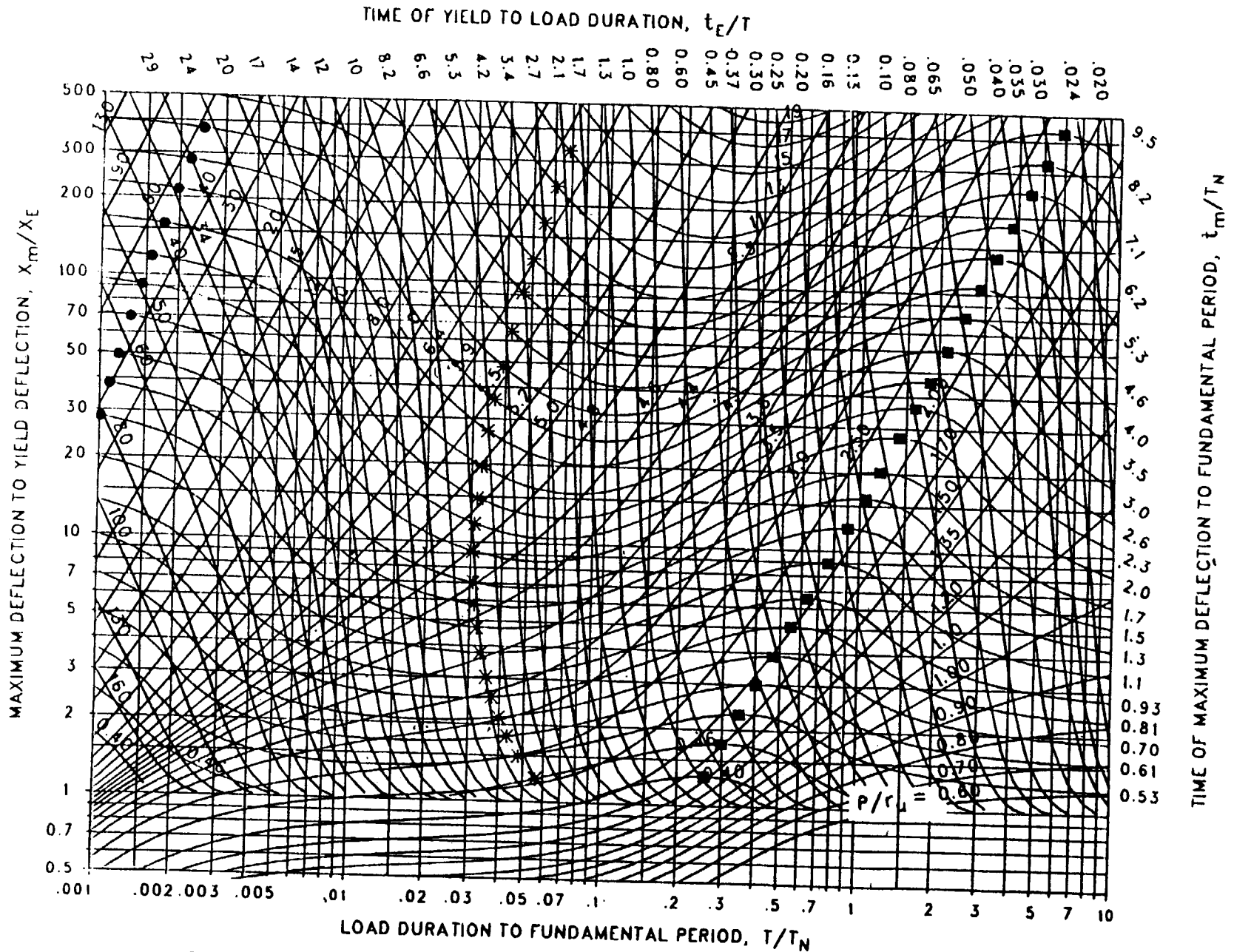


Figure 3-197 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.215$ ,  $C_2 = 300.$ )

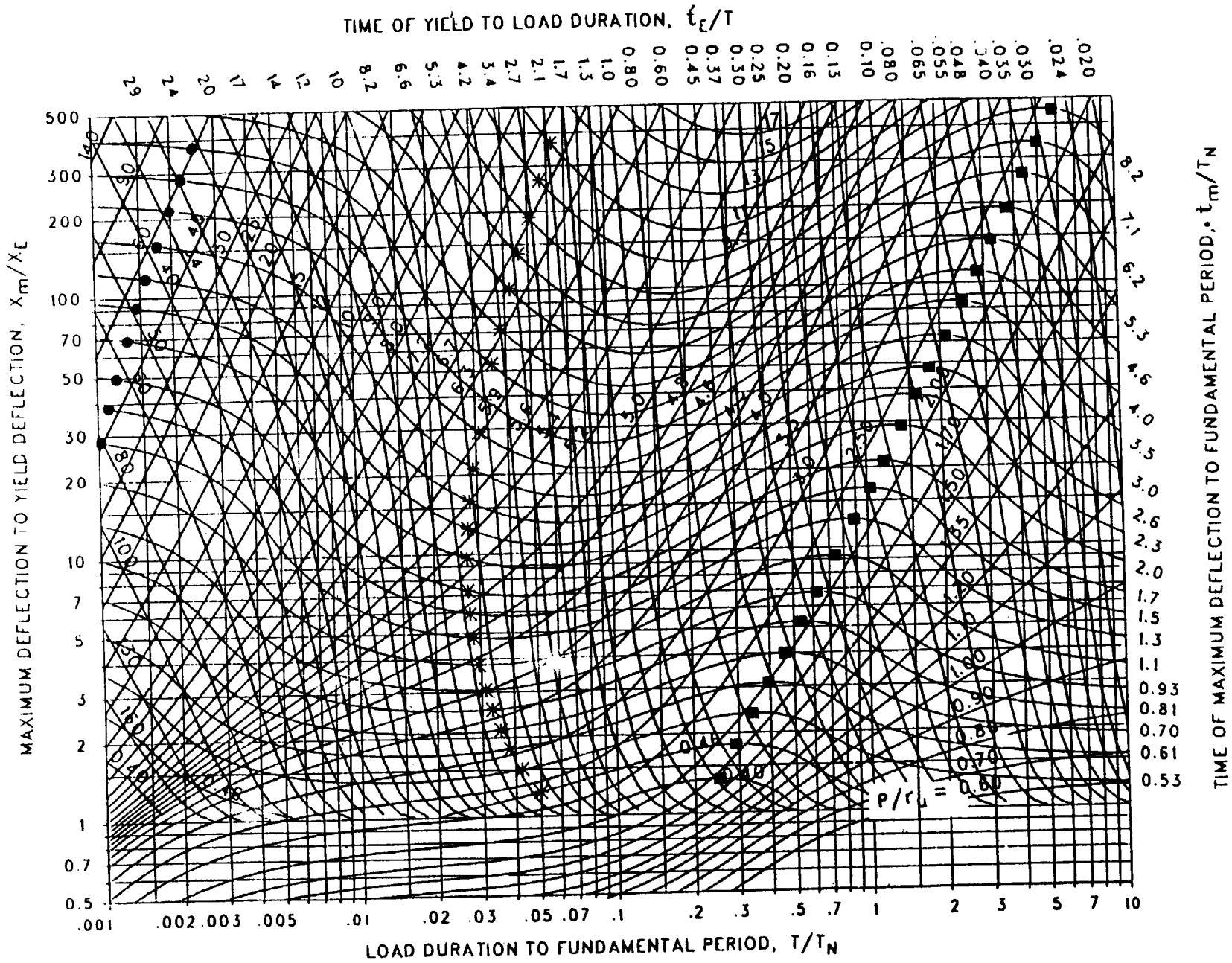


Figure 3-198 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.198$ ,  $C_2 = 300$ .)

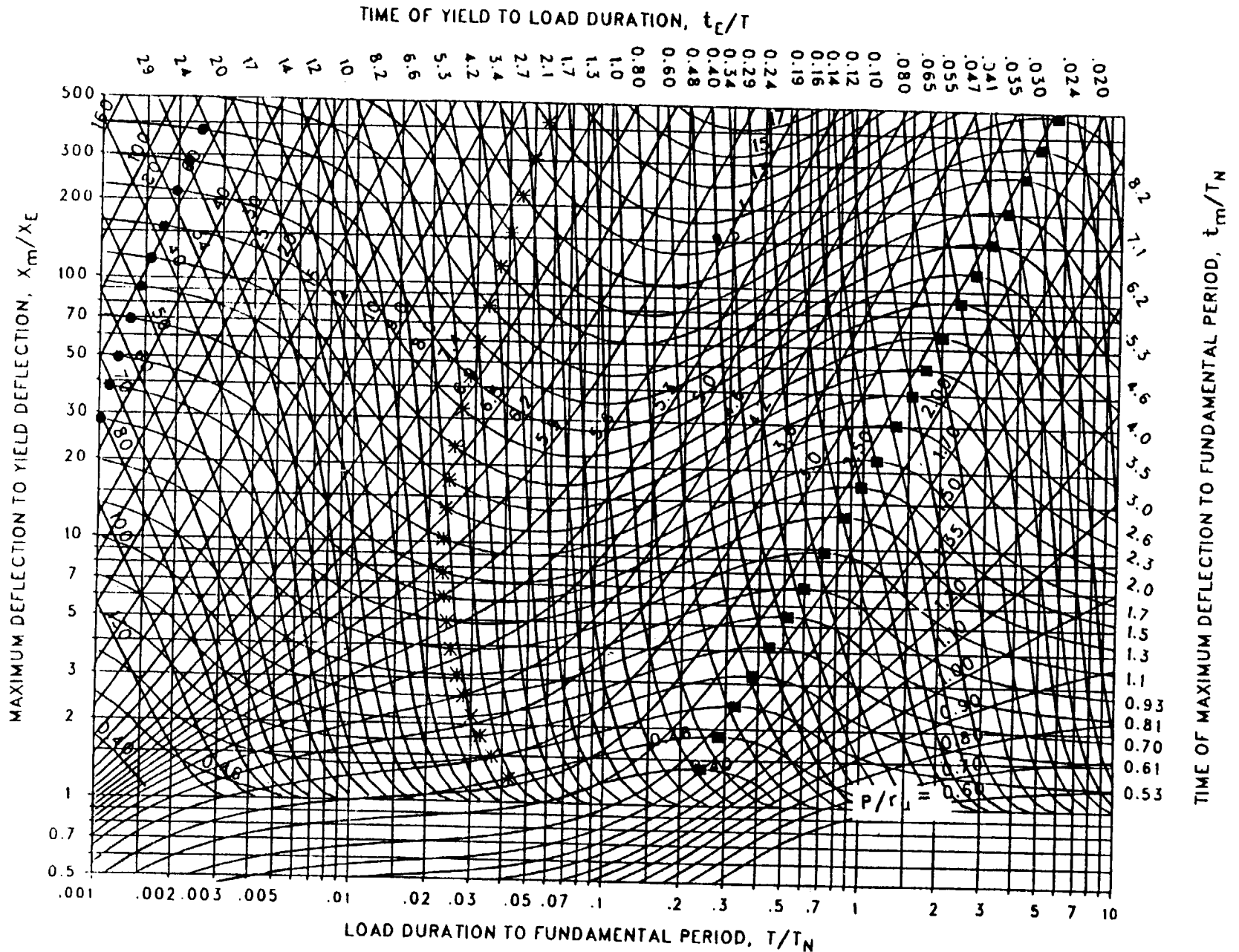


Figure 3-199 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.178$ ,  $C_2 = 300$ .)



3-258

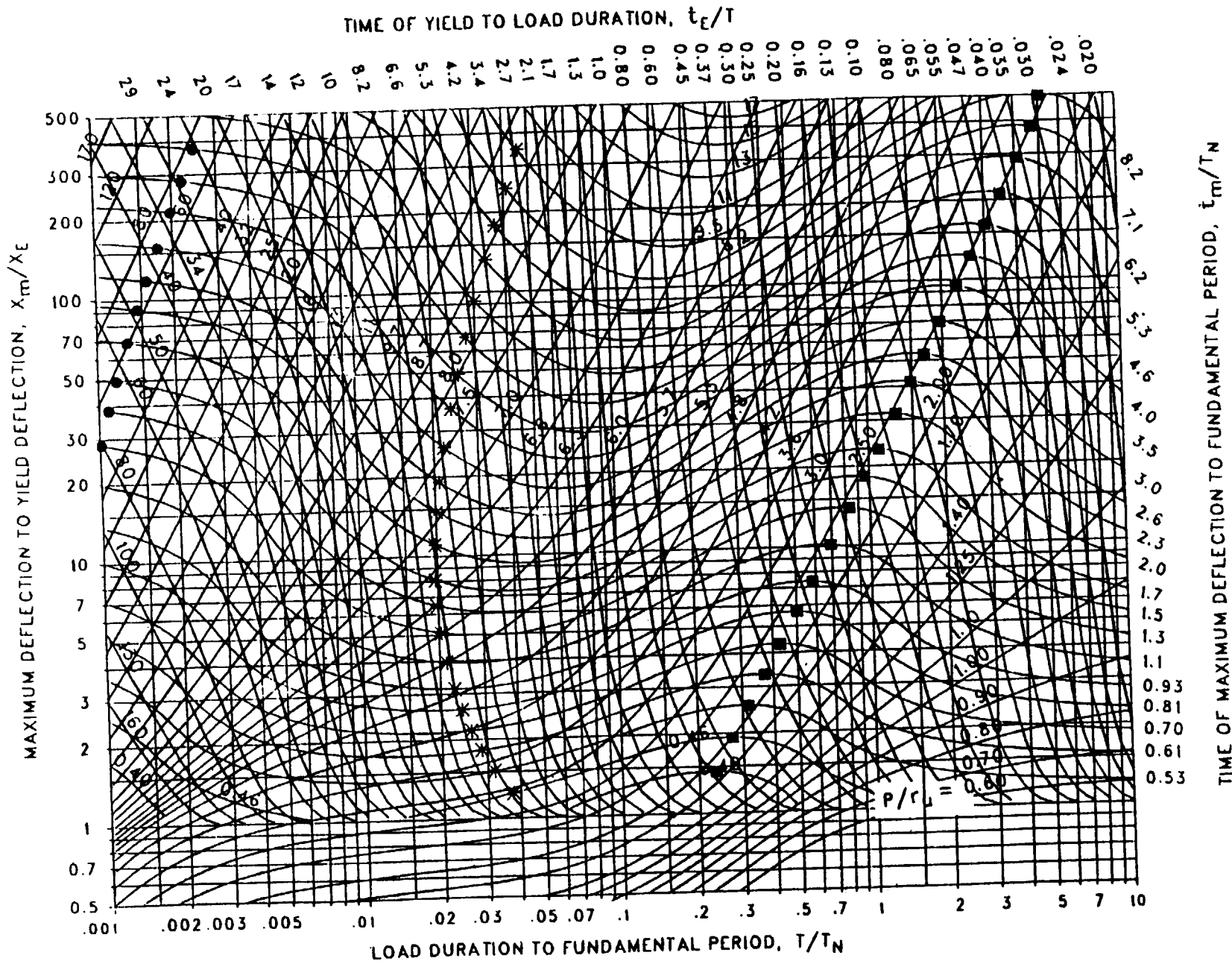


Figure 3-200 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.162$ ,  $C_2 = 300$ .)

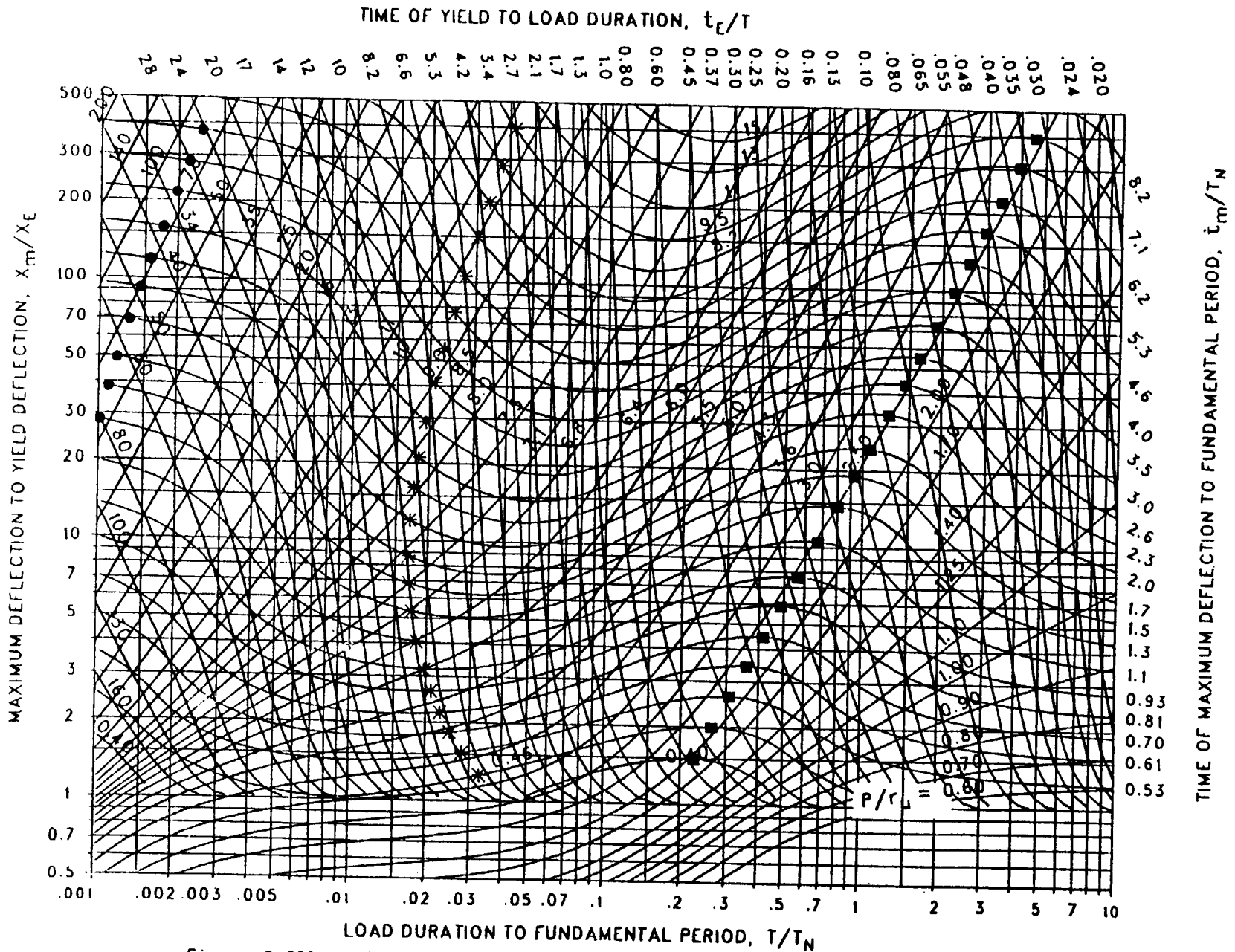


Figure 3-201 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.147, C_2 = 300.$ )

3-260

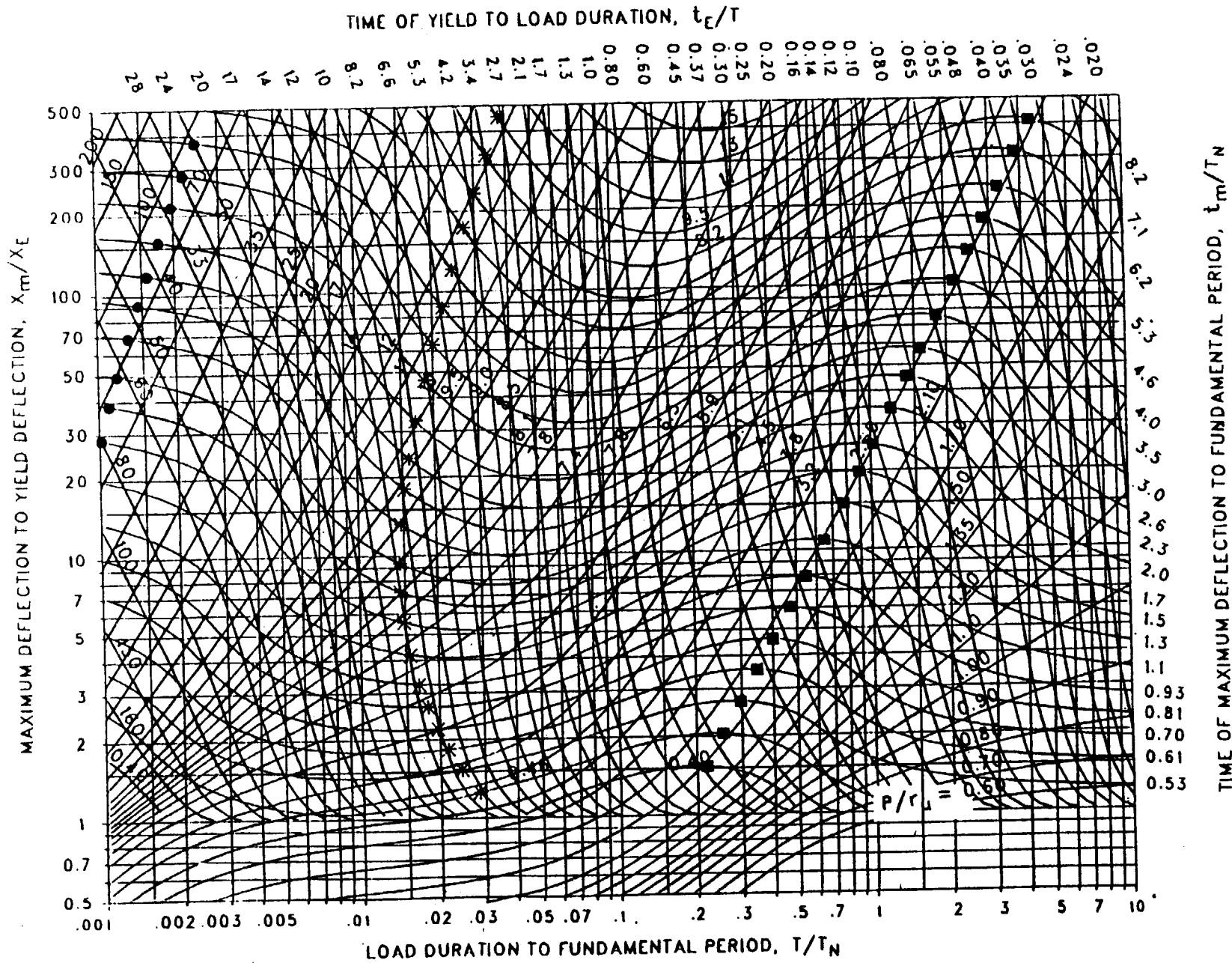


Figure 3-202 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.133$ ,  $C_2 = 300$ .)

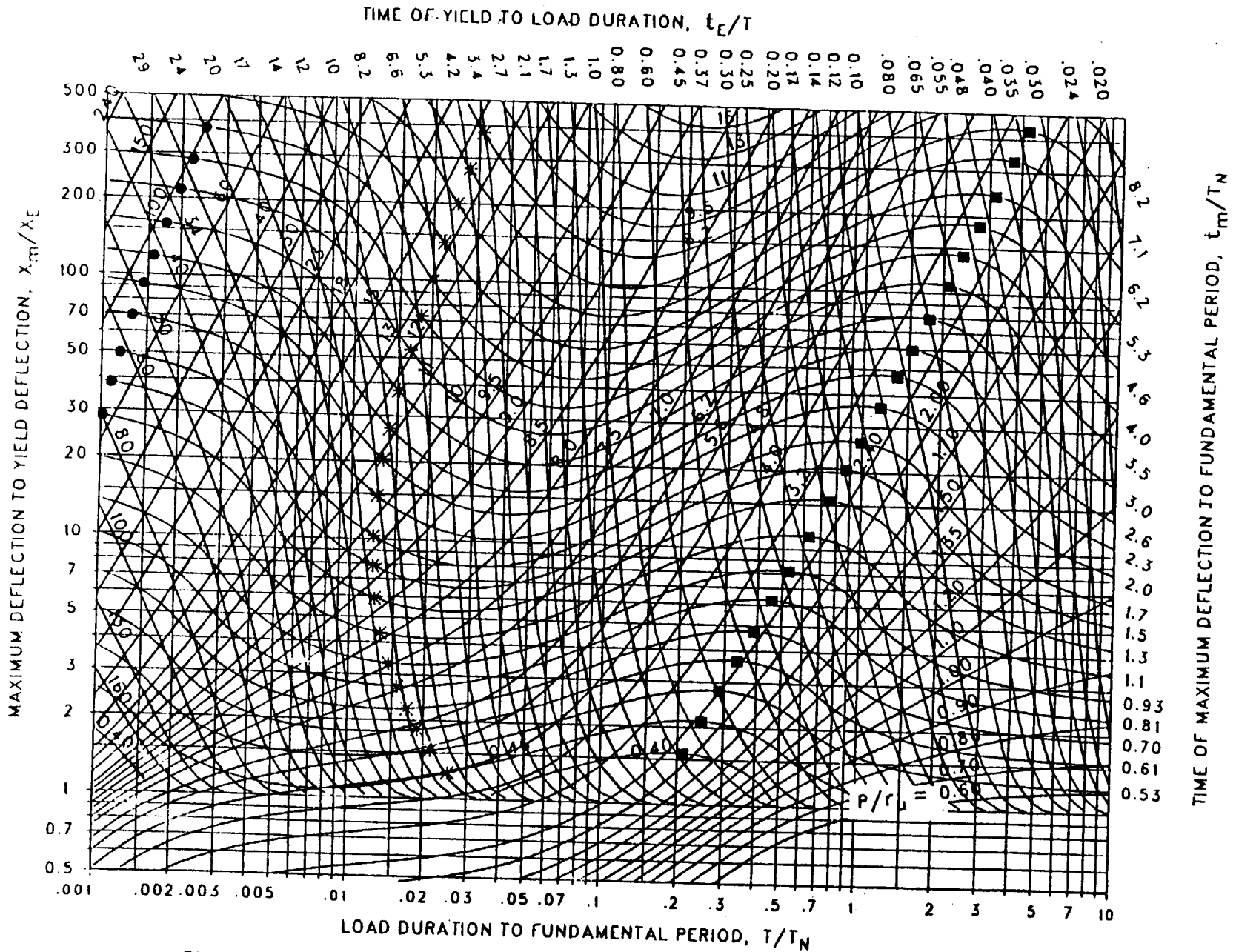


Figure 3-203 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.121$ ,  $C_2 = 300$ .)

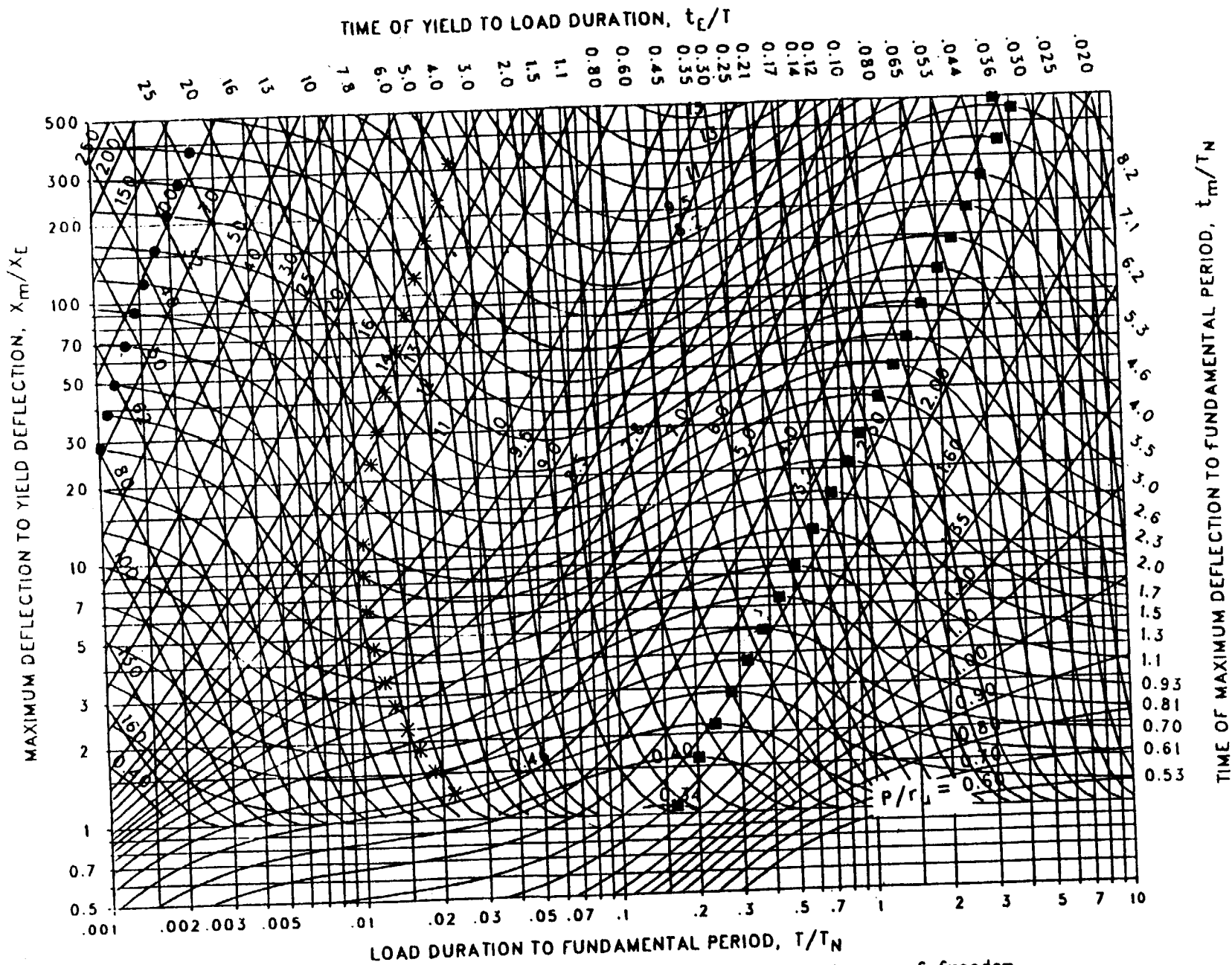


Figure 3-204 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.110$ ,  $C_2 = 300$ .)

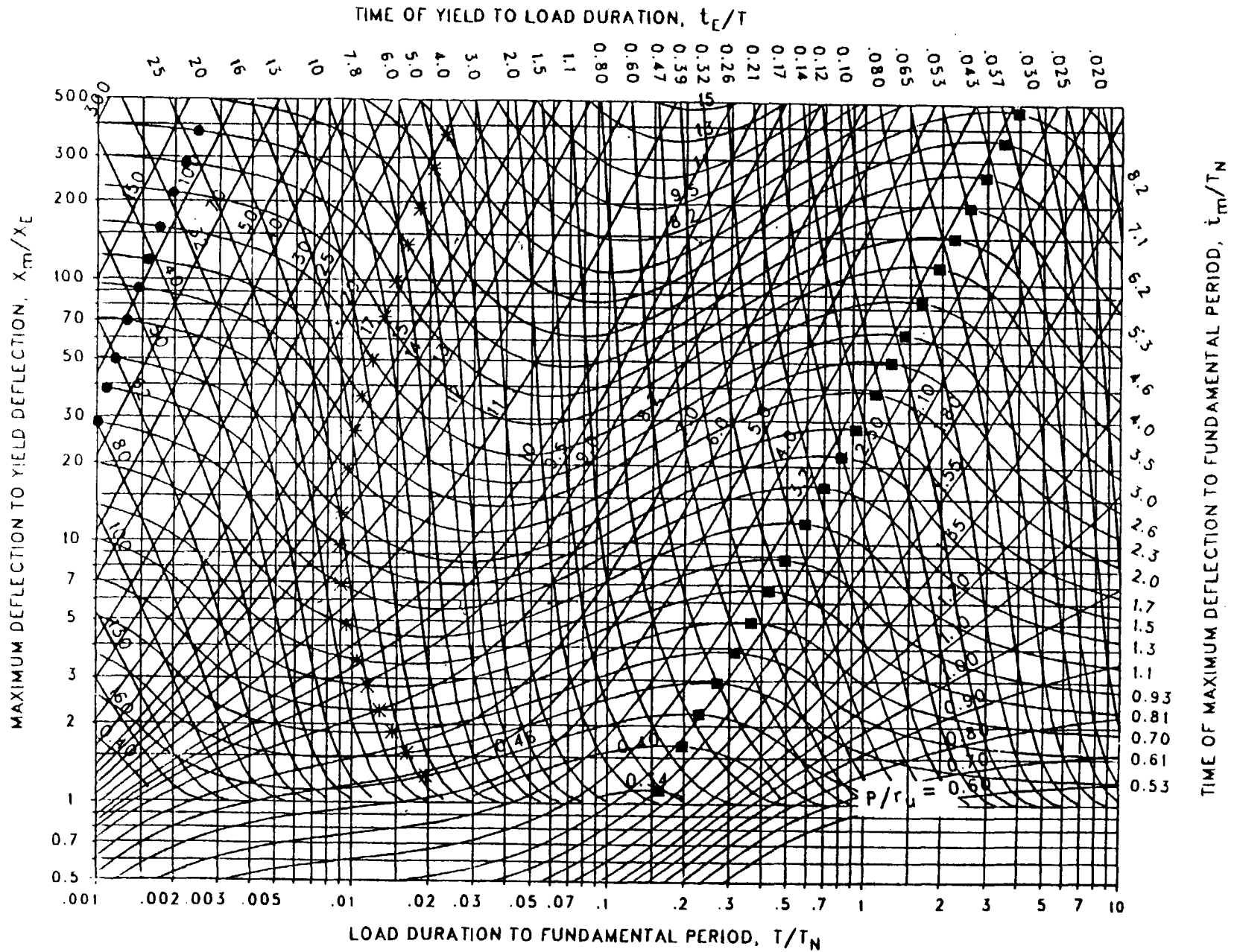


Figure 3-205 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.100$ ,  $C_2 = 300$ .)

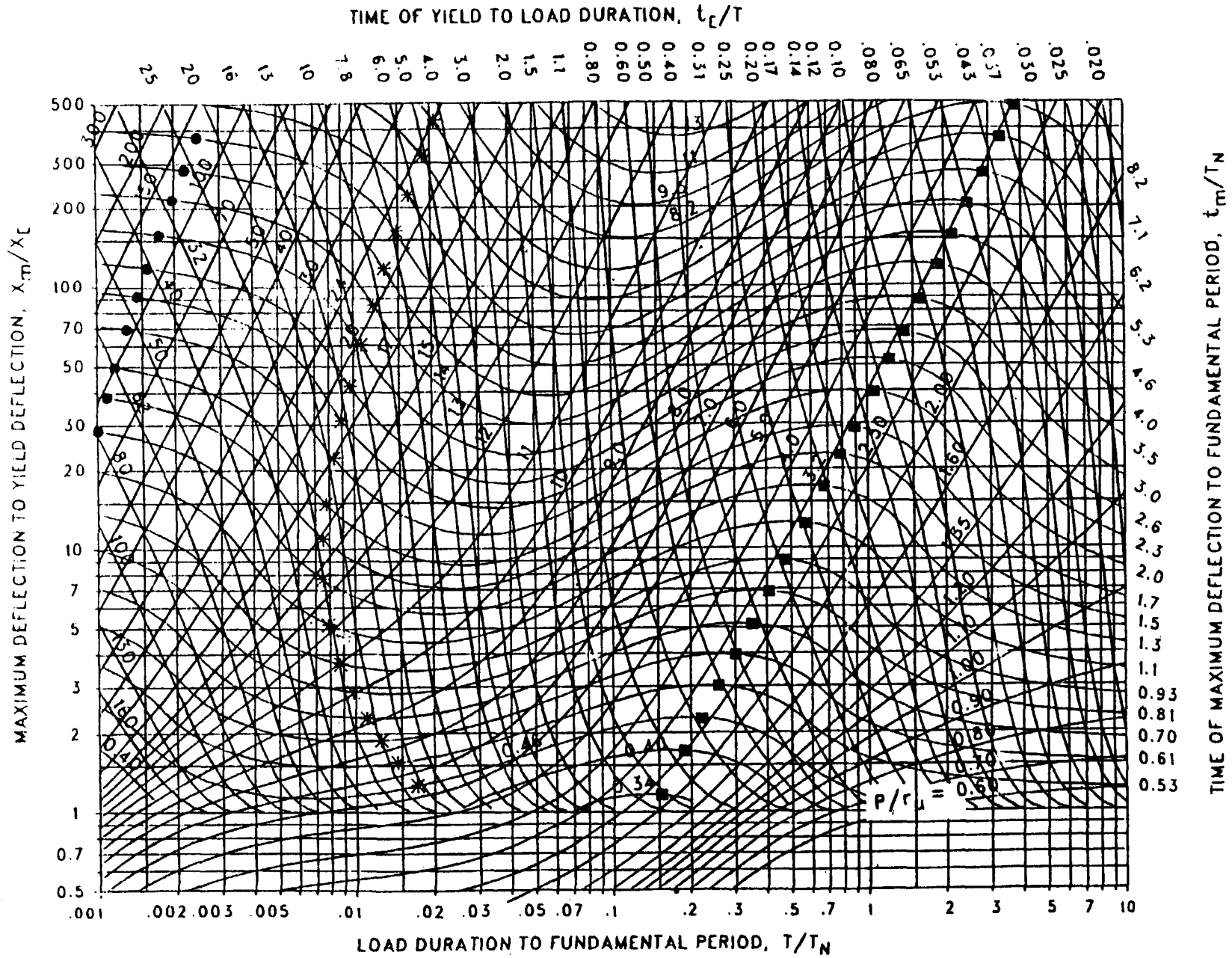


Figure 3-206 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.091$ ,  $C_2 = 300$ .)

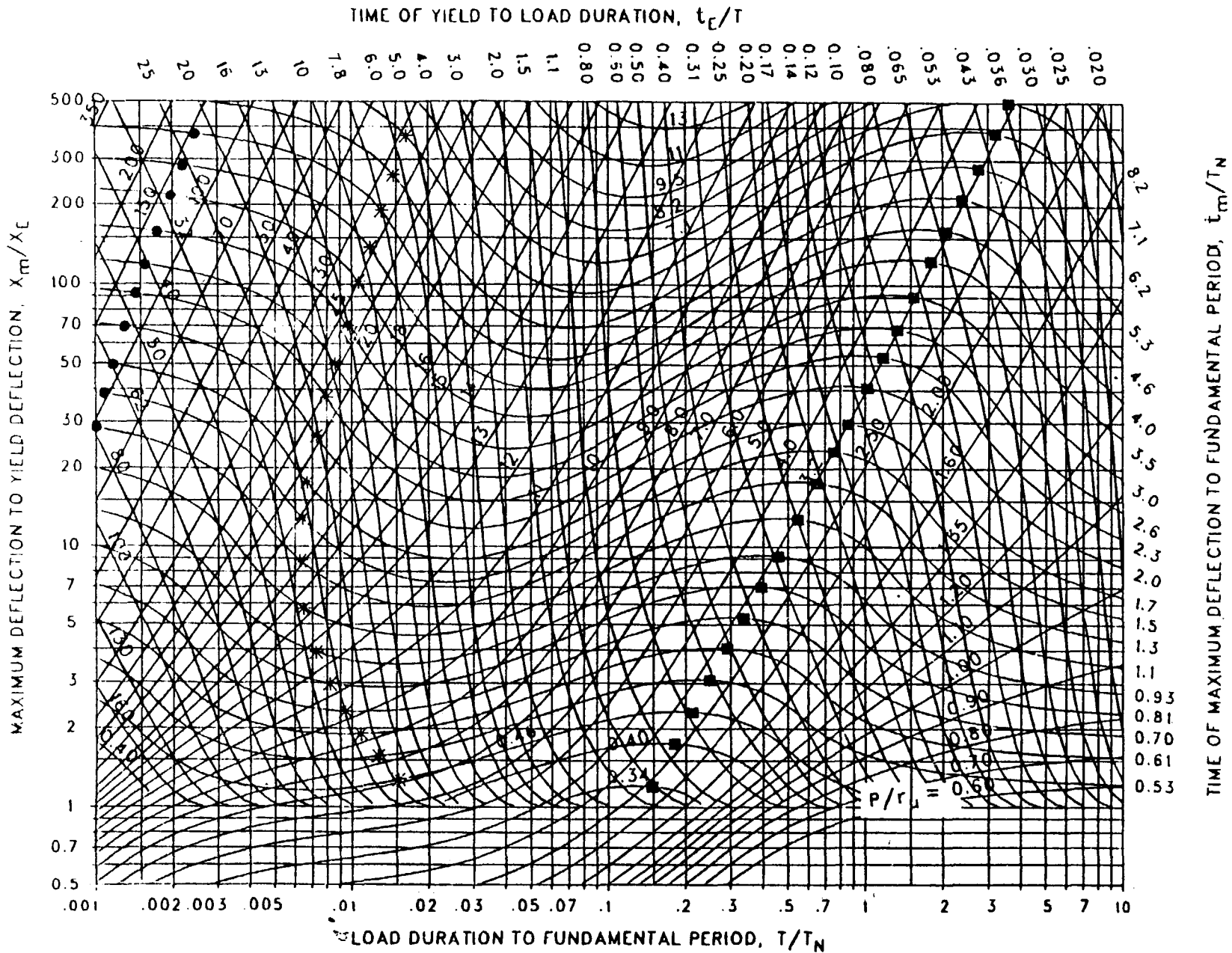


Figure 3-207 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.083$ ,  $C_2 = 300$ .)



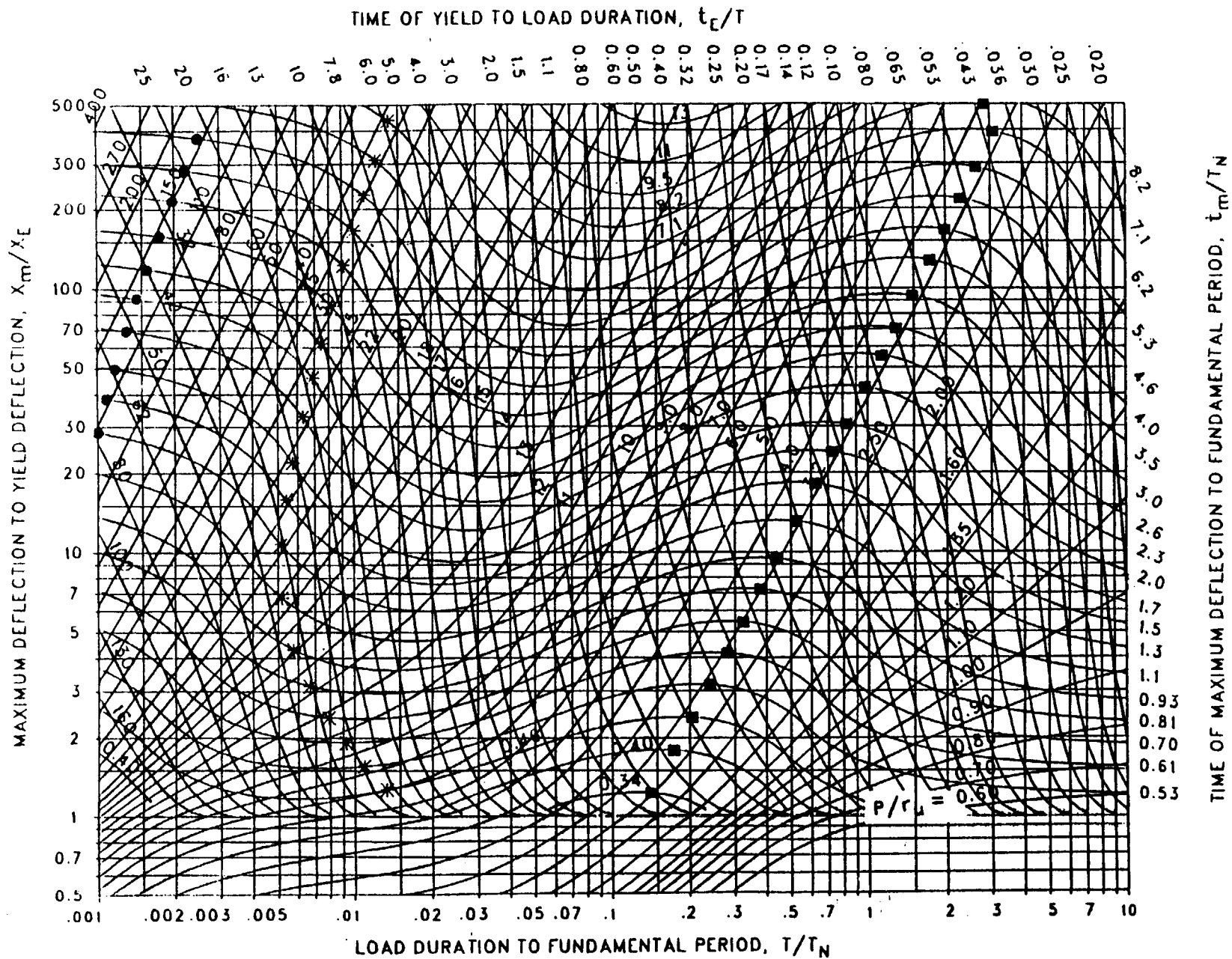


Figure 3-208 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.075$ ,  $C_2 = 300$ .)

3-267

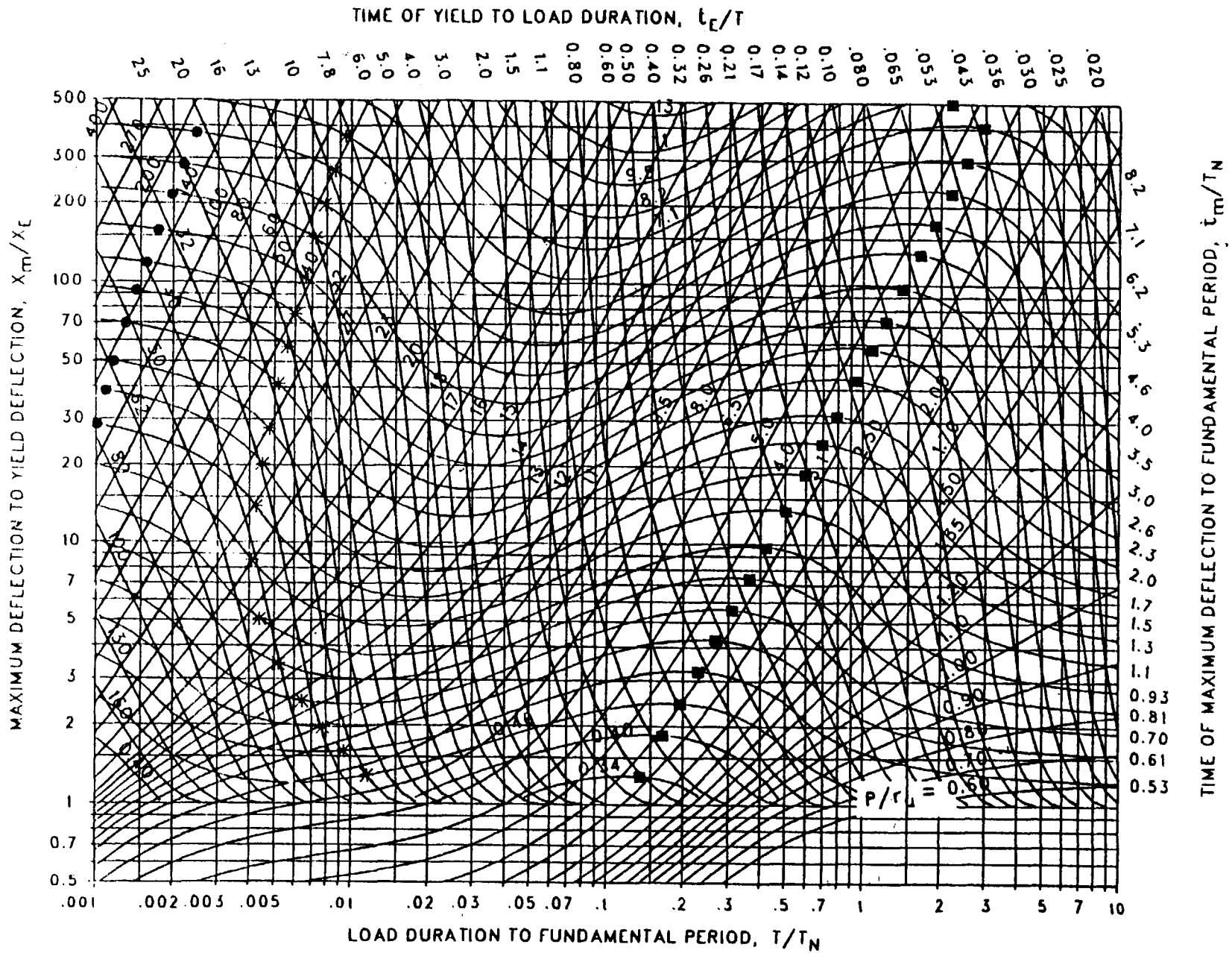


Figure 3-209 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.068$ ,  $C_2 = 300$ .)

3-268

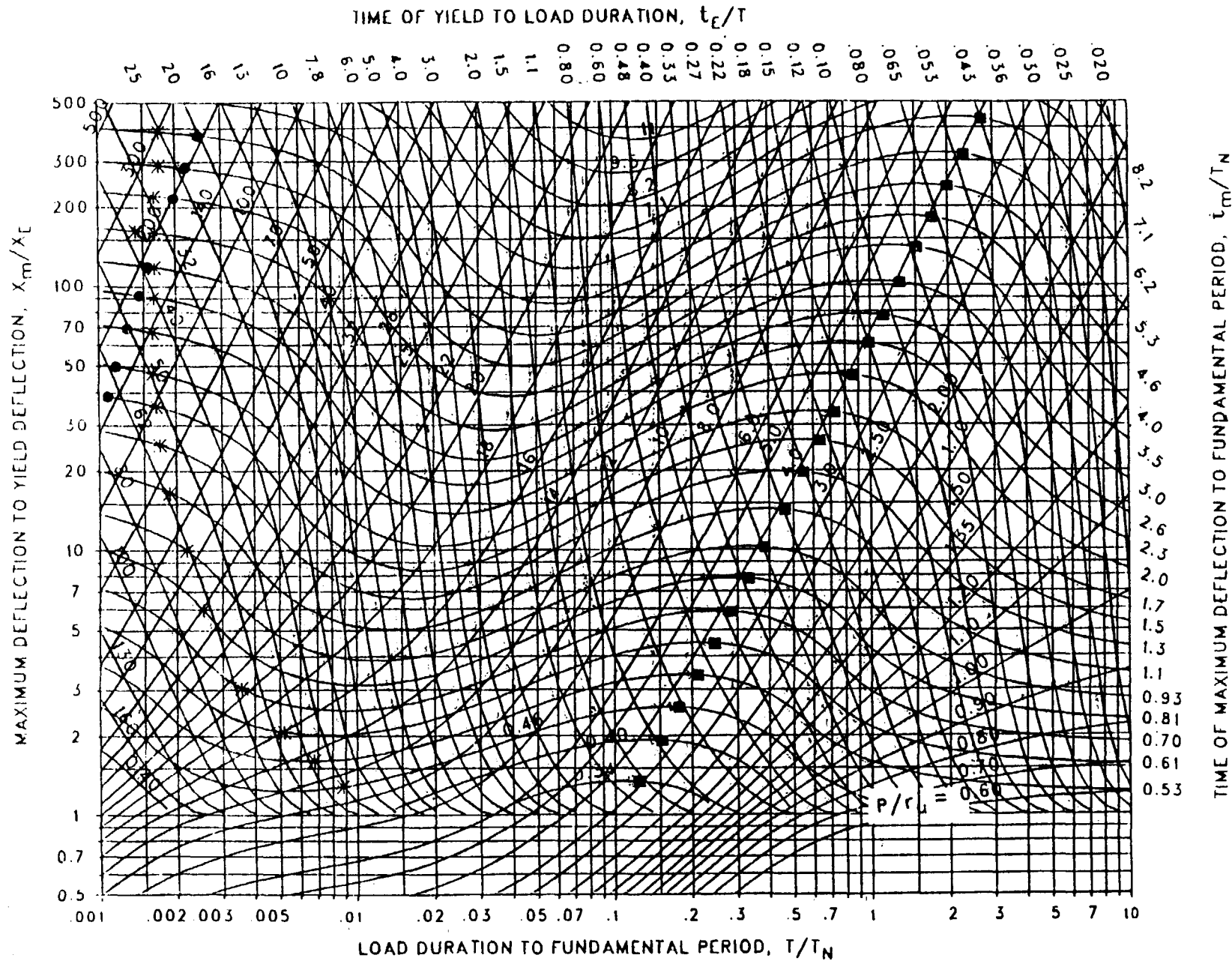


Figure 3-210 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.056$ ,  $C_2 = 300$ .)

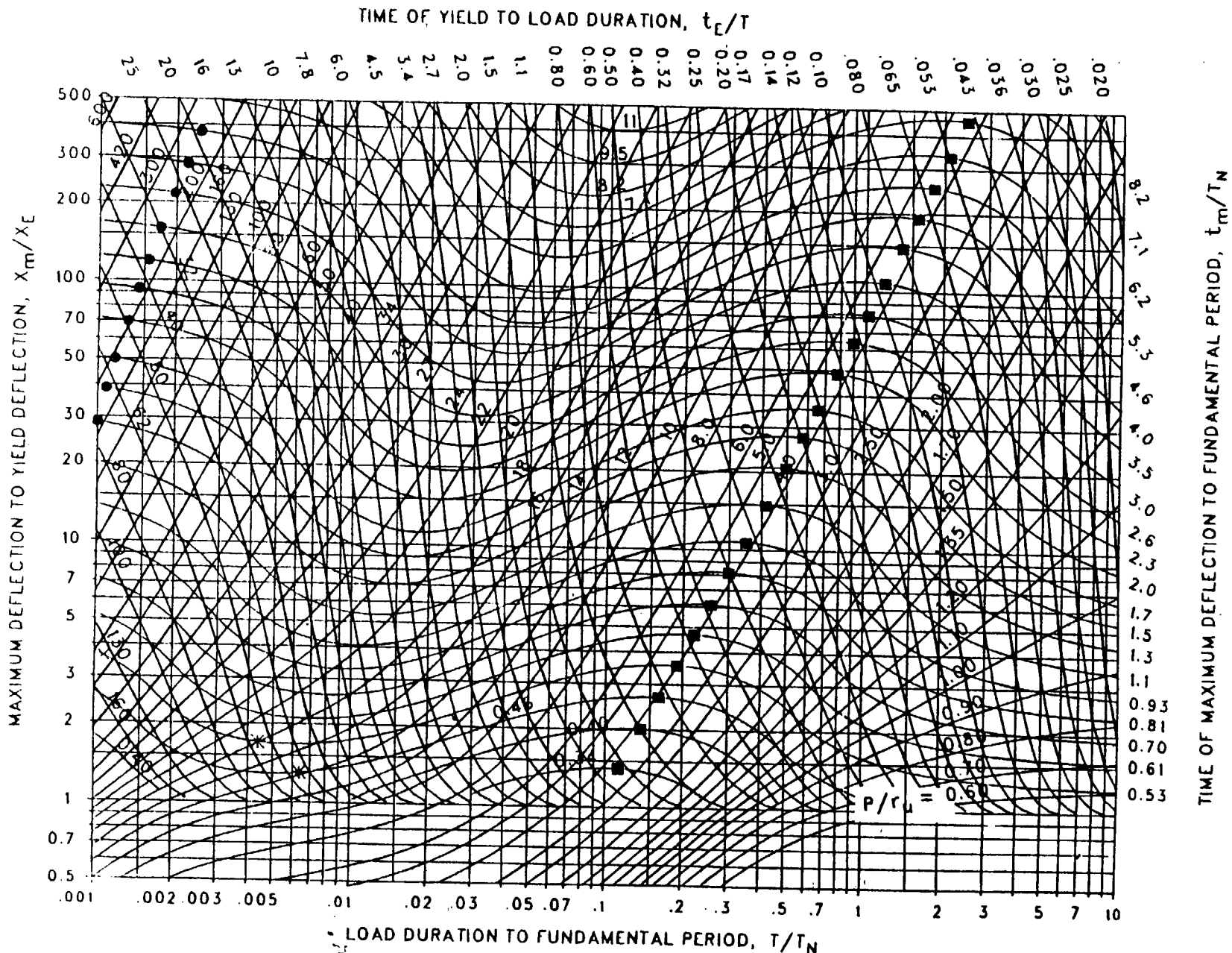


Figure 3-211 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.046$ ,  $C_2 = 300$ .)

3-270

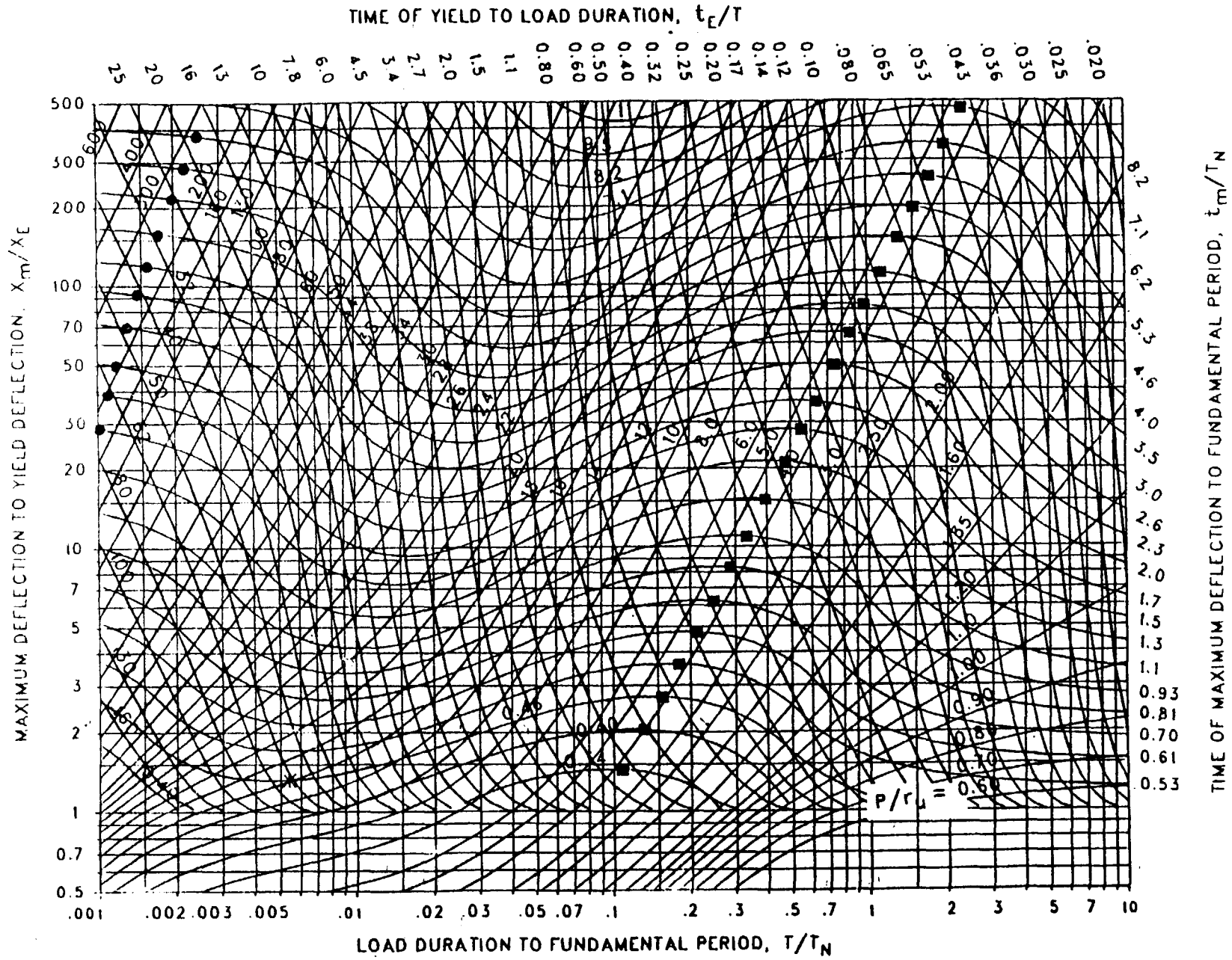


Figure 3-212 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.042$ ;  $C_2 = 300$ .)

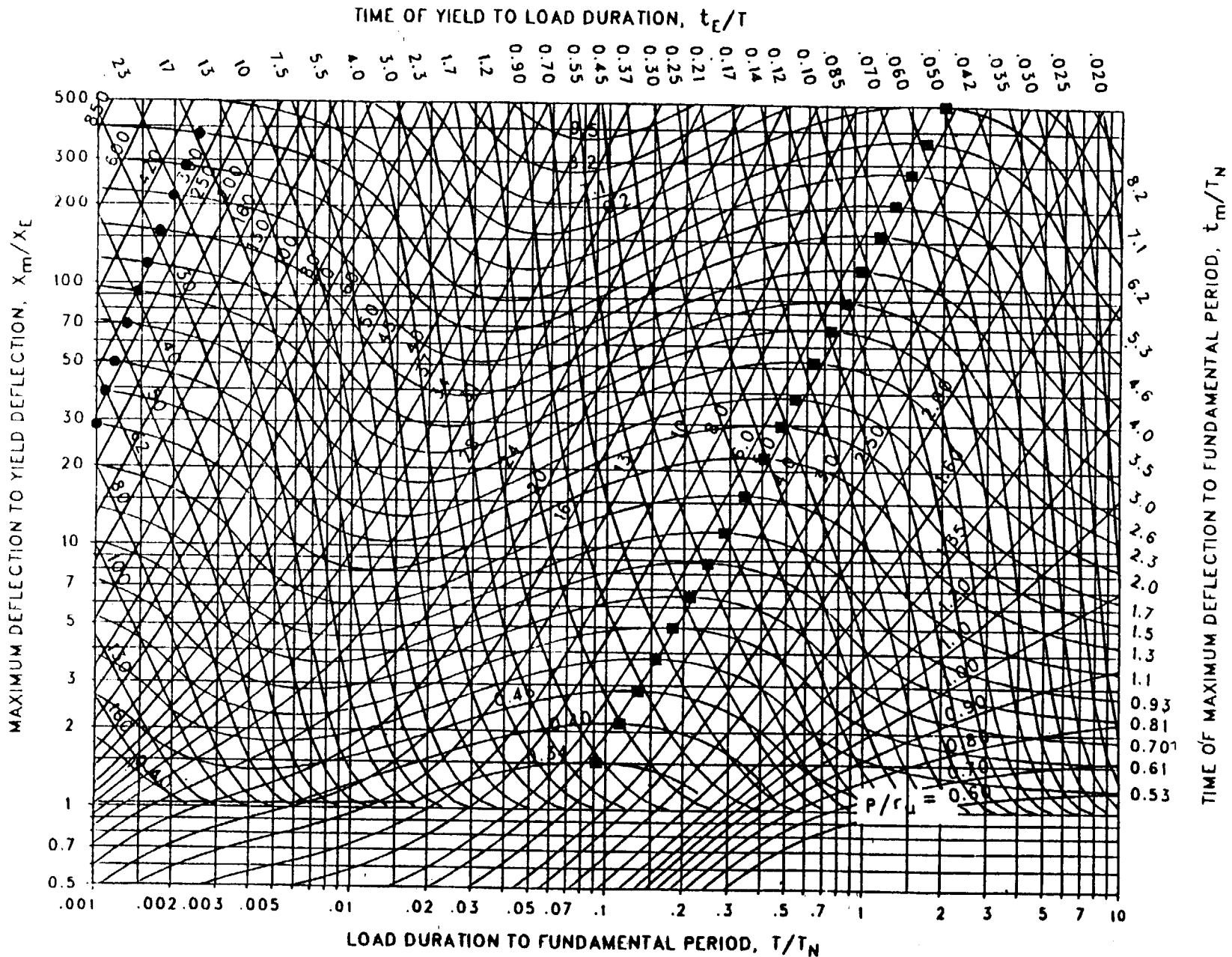


Figure 3-213 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.032$ ,  $C_2 = 300.$ )

3-272

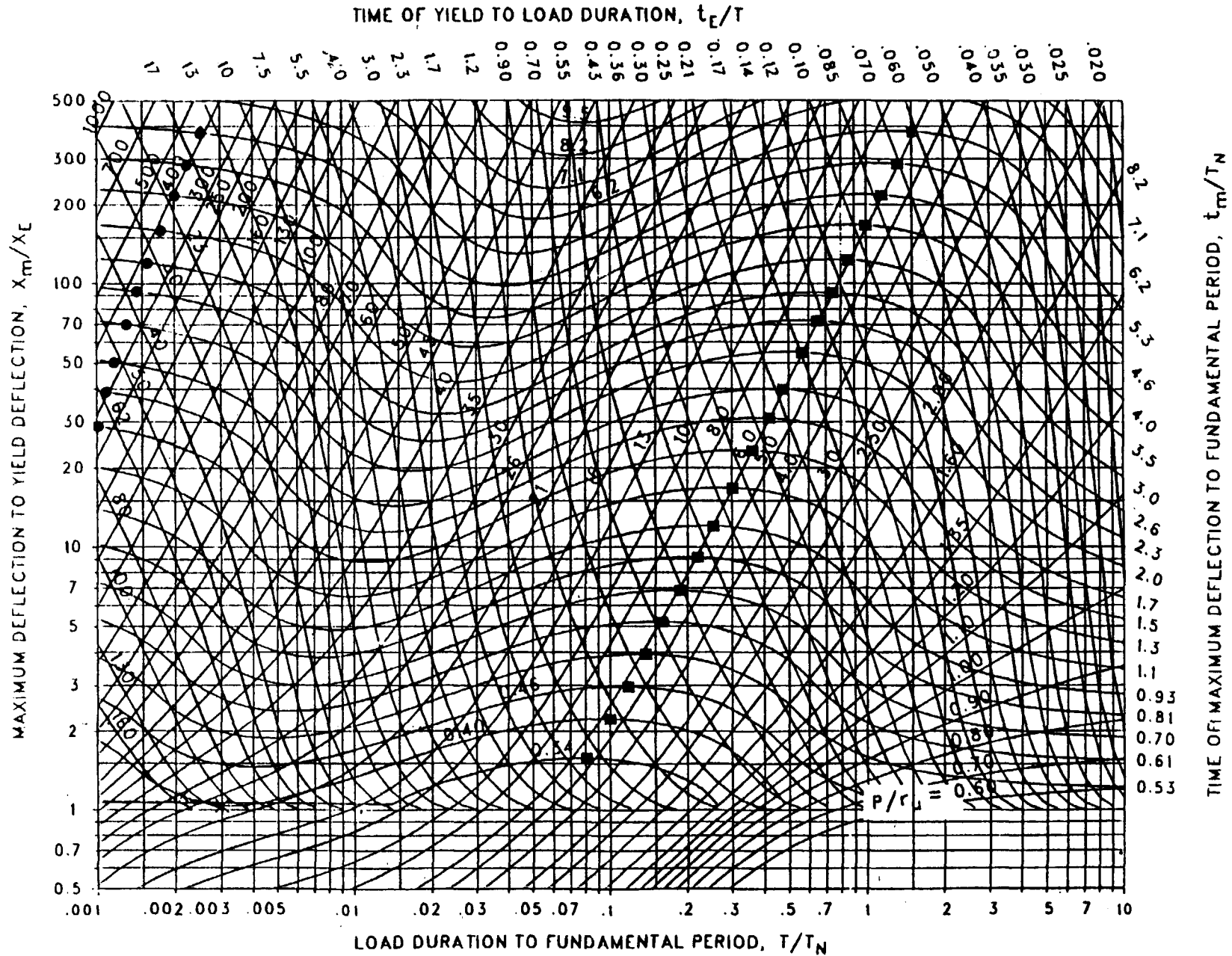


Figure 3-214 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.026$ ,  $C_2 = 300$ .)

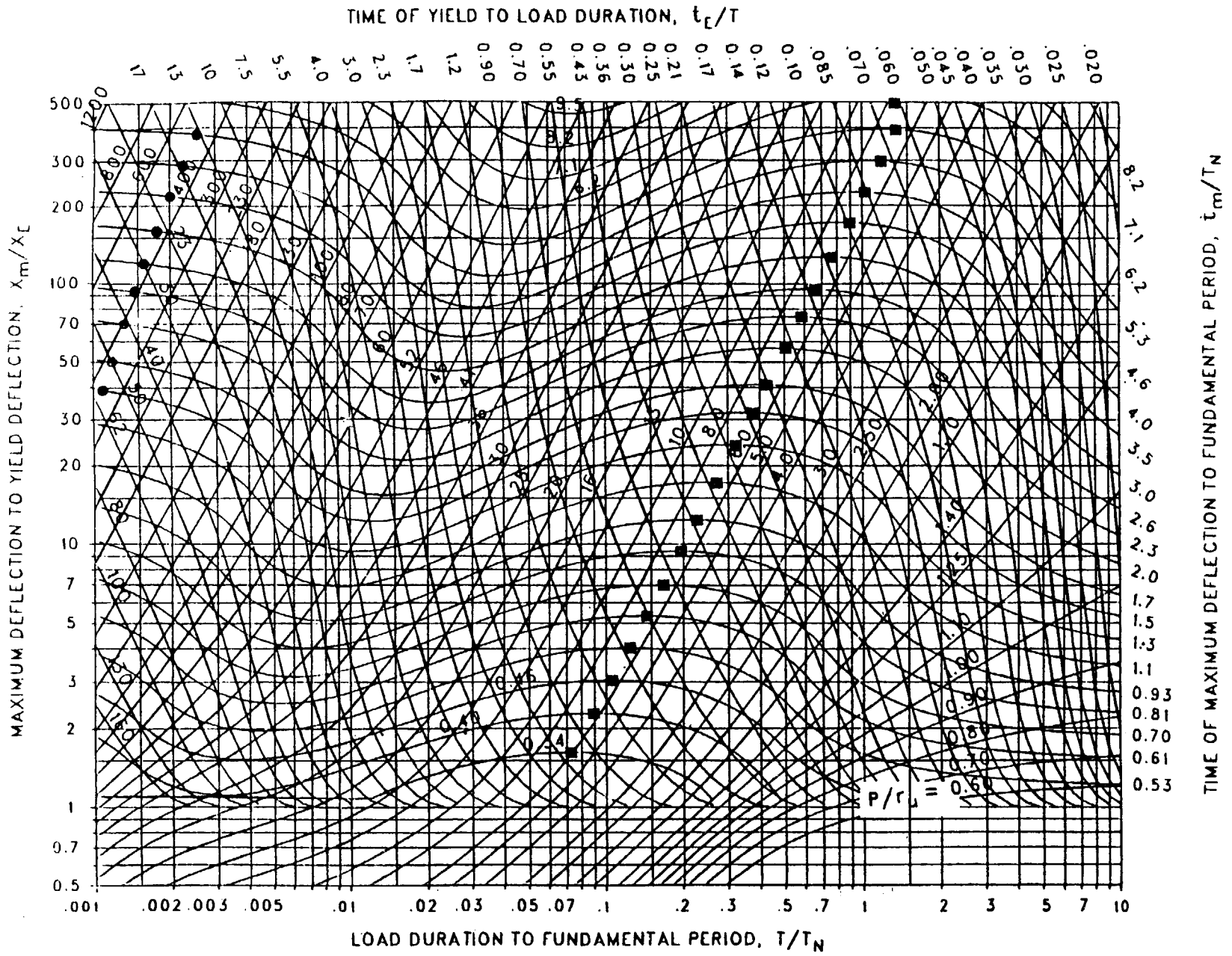


Figure 3-215 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.022$ ,  $C_2 = 300$ .)



3-274

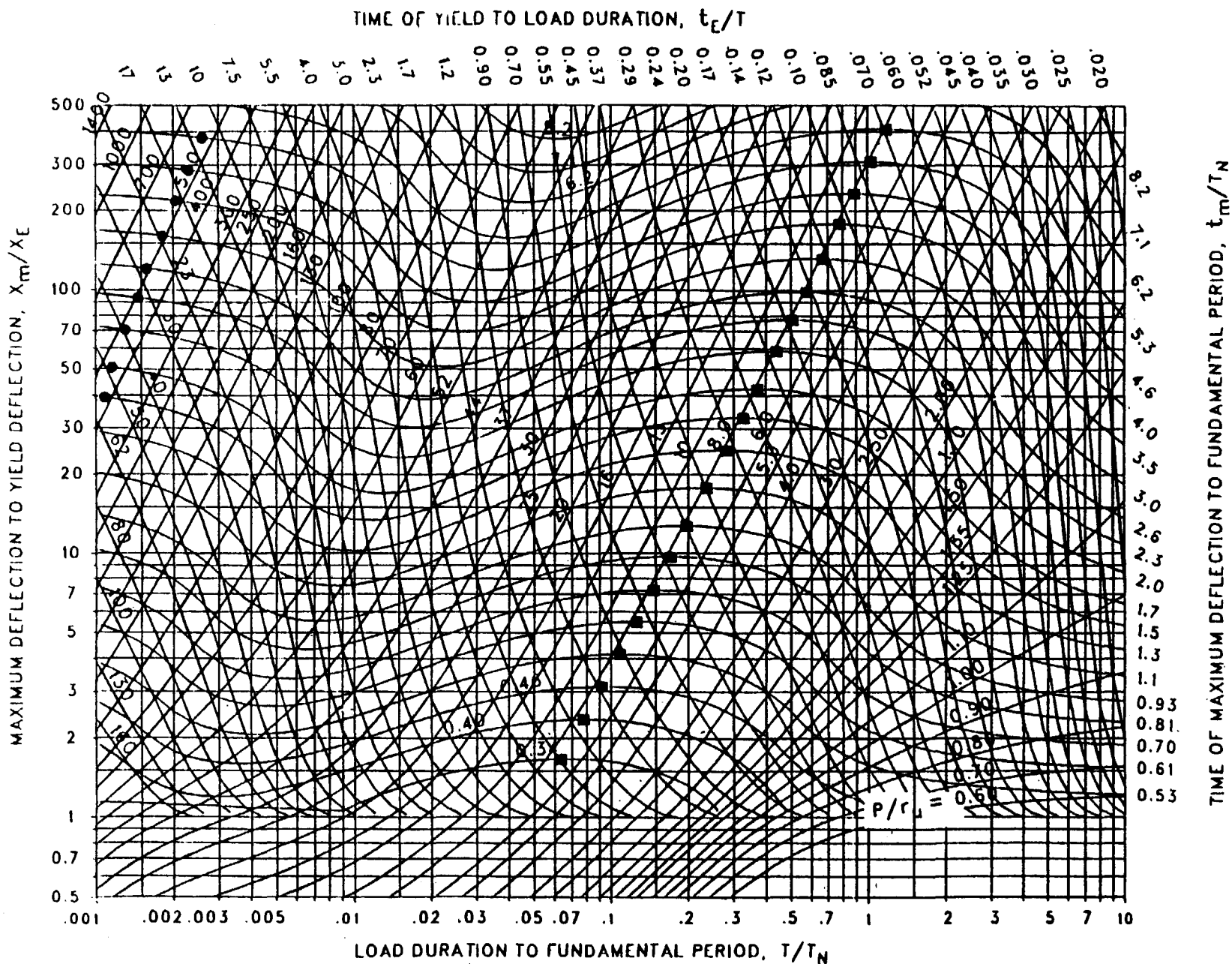


Figure 3-216 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.018$ ,  $C_2 = 300$ .)

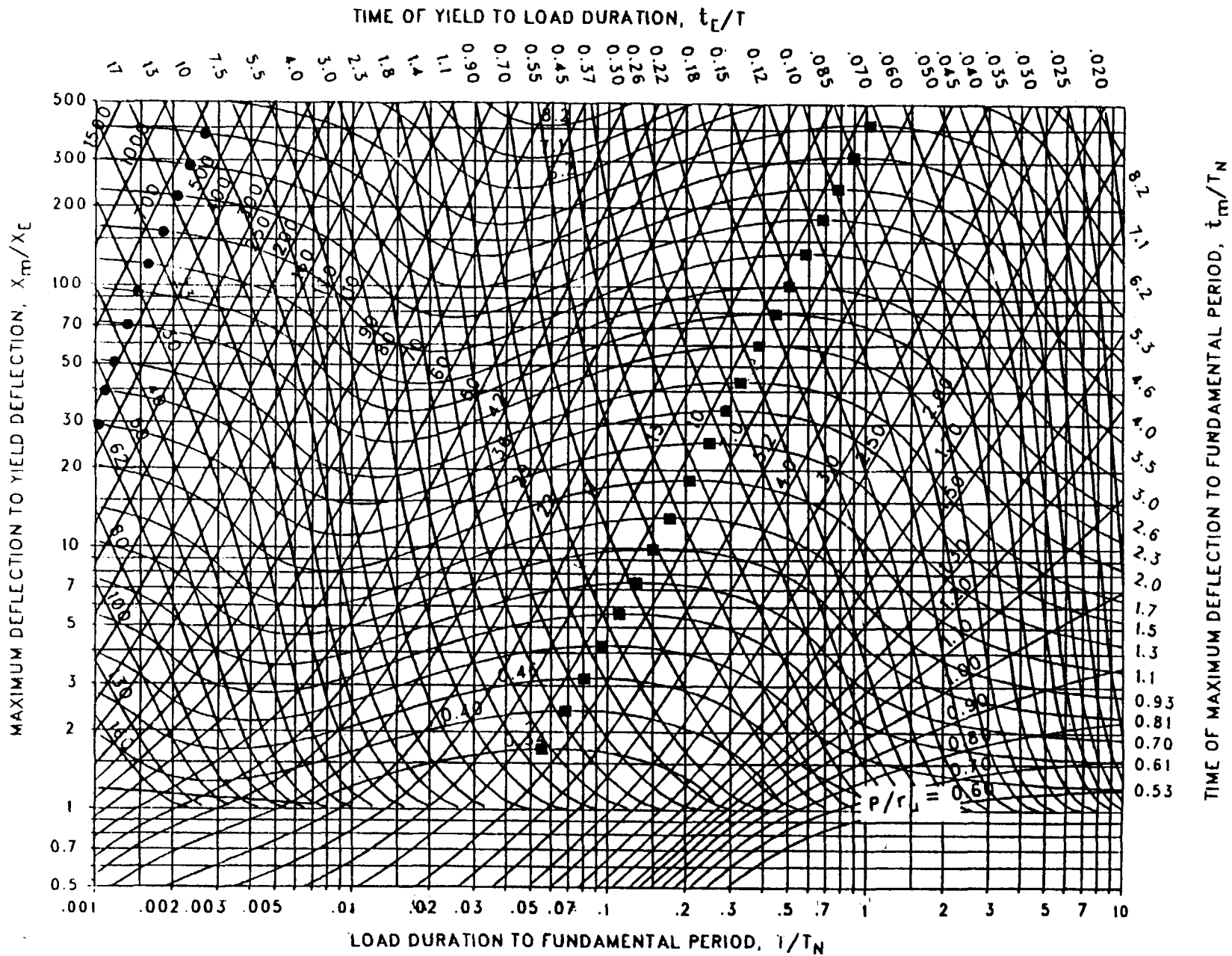


Figure 3-217 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.015$ ,  $C_2 = 300$ .)

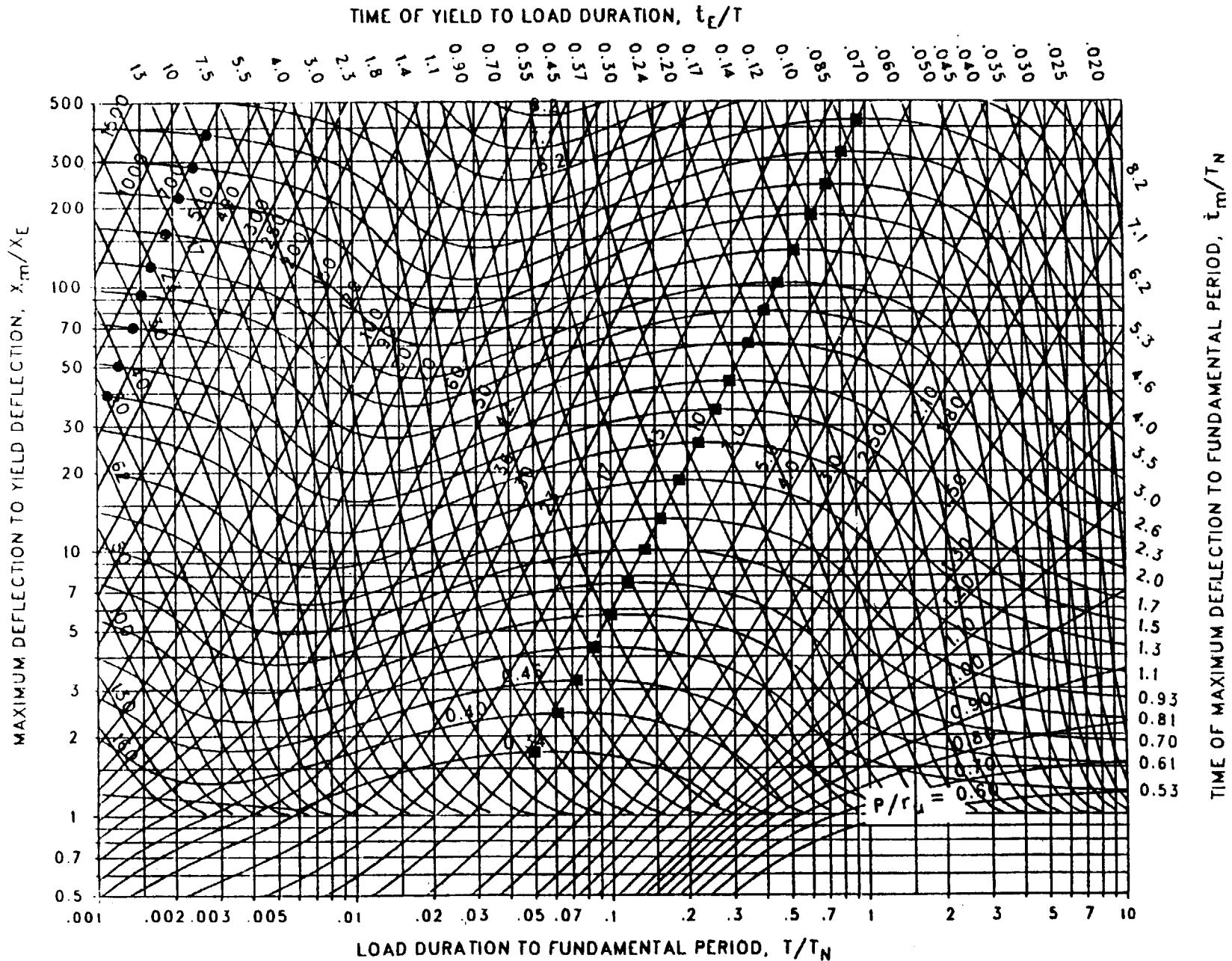


Figure 3-218 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.013$ ,  $C_2 = 300$ .)

3-277

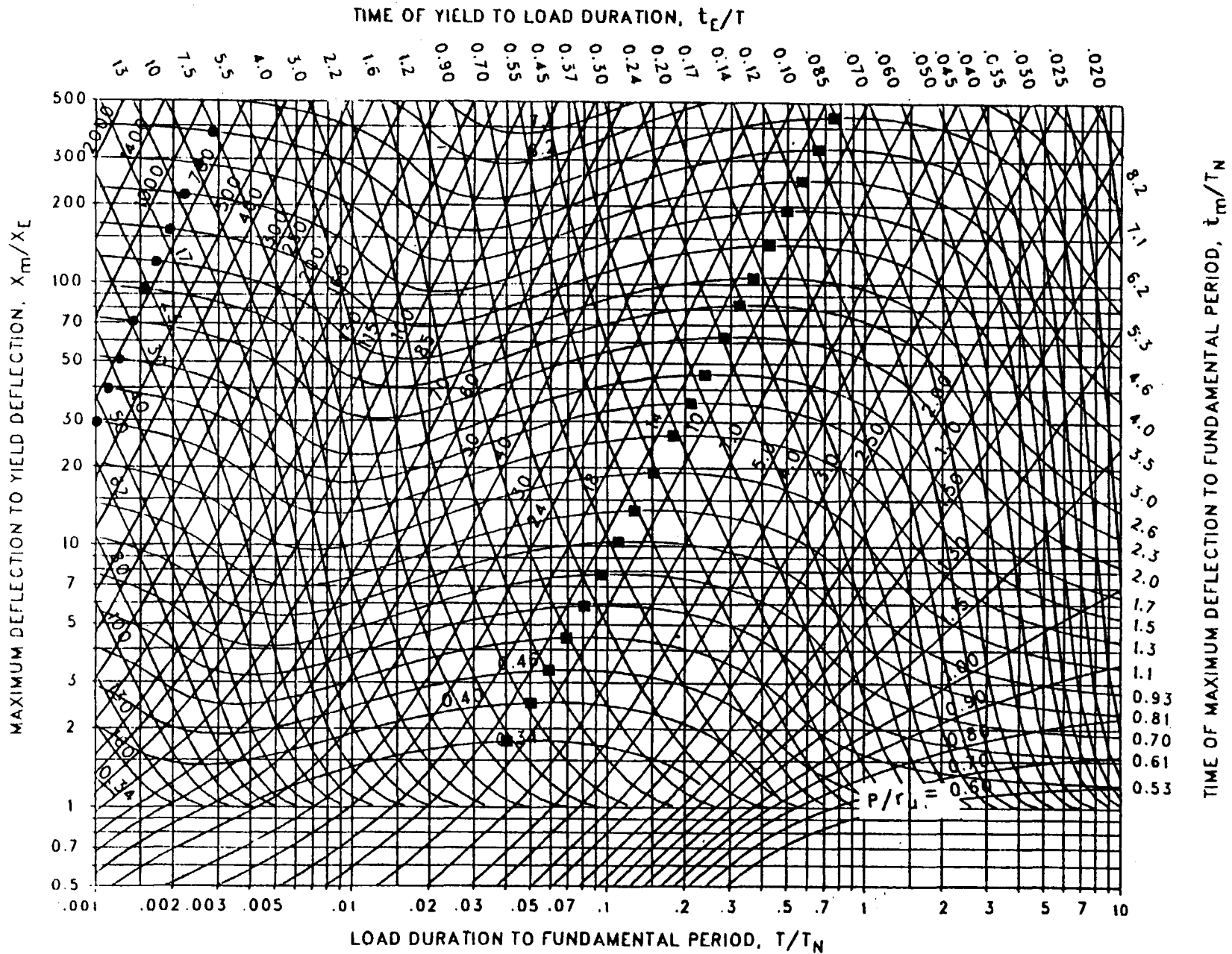


Figure 3-219 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.010$ ,  $C_2 = 300$ .)

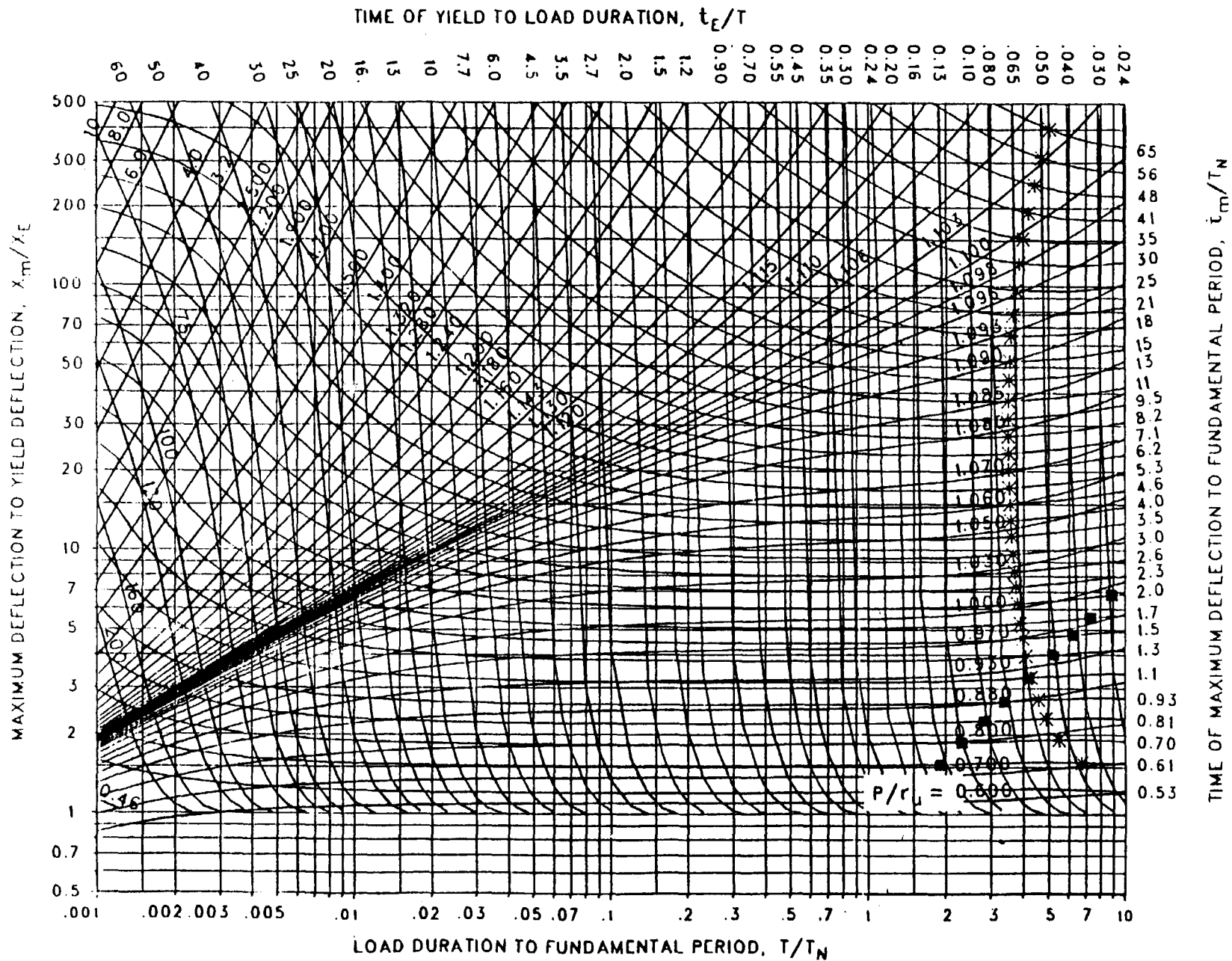


Figure 3-220 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.909$ ,  $C_2 = 1000$ .)

3-279

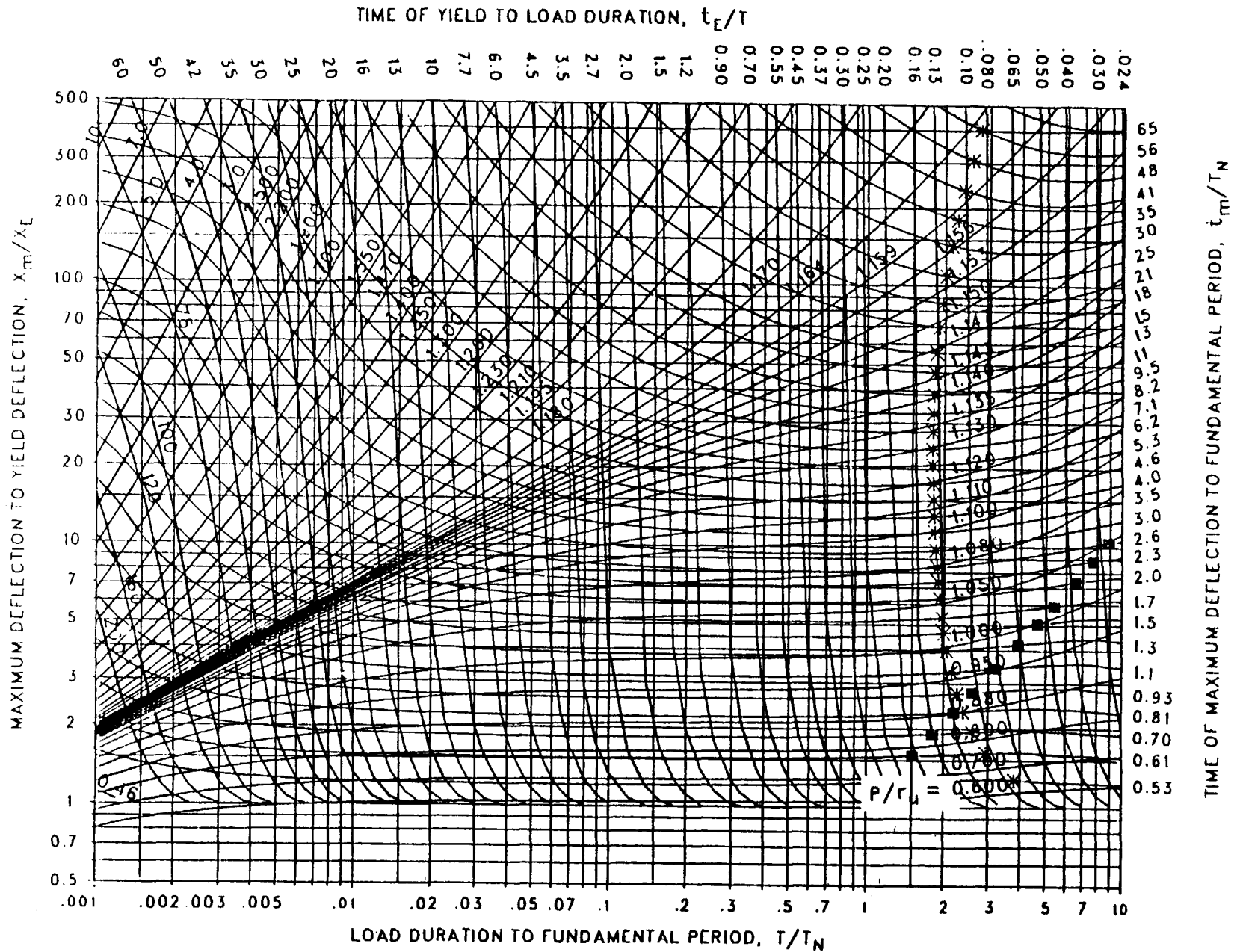


Figure 3-221 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.866$ ,  $C_2 = 1000$ .)

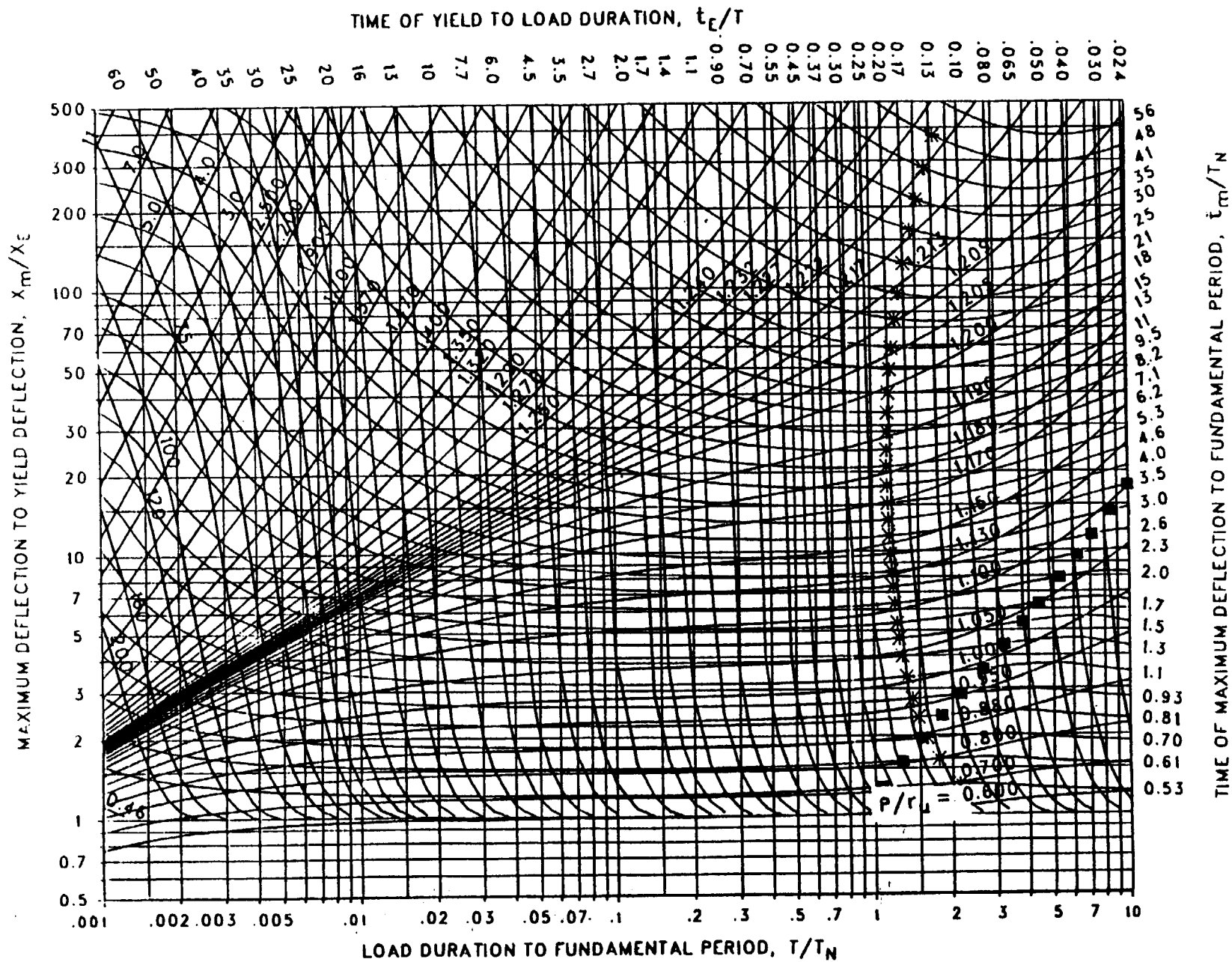


Figure 3-222 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.825$ ,  $C_2 = 1000$ .)

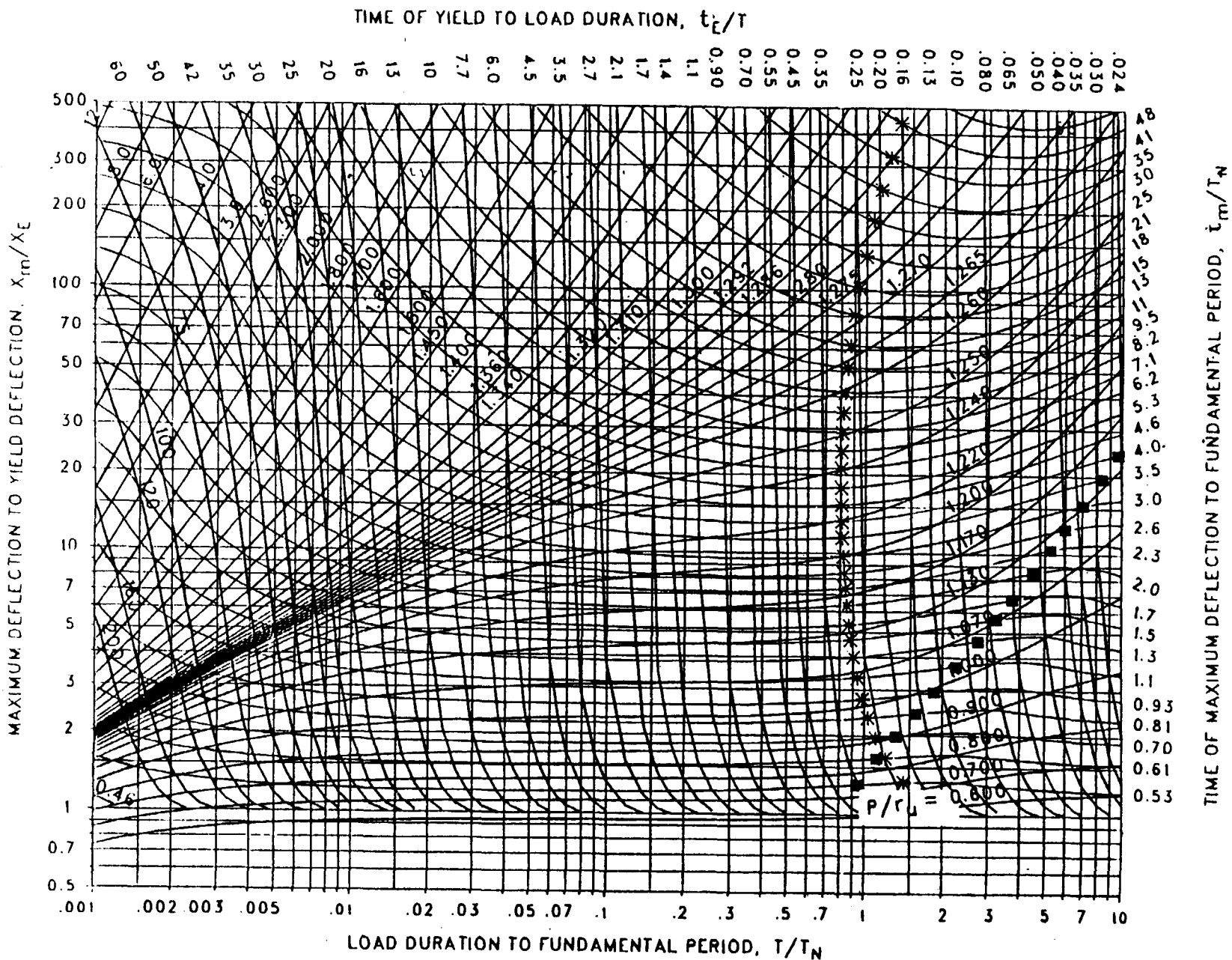


Figure 3-223 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.787$ ,  $C_2 = 1000$ .)



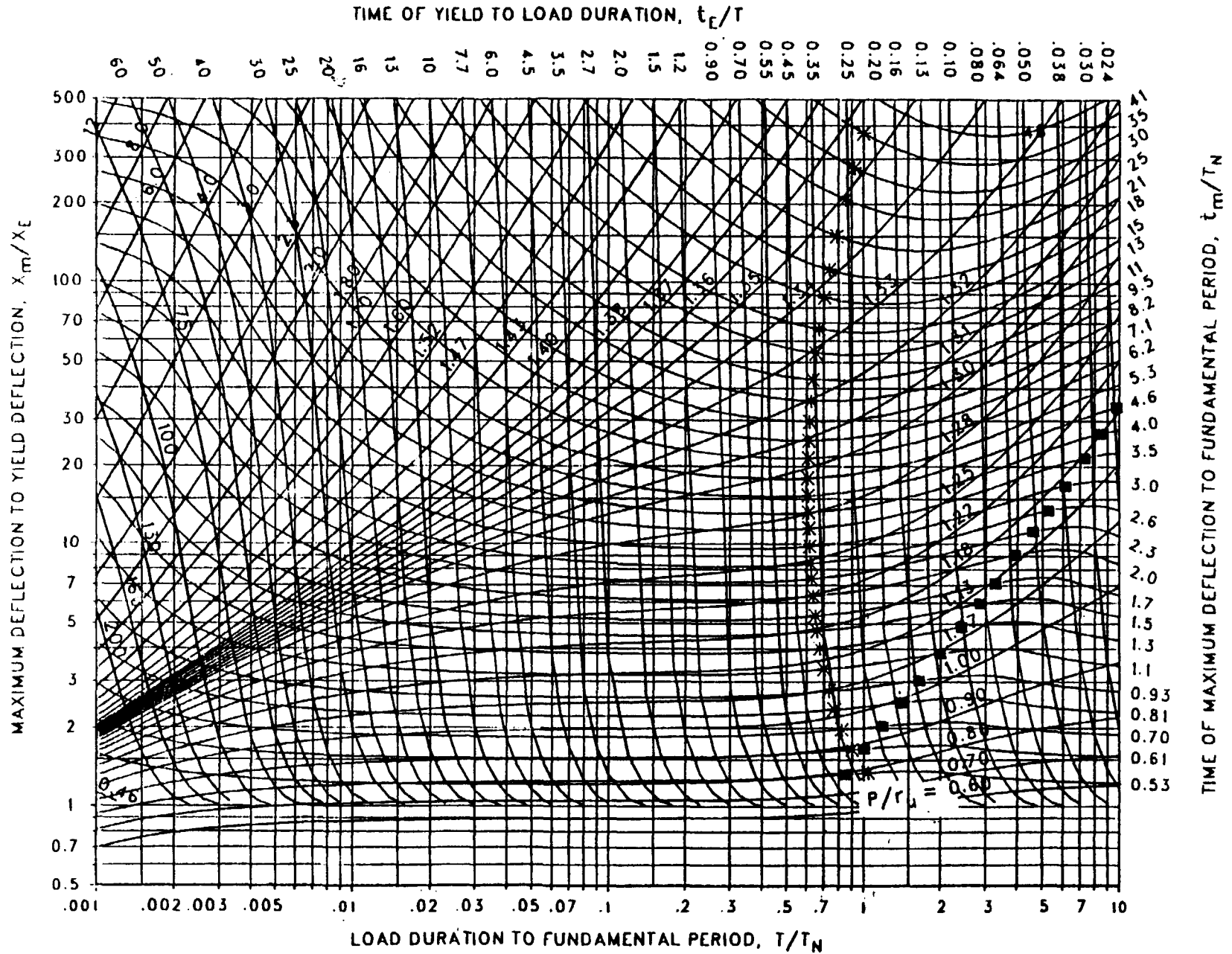


Figure 3-224 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.750$ ,  $C_2 = 1000$ .)

3-283

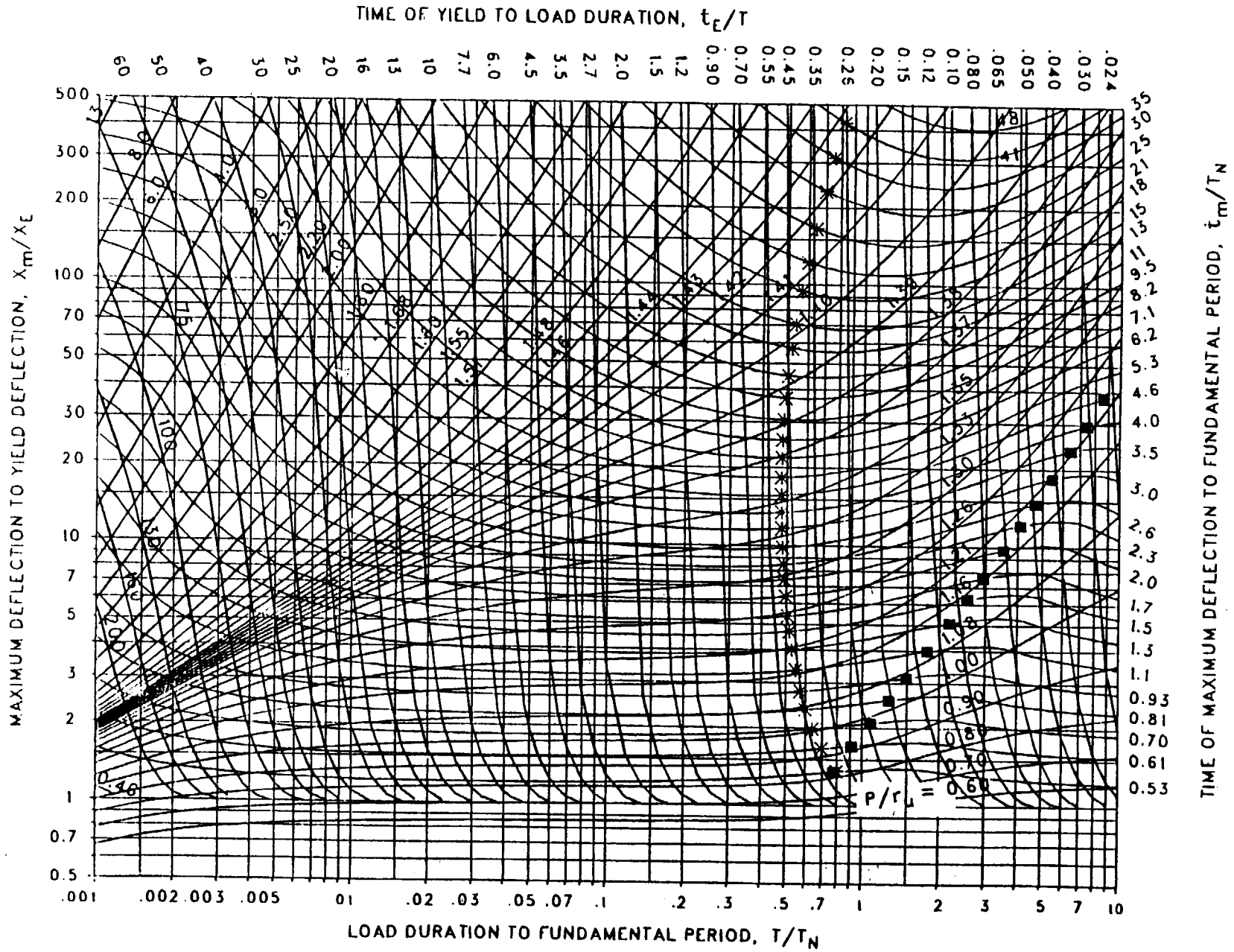


Figure 3-225 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.715$ ,  $C_2 = 1000$ .)

3-284

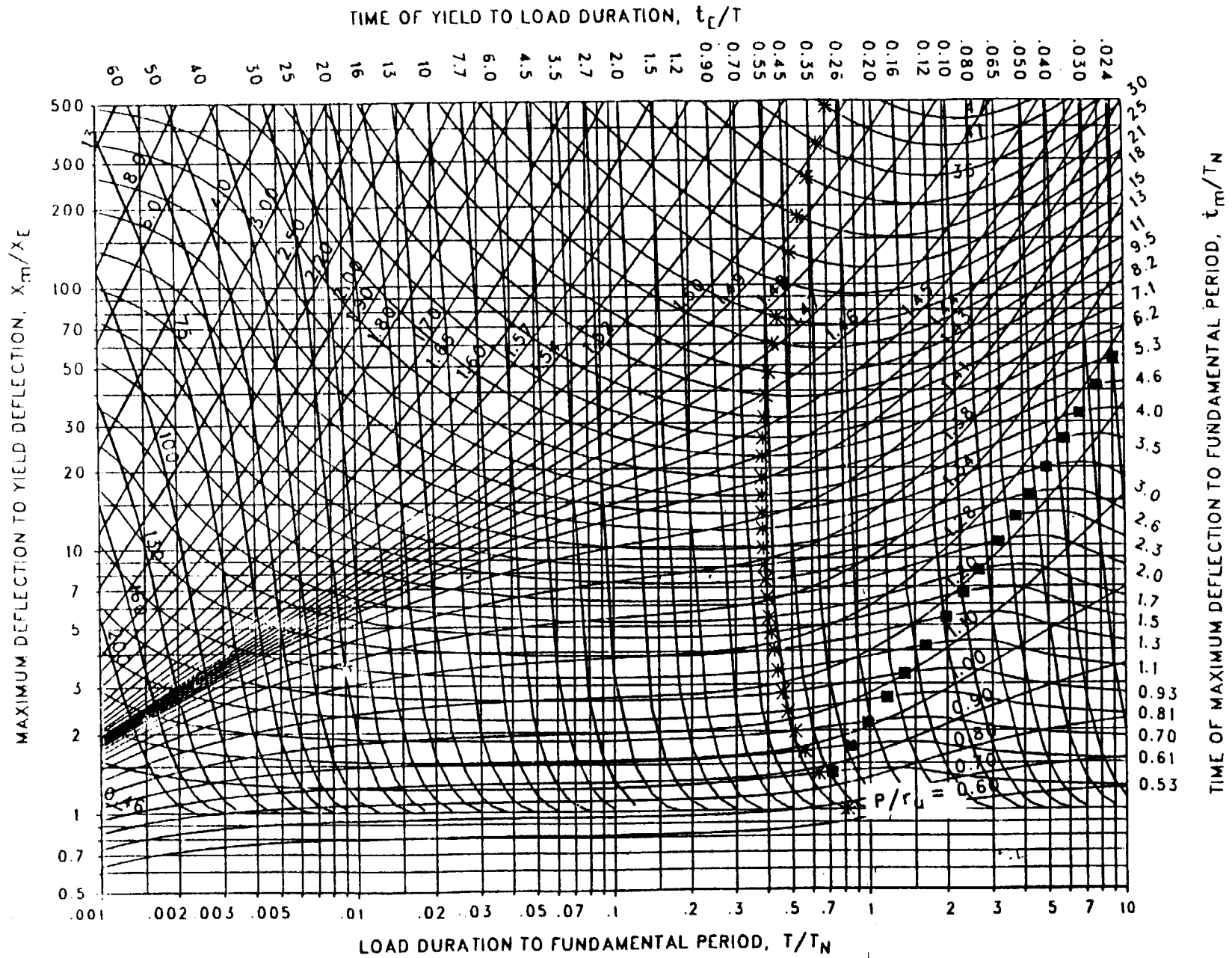


Figure 3-226 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.681$ ,  $C_2 = 1000$ .)

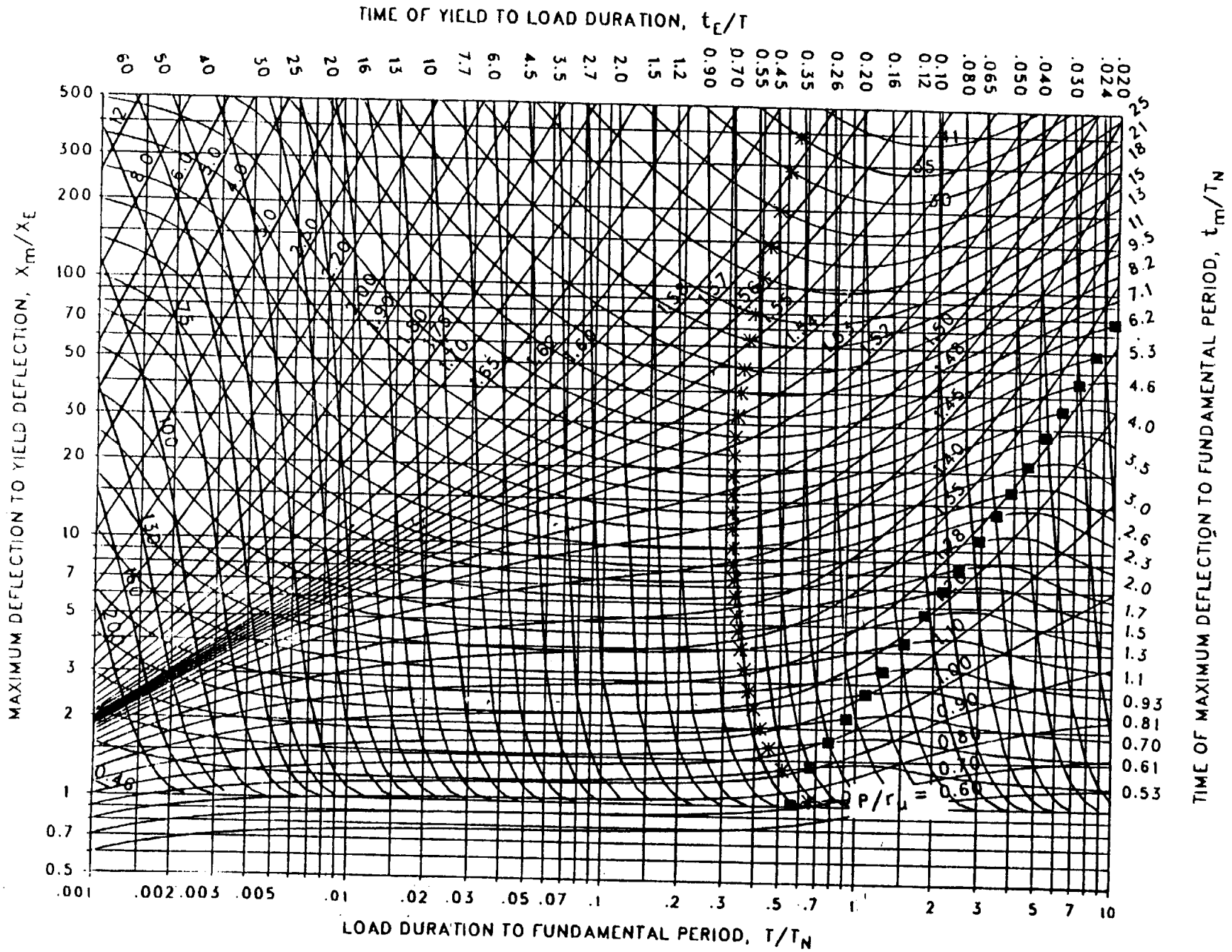


Figure 3-227 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.648$ ,  $C_2 = 1000$ .)

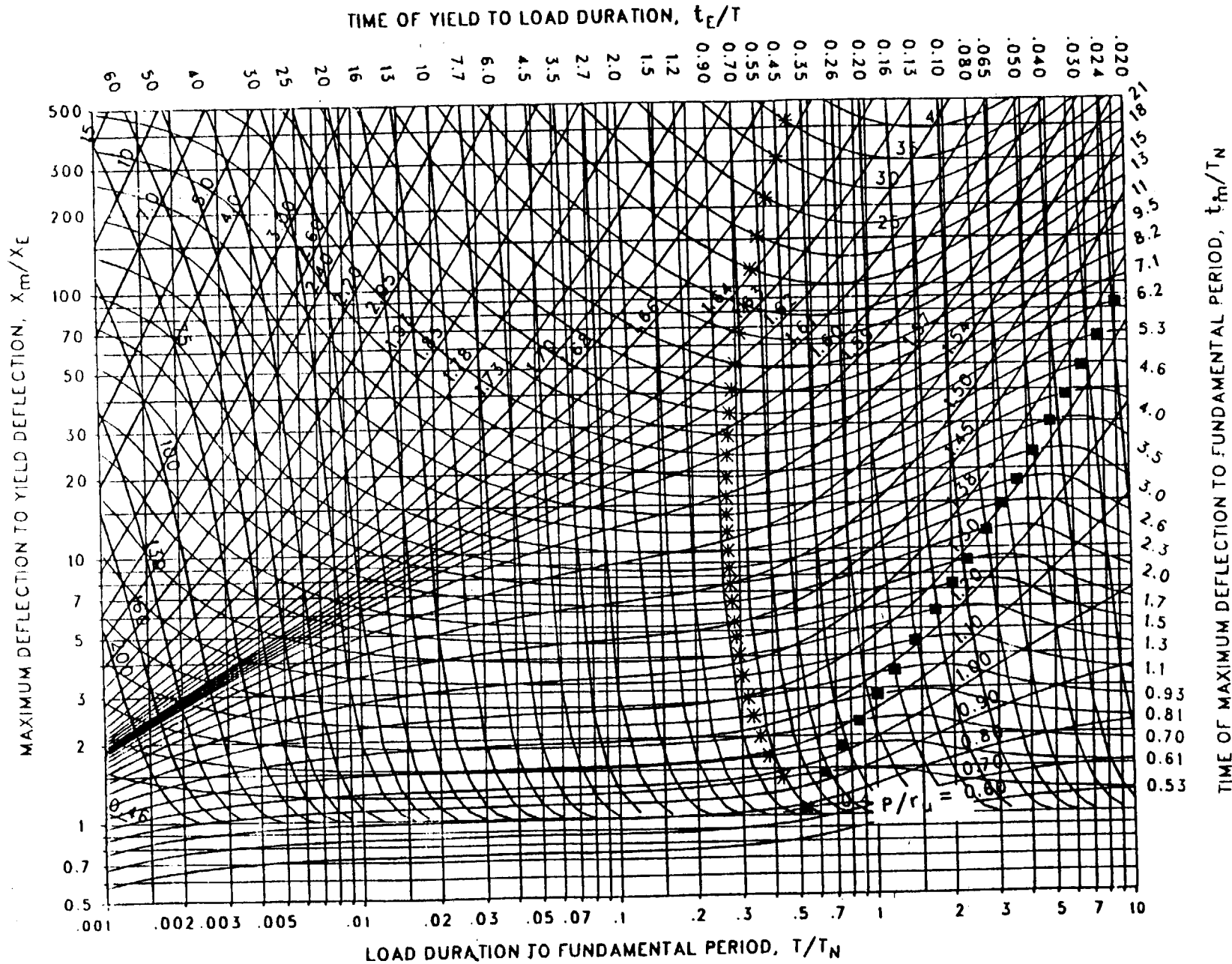


Figure 3-228 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.619$ ,  $C_2 = 1000$ .)

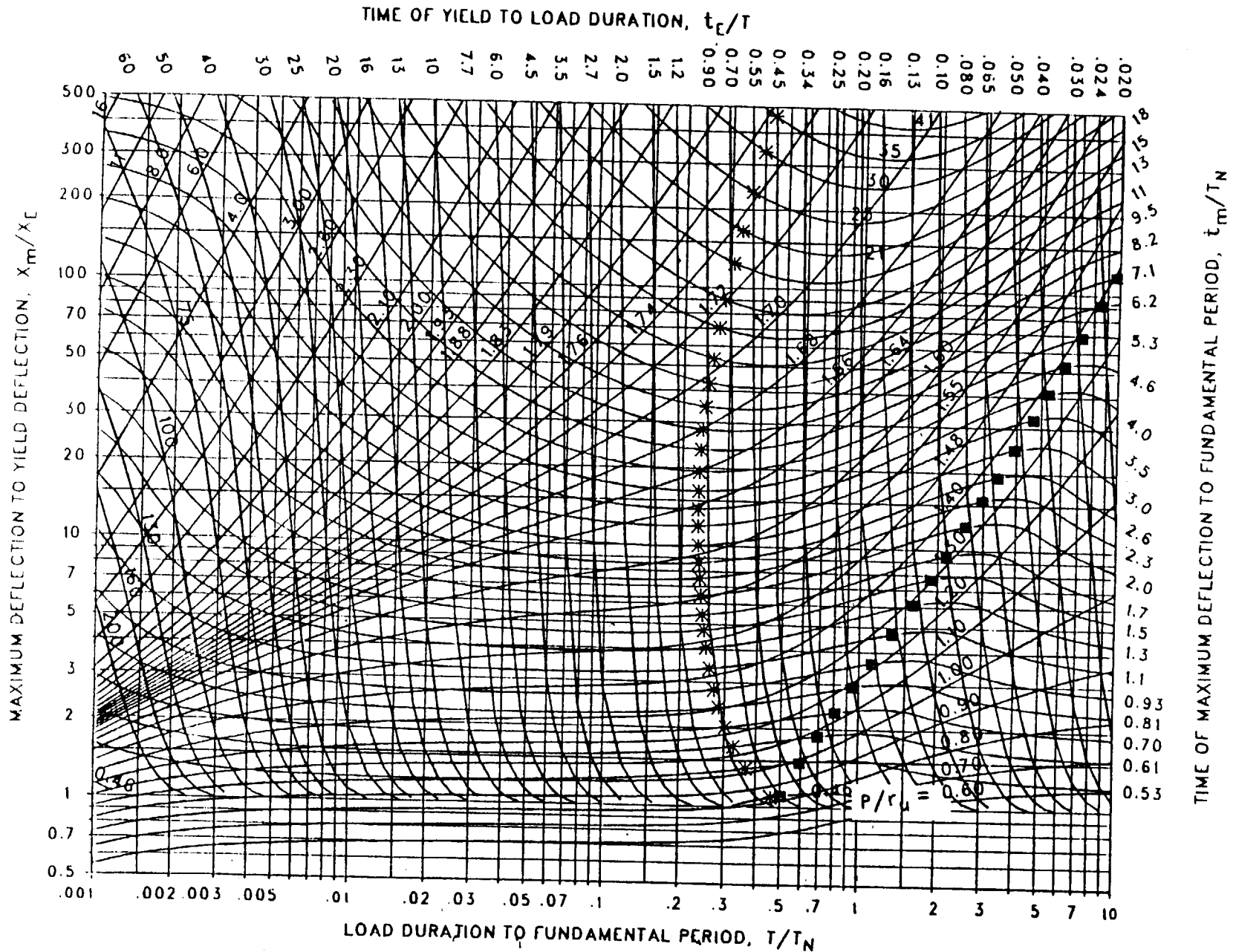


Figure 3-229 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.590$ ,  $C_2 = 1000$ .)

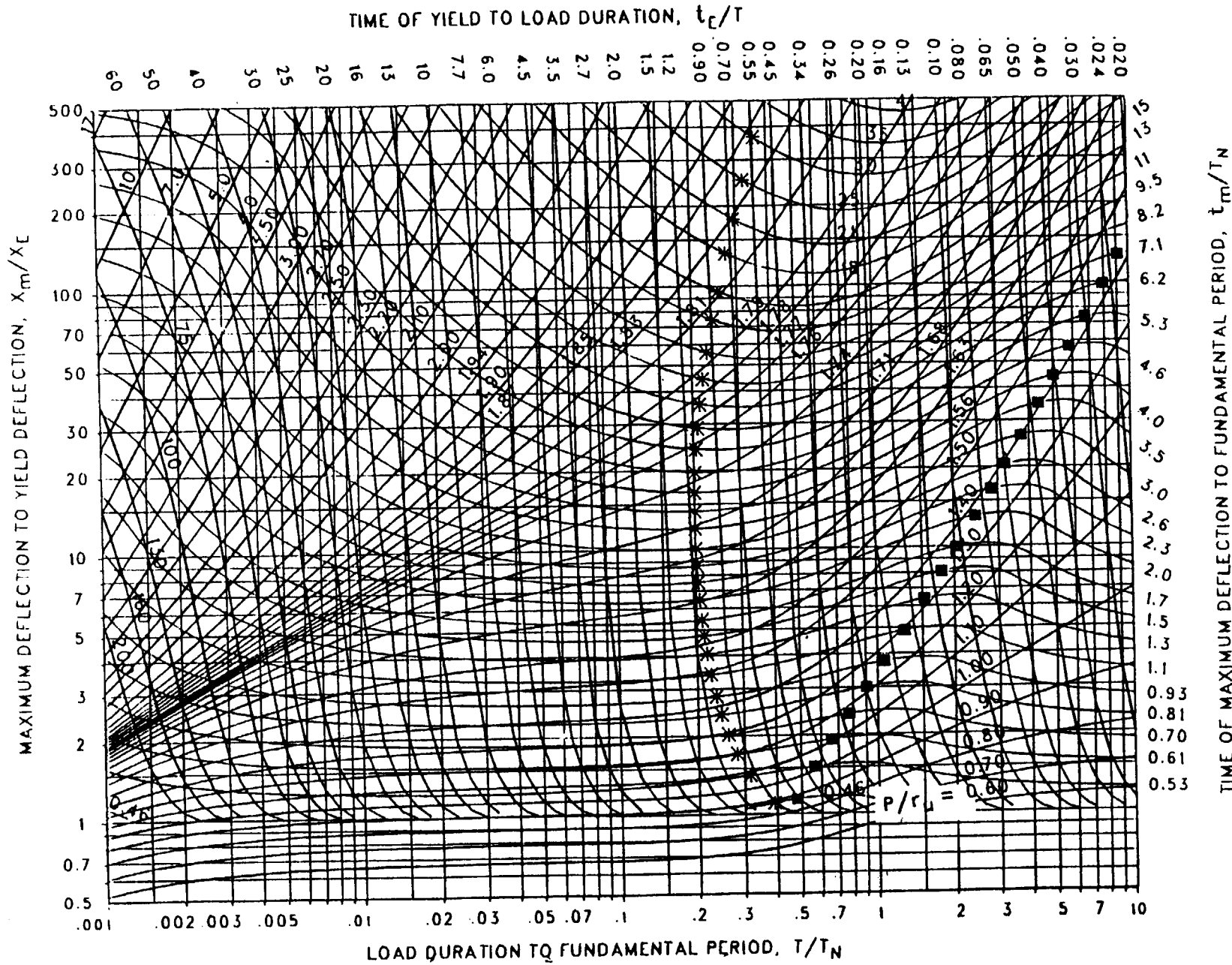


Figure 3-230 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.562$ ,  $C_2 = 1000$ .)

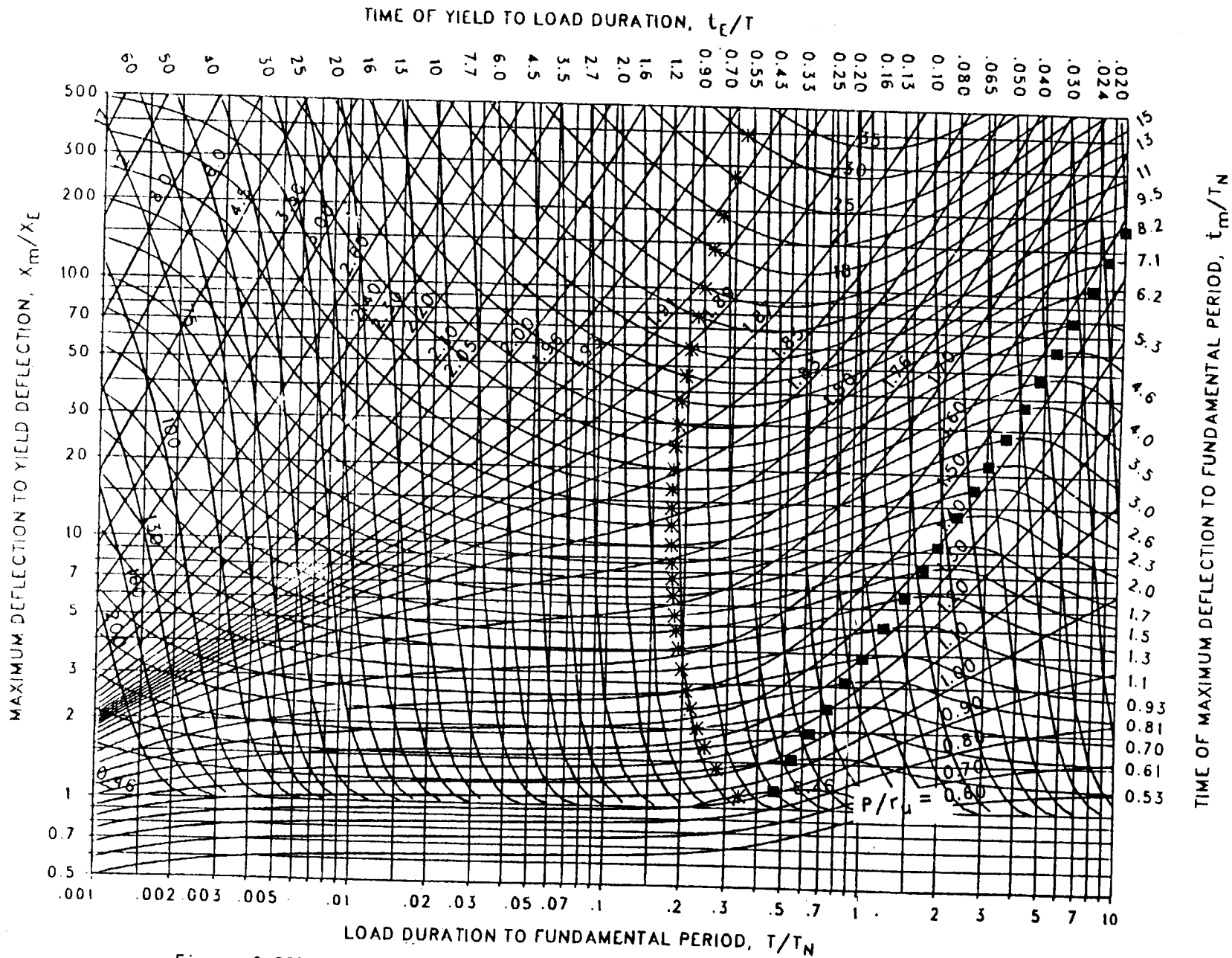


Figure 3-231 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.536$ ,  $C_2 = 1000$ .)



3-290

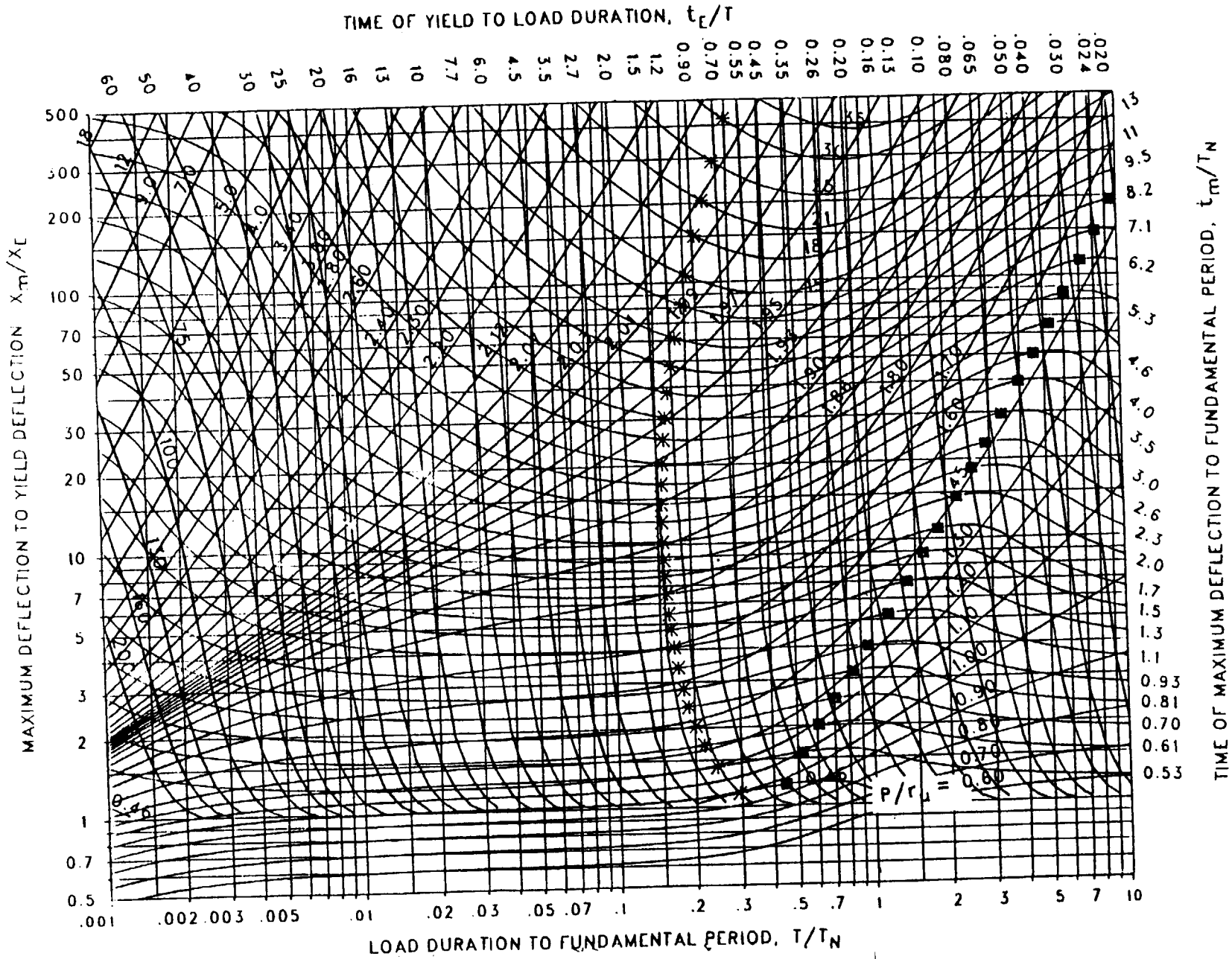


Figure 3-232 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.511$ ,  $C_2 = 1000$ .)

3-291

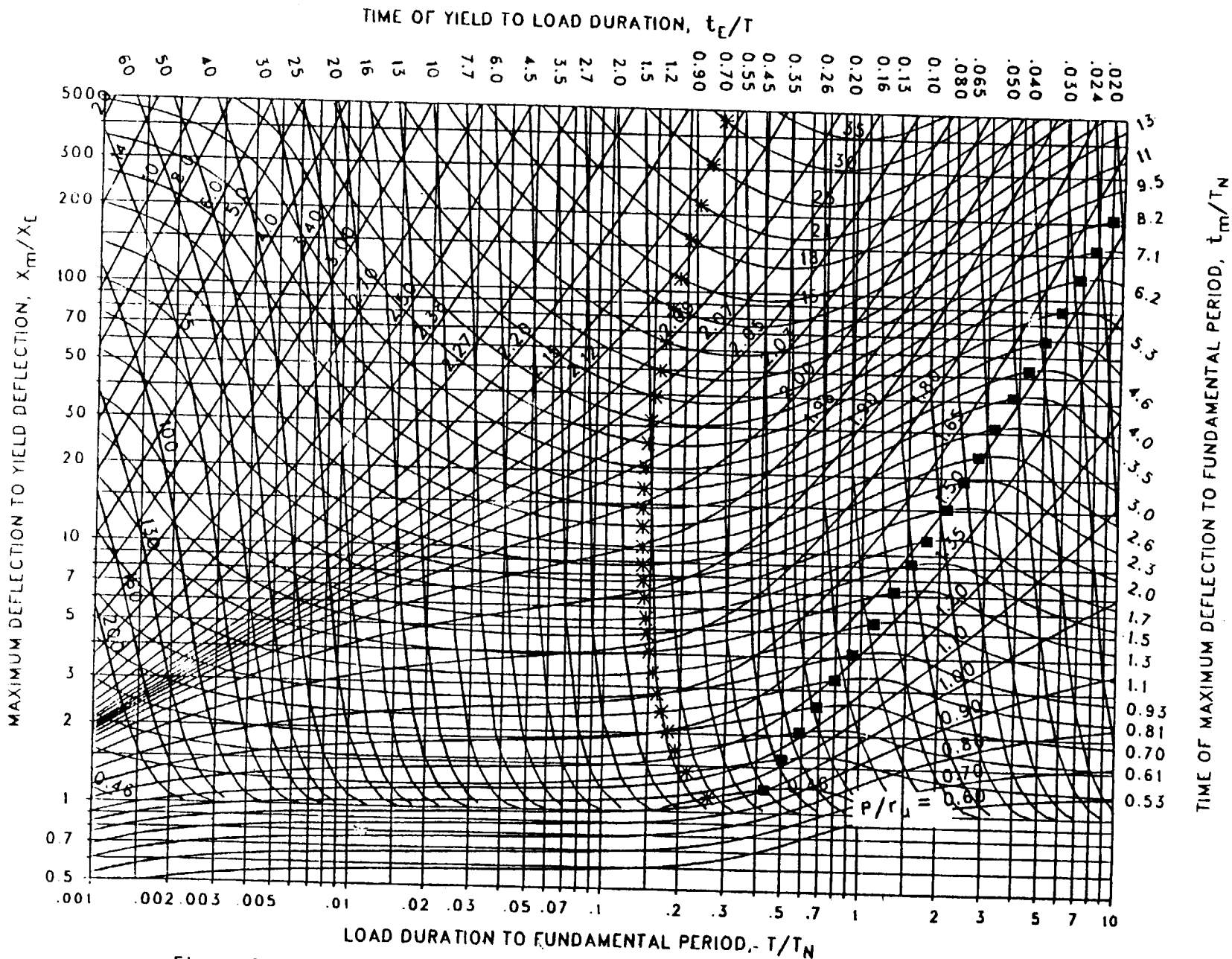


Figure 3-233 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.487$ ,  $C_2 = 1000$ .)

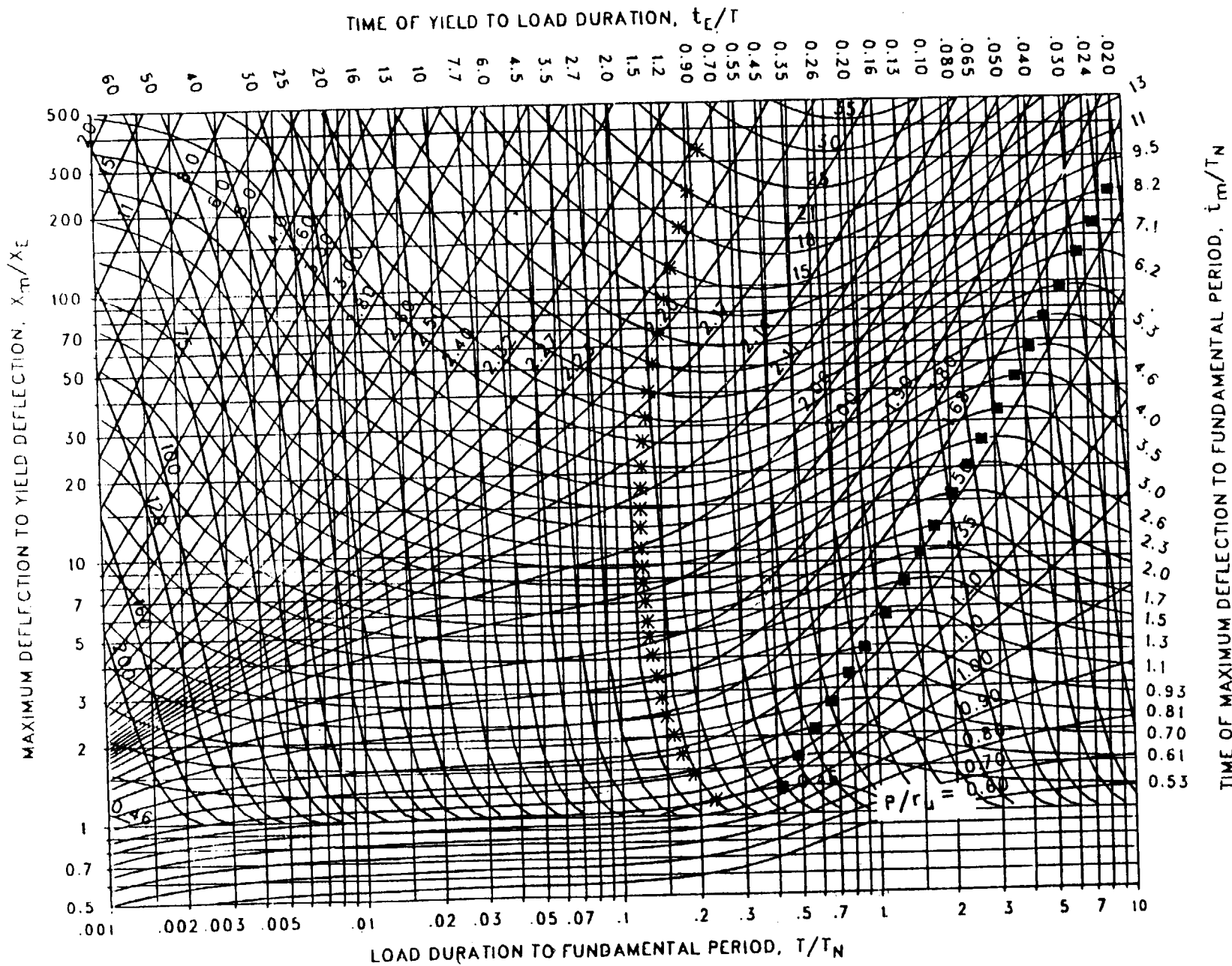


Figure 3-234 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.464$ ,  $C_2 = 1000$ .)

3-293

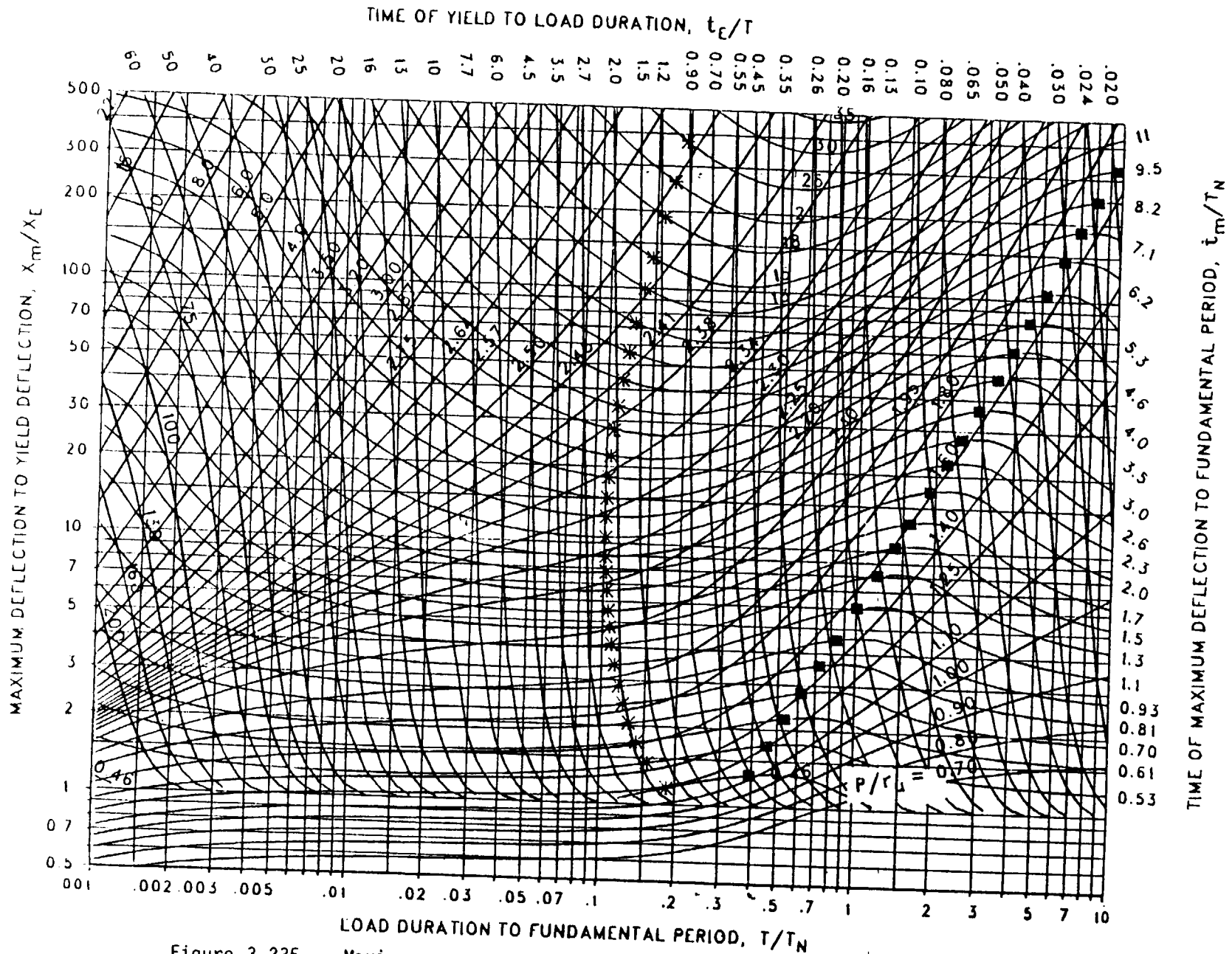


Figure 3-235

Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.422$ ,  $C_2 = 1000$ .)

3-294

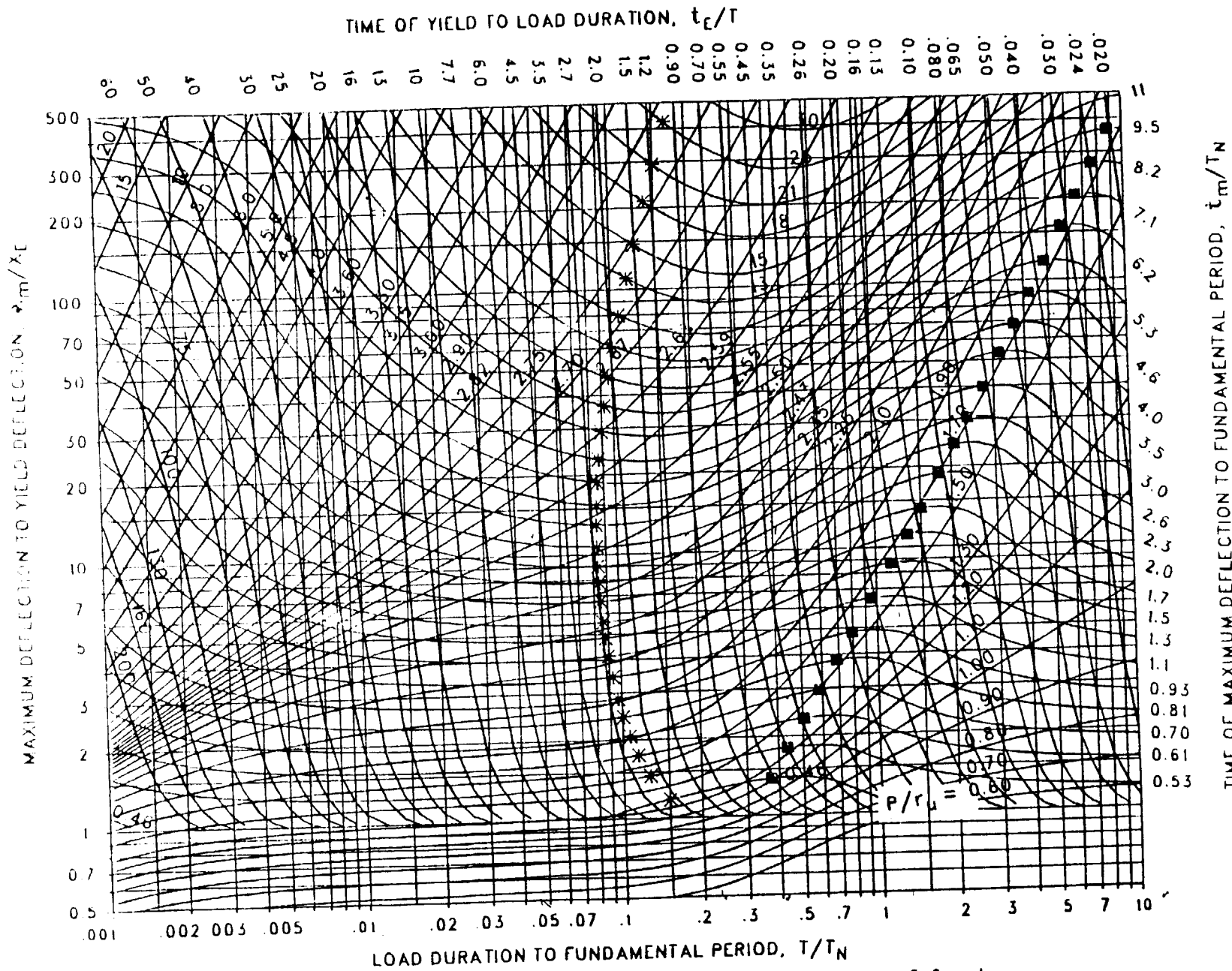


Figure 3-236 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.383$ ,  $C_2 = 1000$ .)

3-295

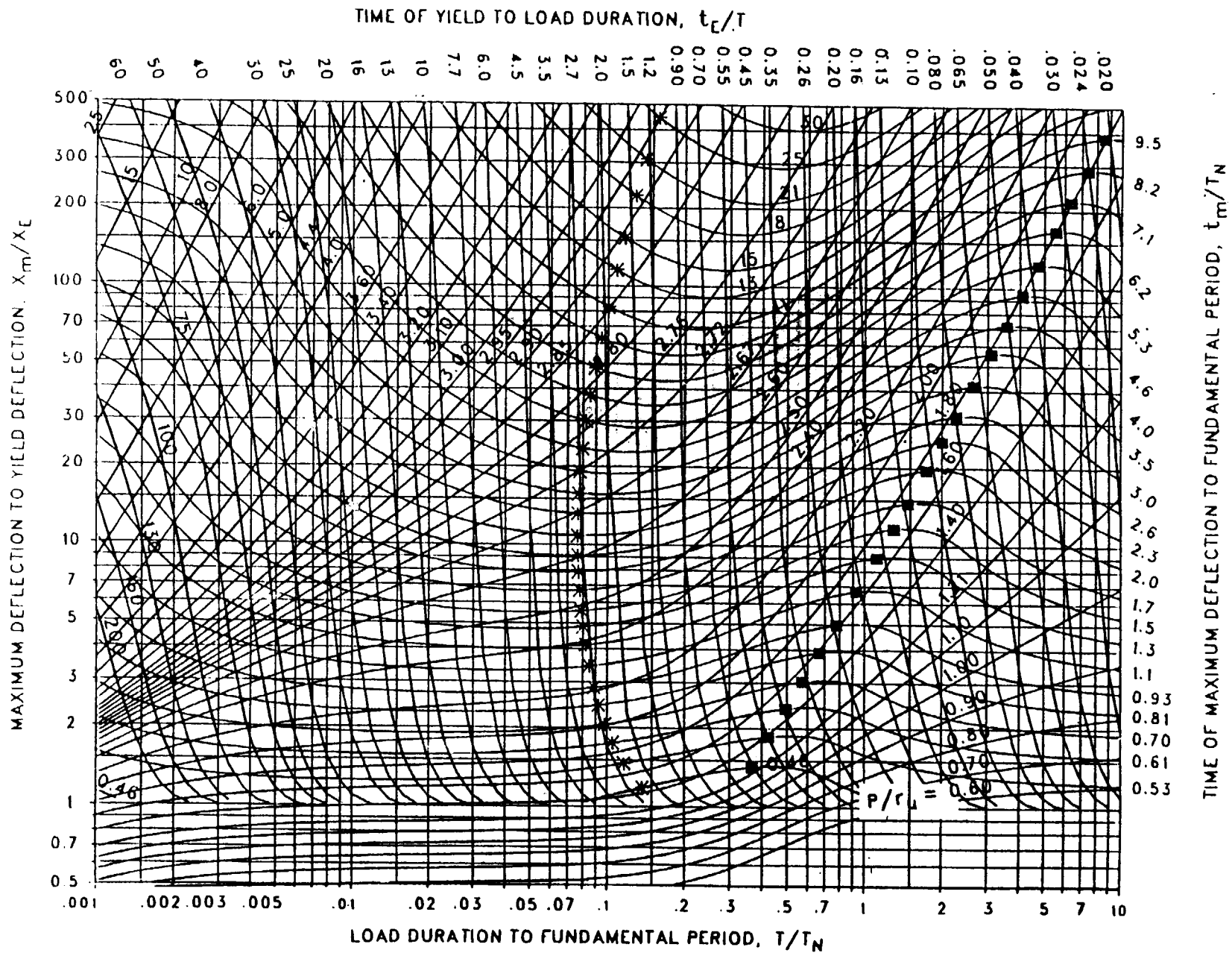


Figure 3-237 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.365$ ,  $C_2 = 1000$ .)

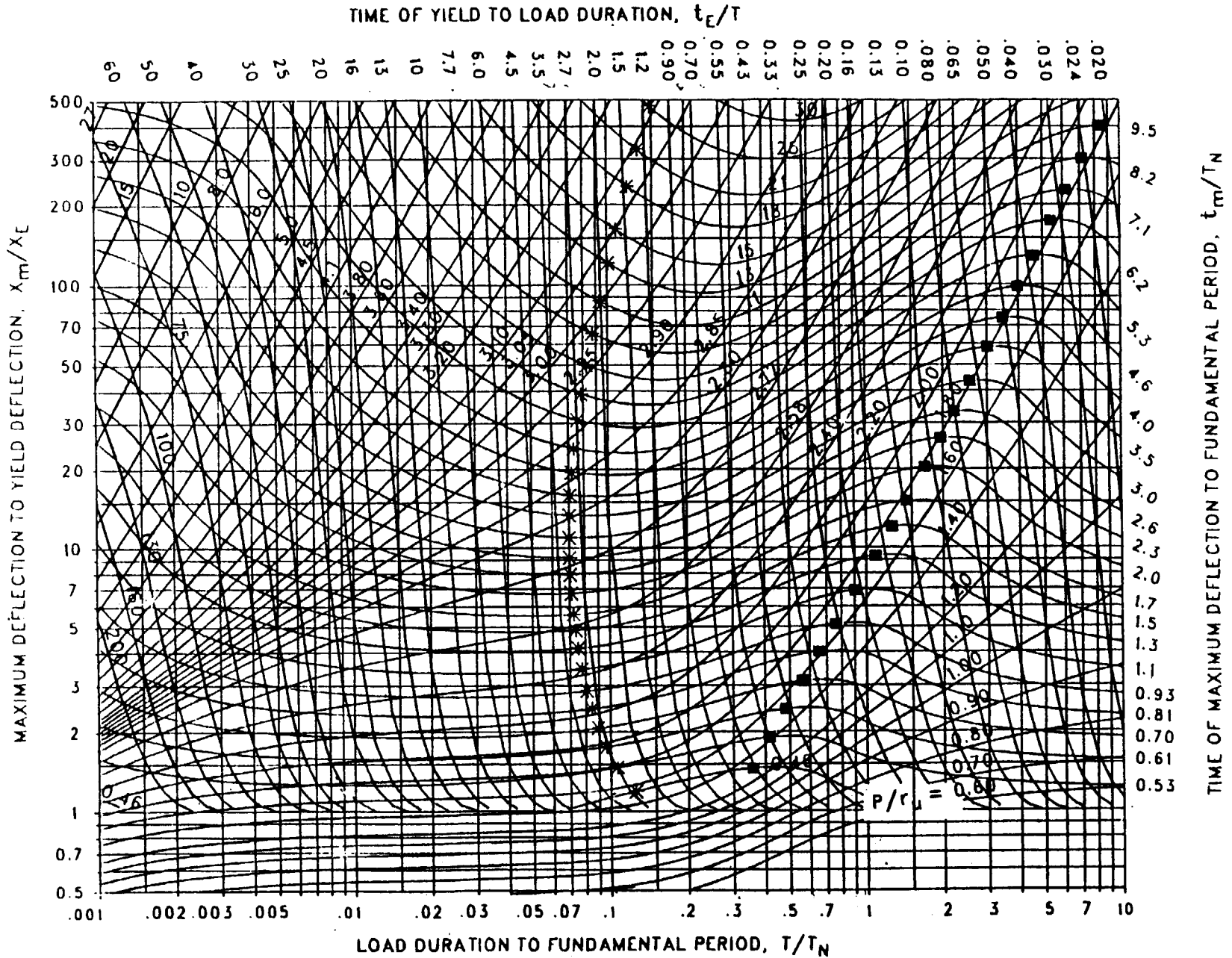


Figure 3-238 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.348$ ,  $C_2 = 1000$ .)

3-297

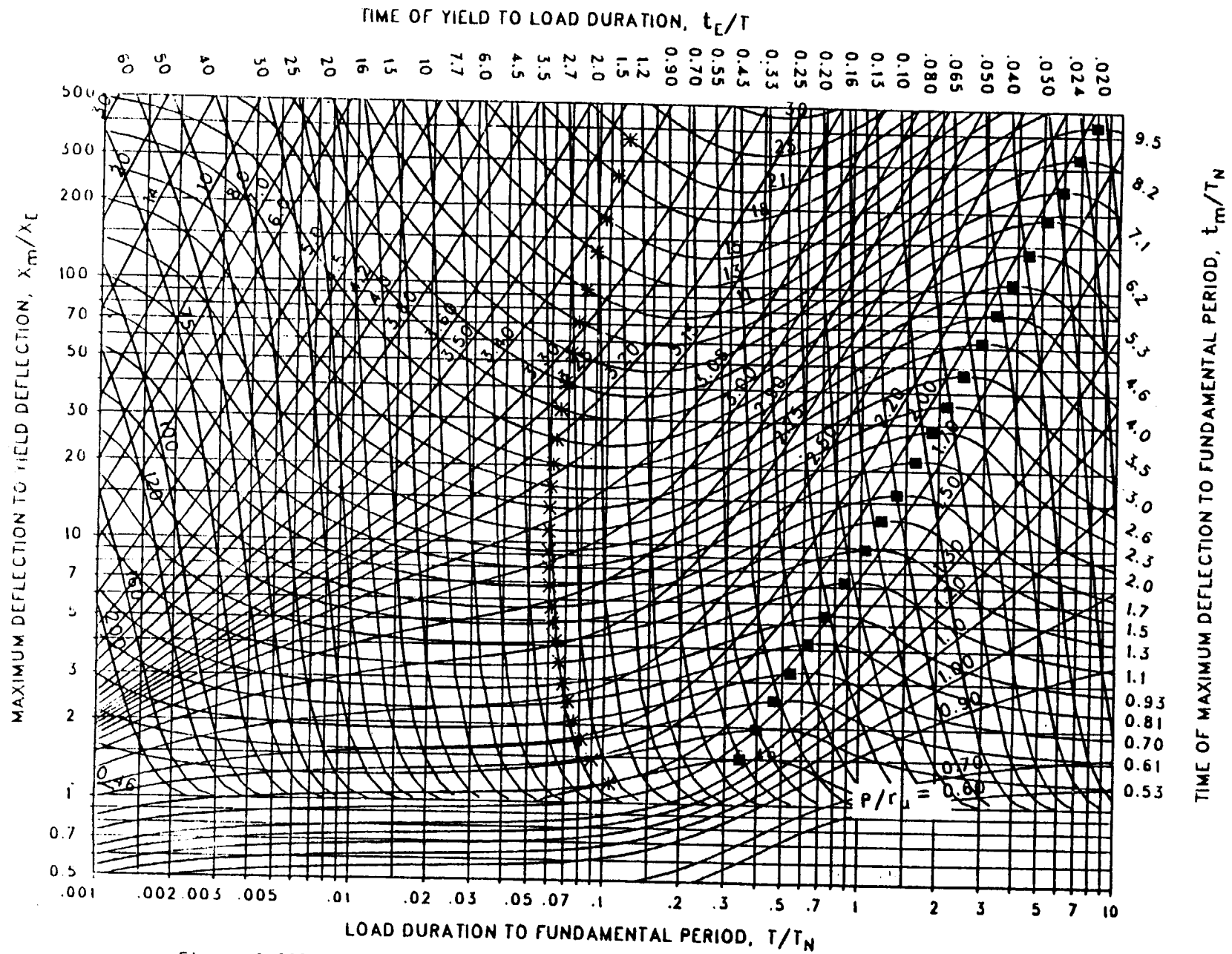


Figure 3-239

Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.316$ ,  $C_2 = 1000$ .)



3-298

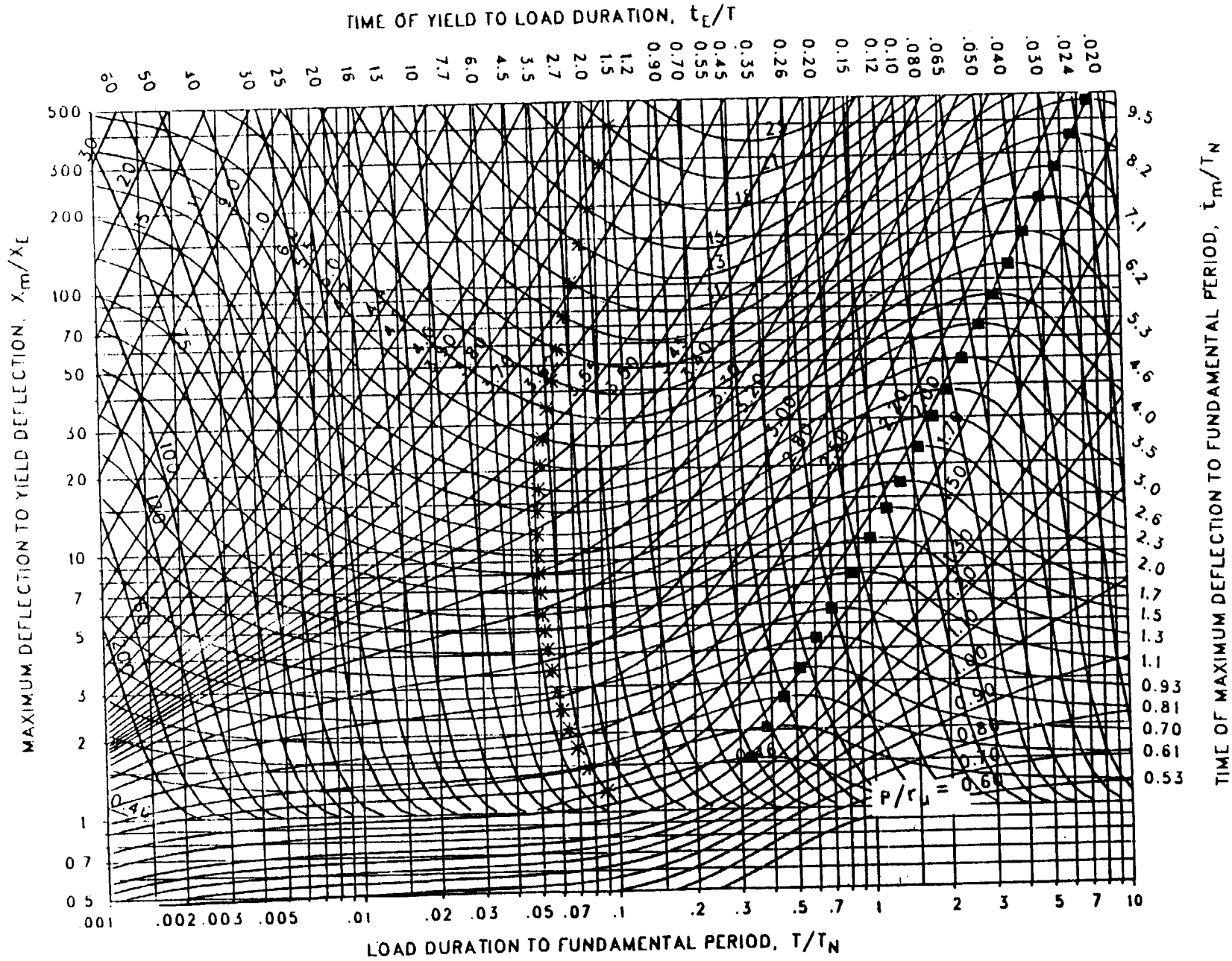


Figure 3-240 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.287$ ,  $C_2 = 1000$ .)

3-299

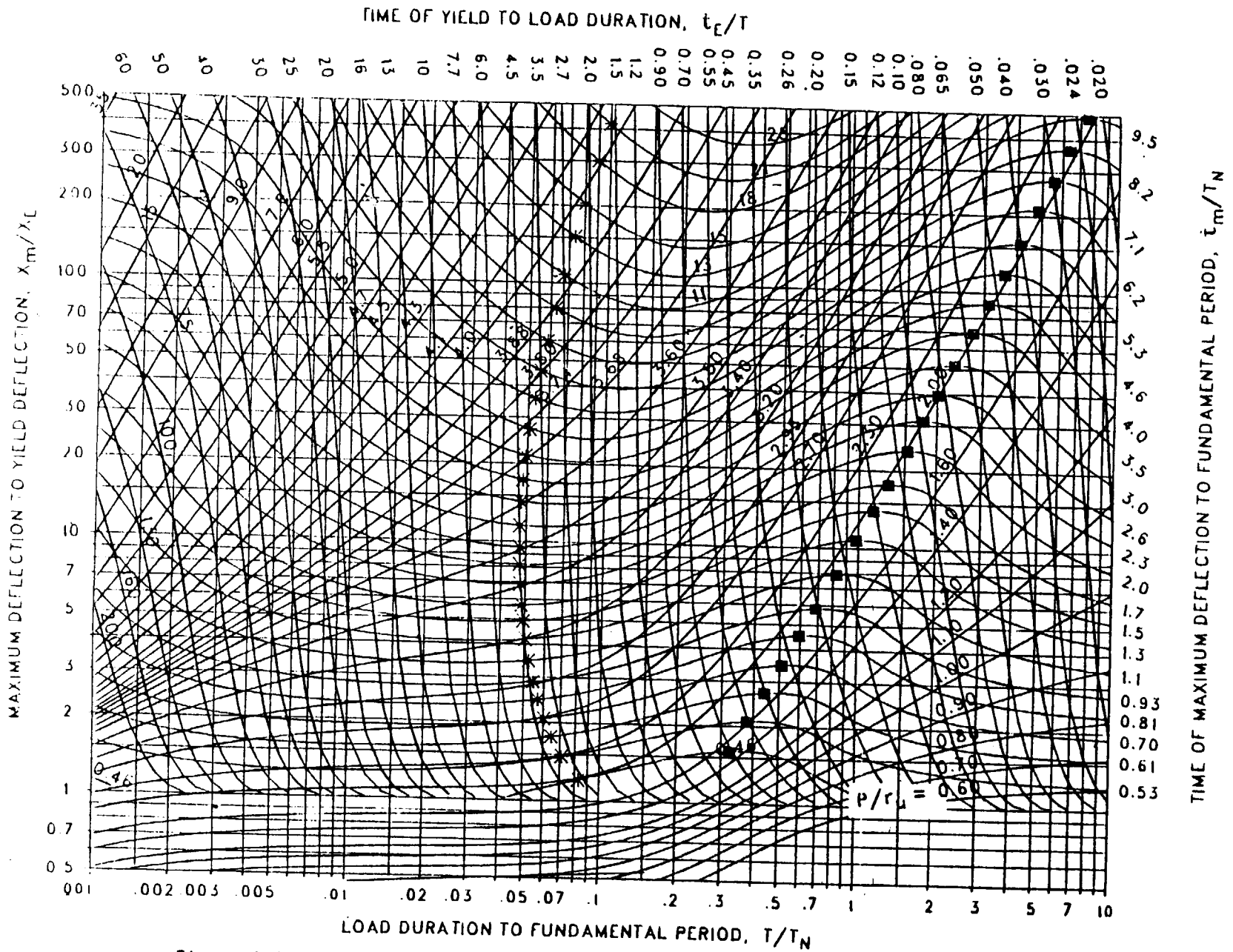


Figure 3-241 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.274$ ,  $C_2 = 1000$ .)

3-300

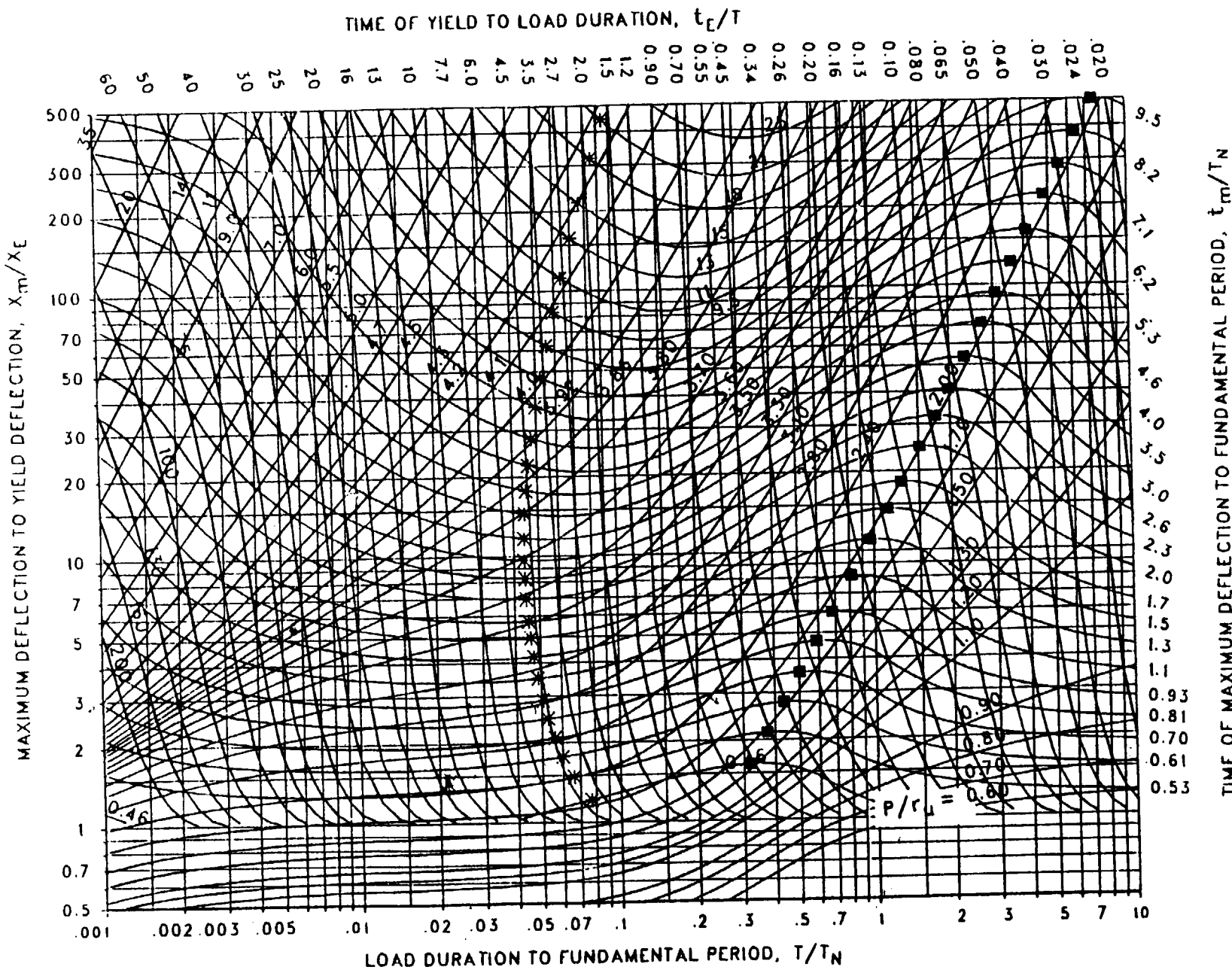


Figure 3-242 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.261$ ,  $C_2 = 1000$ .)

3-301

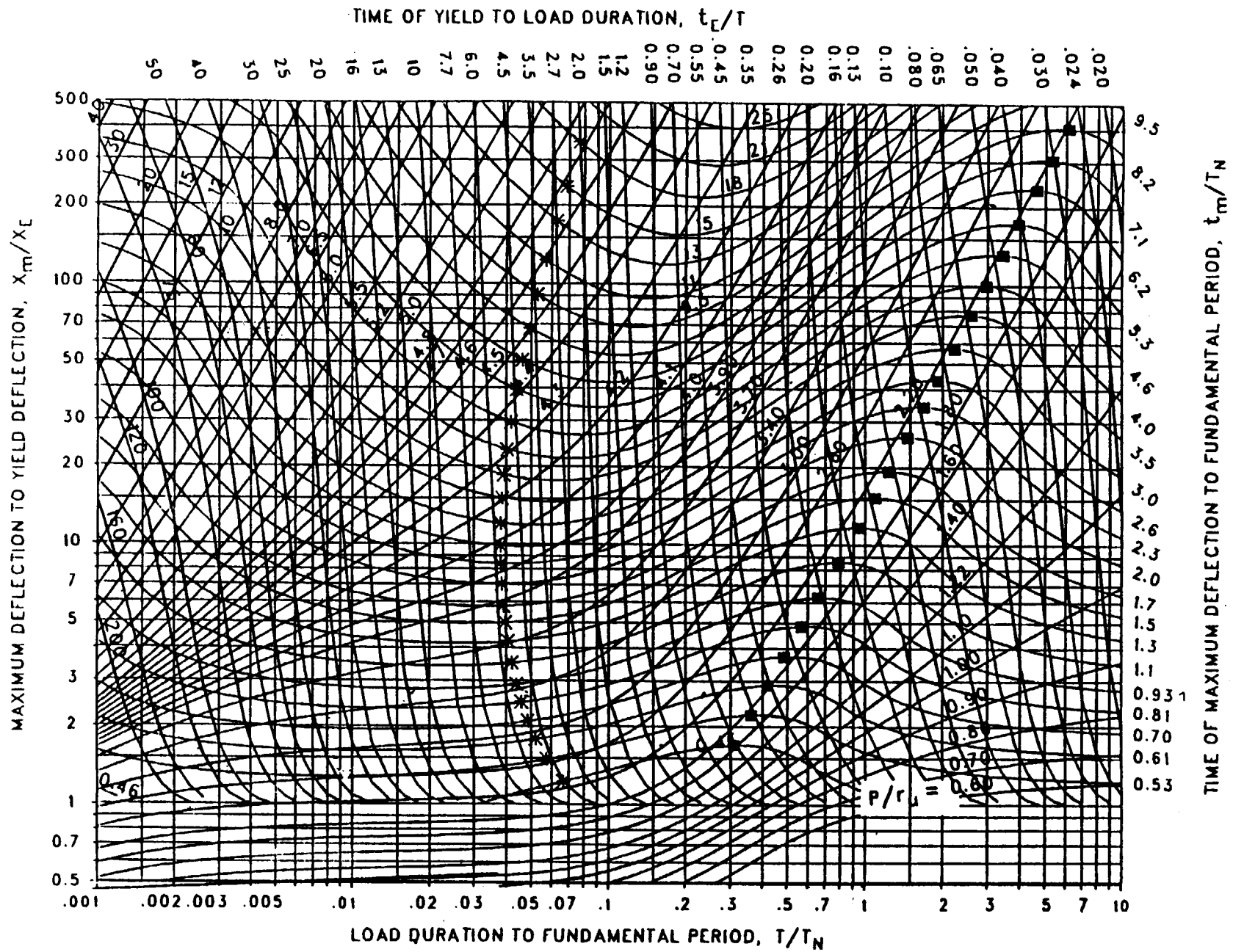


Figure 3-243 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.237$ ,  $C_2 = 1000.$ )

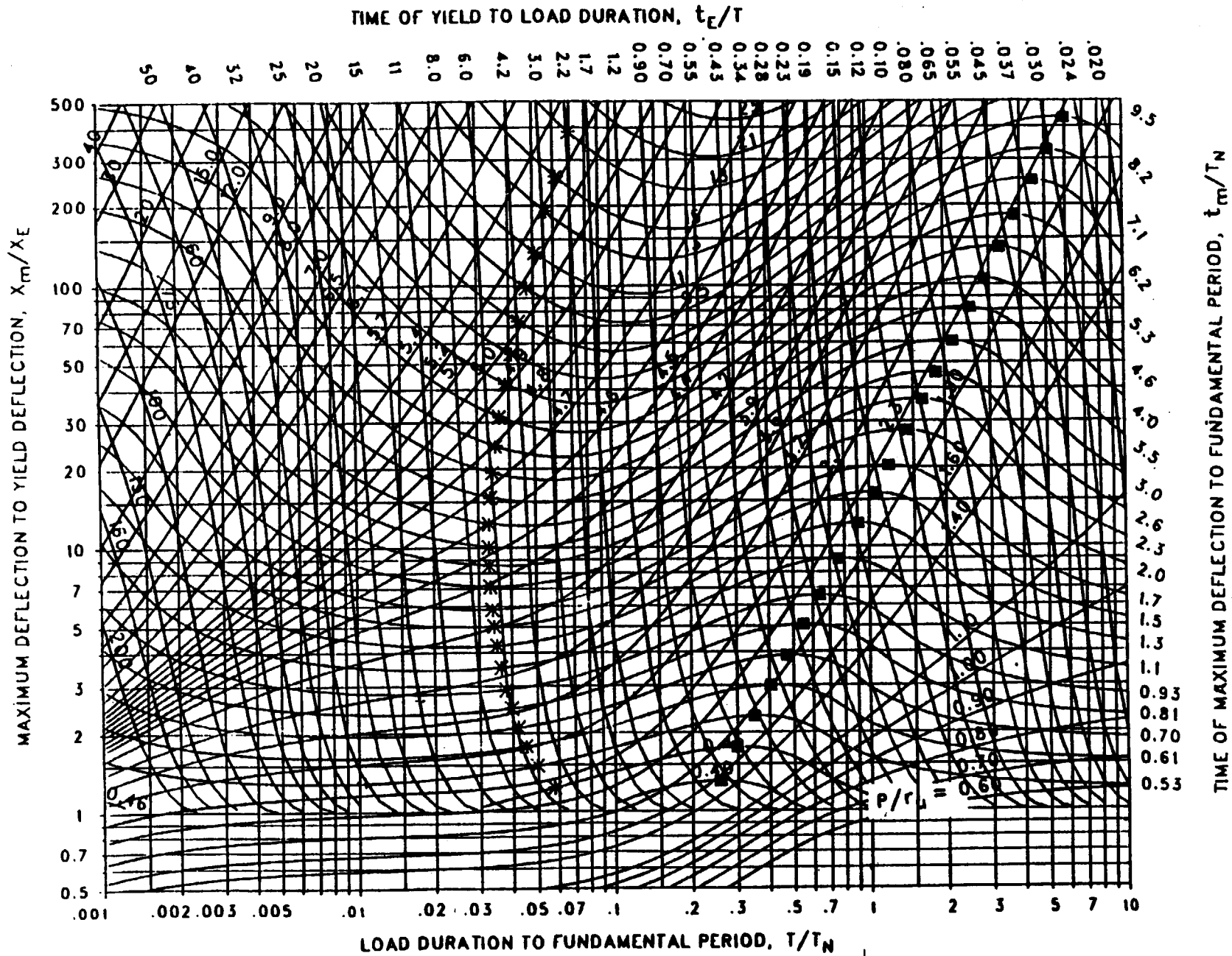


Figure 3-244 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.215$ ,  $C_2 = 1000$ .)

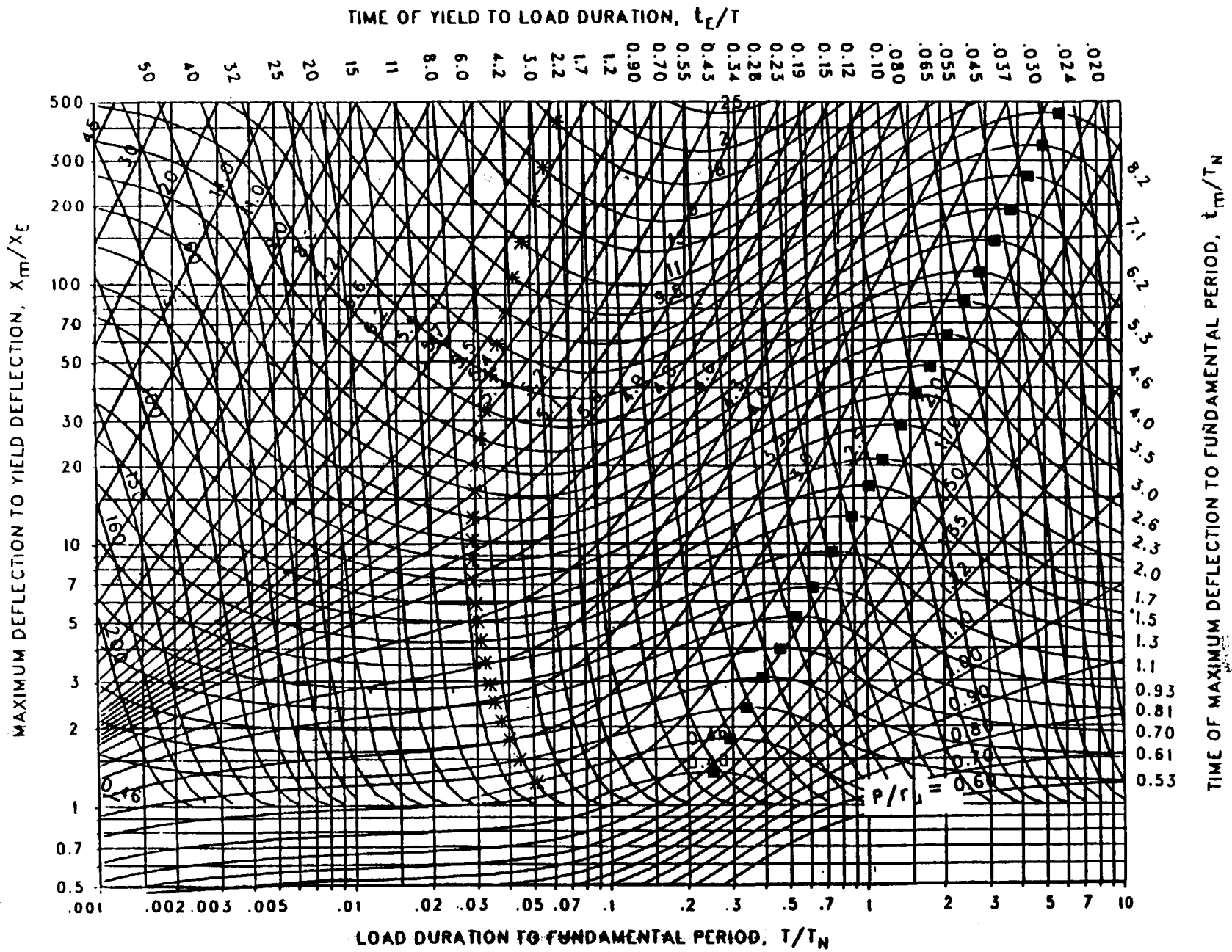


Figure 3-245 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.198$ ,  $C_2 = 1000$ .)

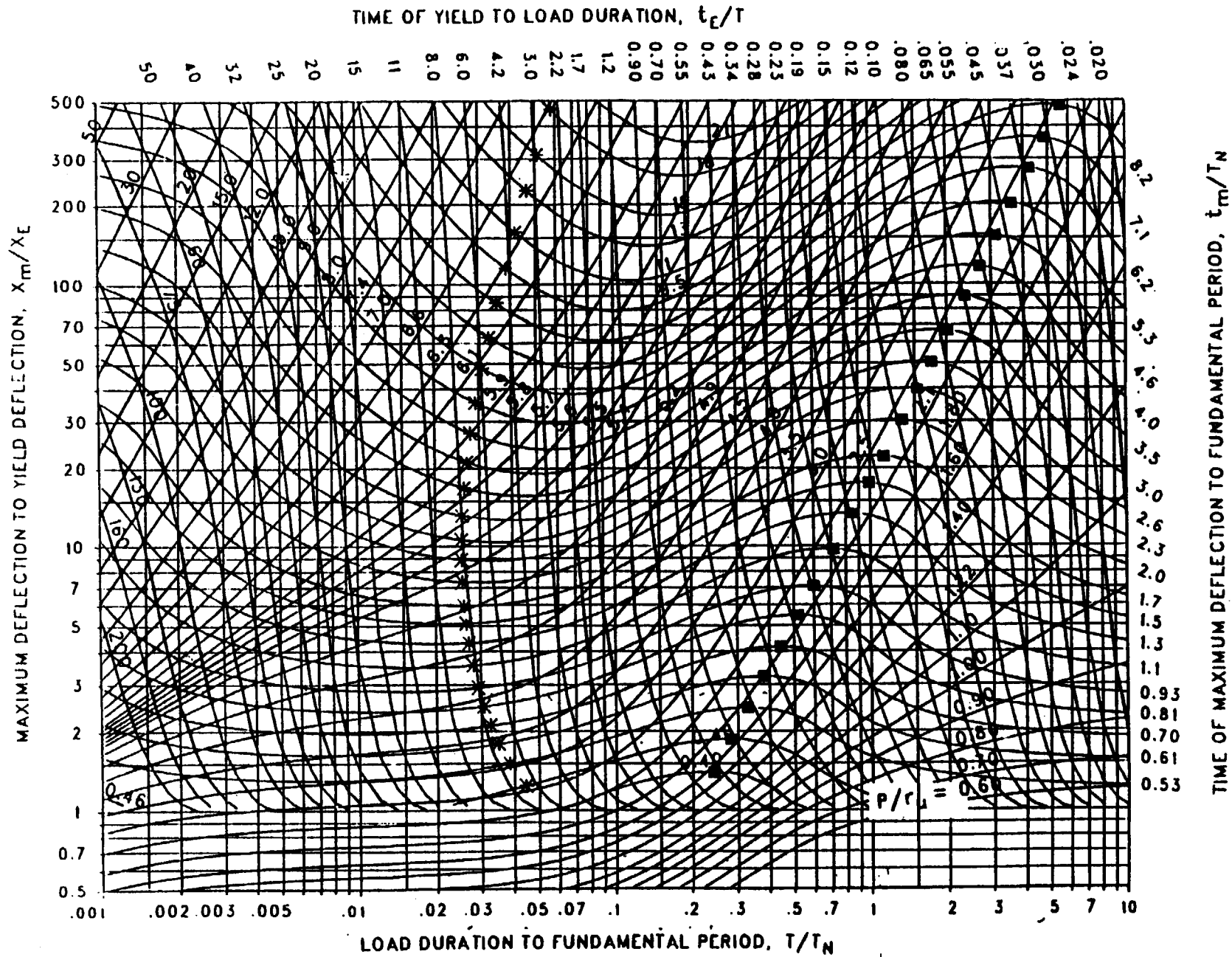


Figure 3-246 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.178$ ,  $C_2 = 1000$ .)

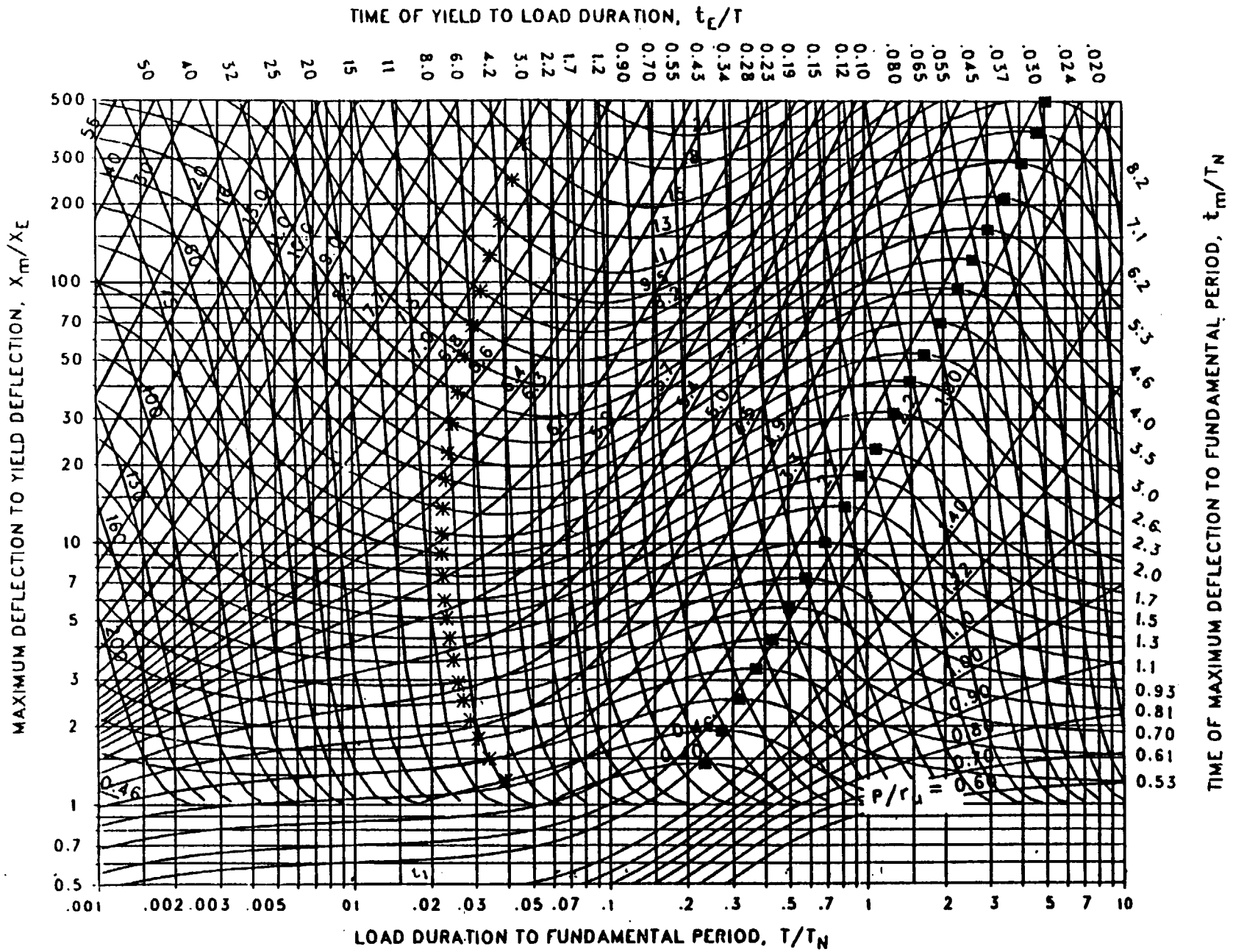


Figure 3-247 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.162$ ,  $C_2 = 1000$ .)



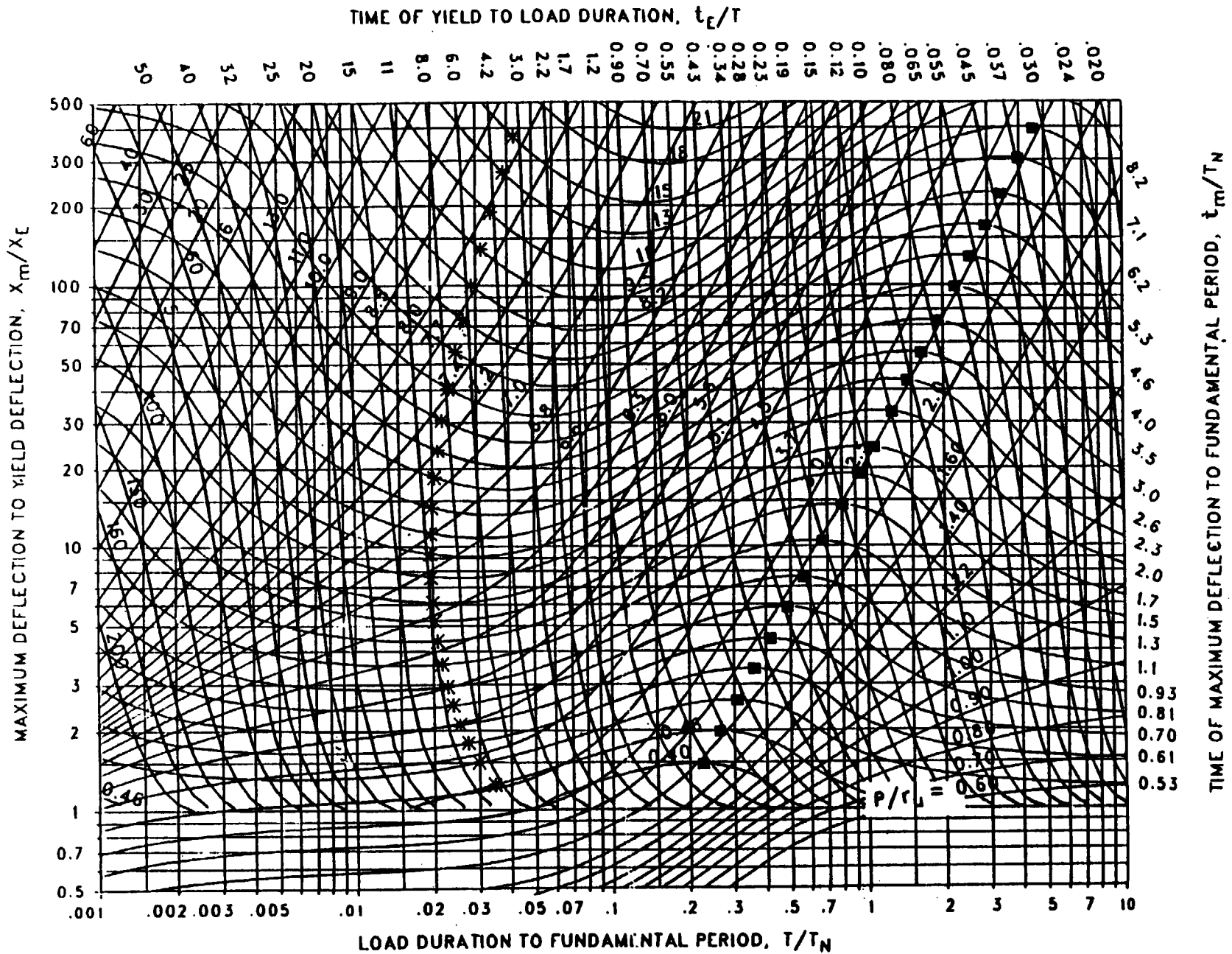


Figure 3-248 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.147$ ,  $C_2 = 1000$ .)

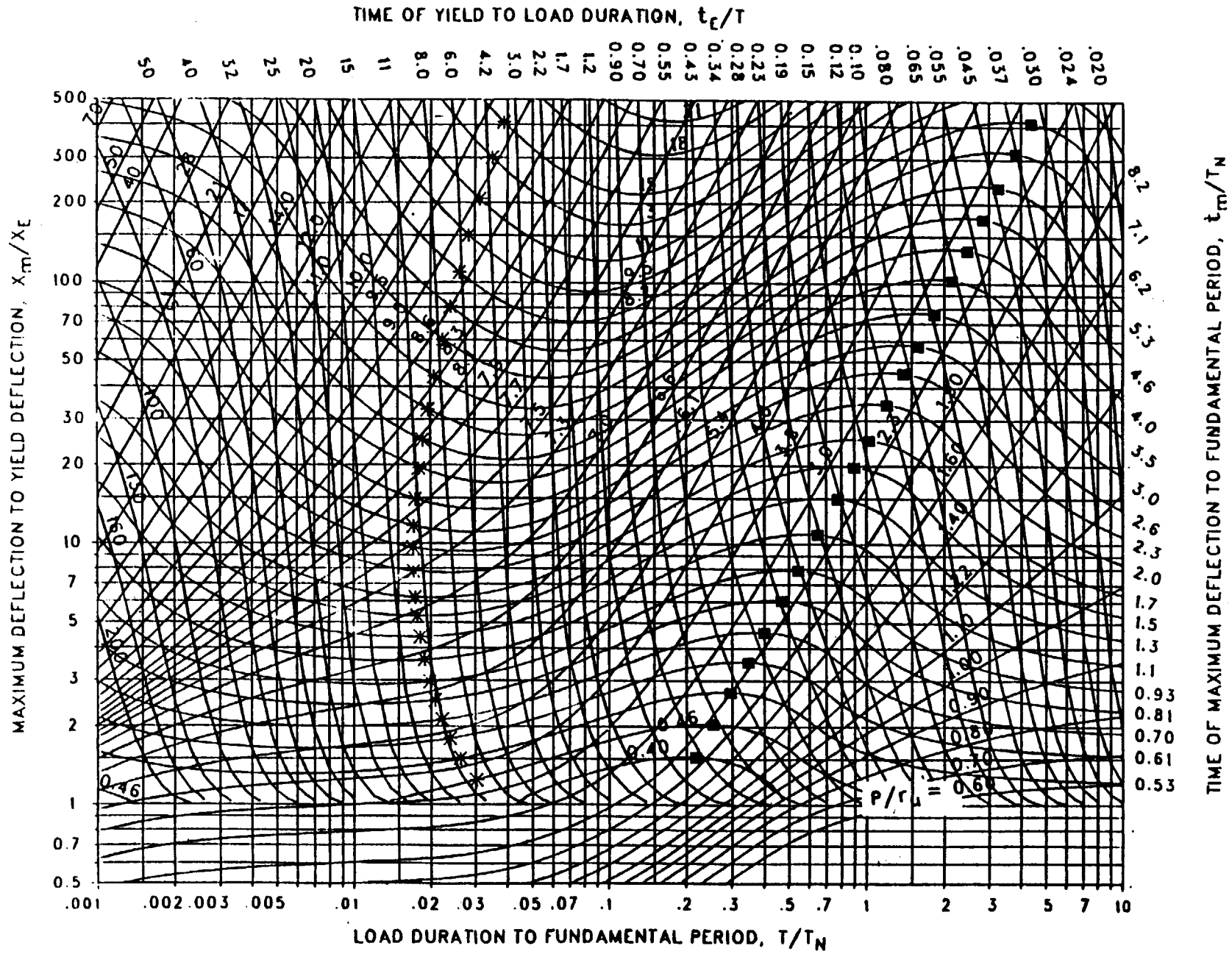


Figure 3-249 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.133$ ,  $C_2 = 1000$ .)

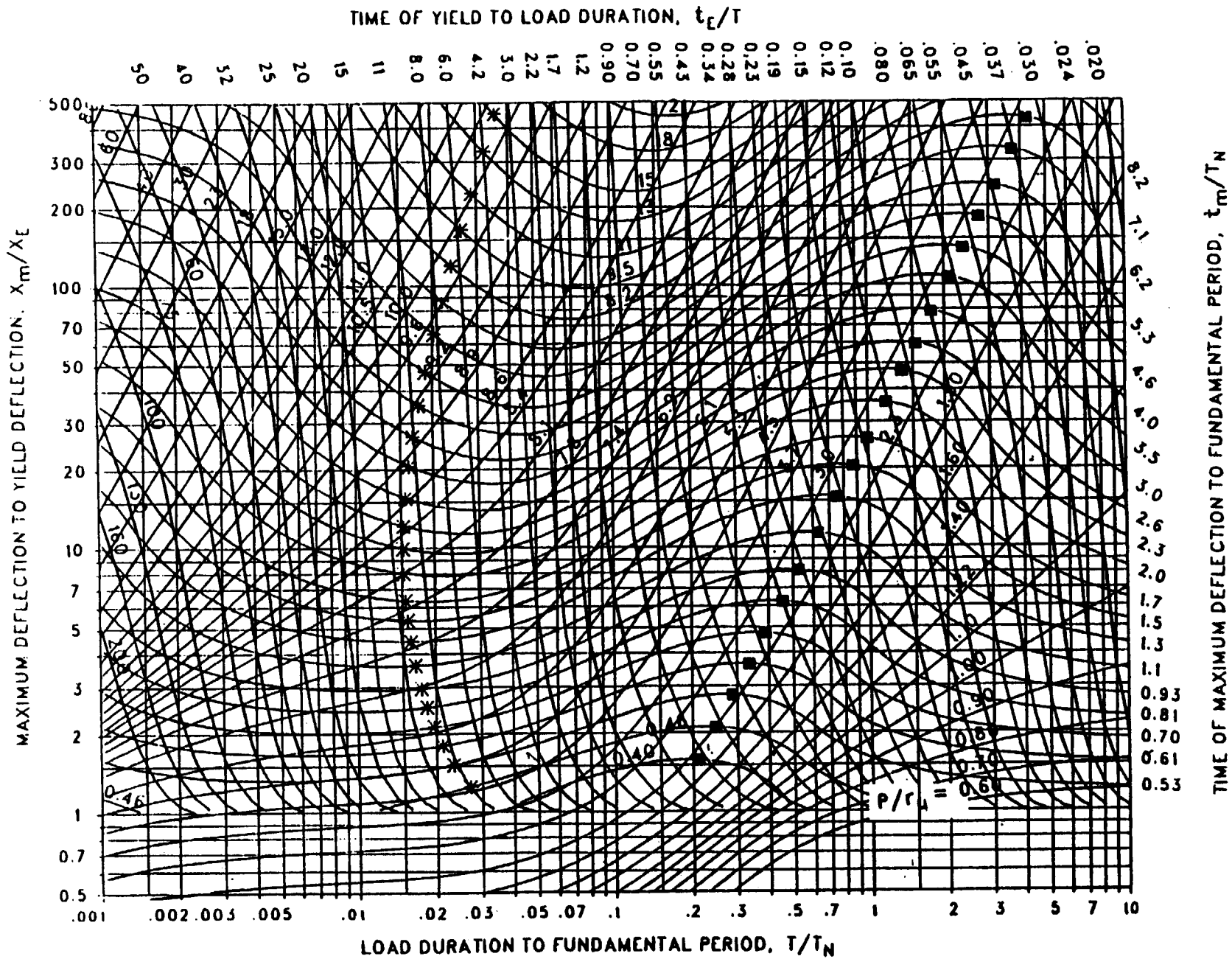


Figure 3-250 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.121$ ,  $C_2 = 1000.$ )

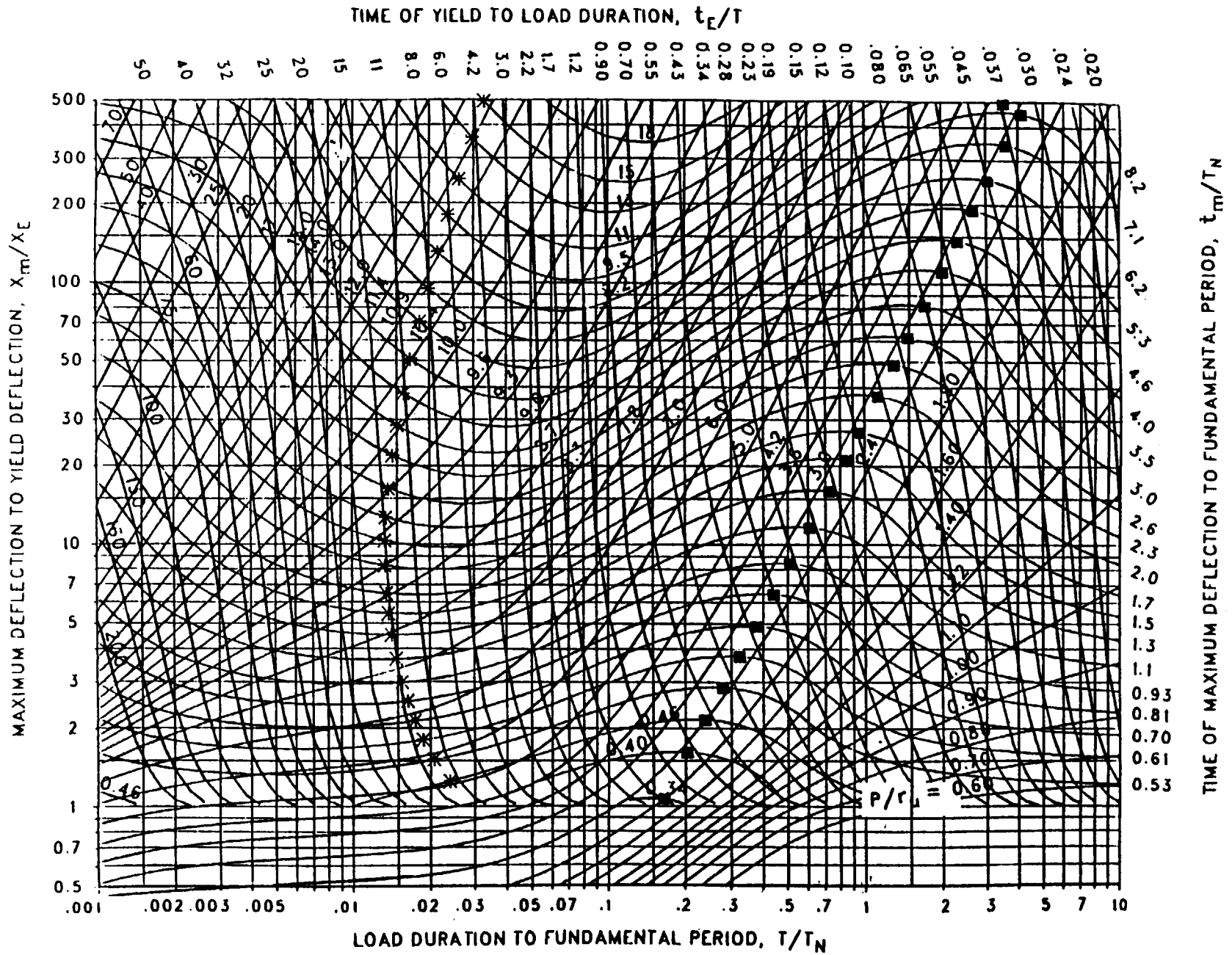


Figure 3-251 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.110$ ,  $C_2 = 1000$ .)

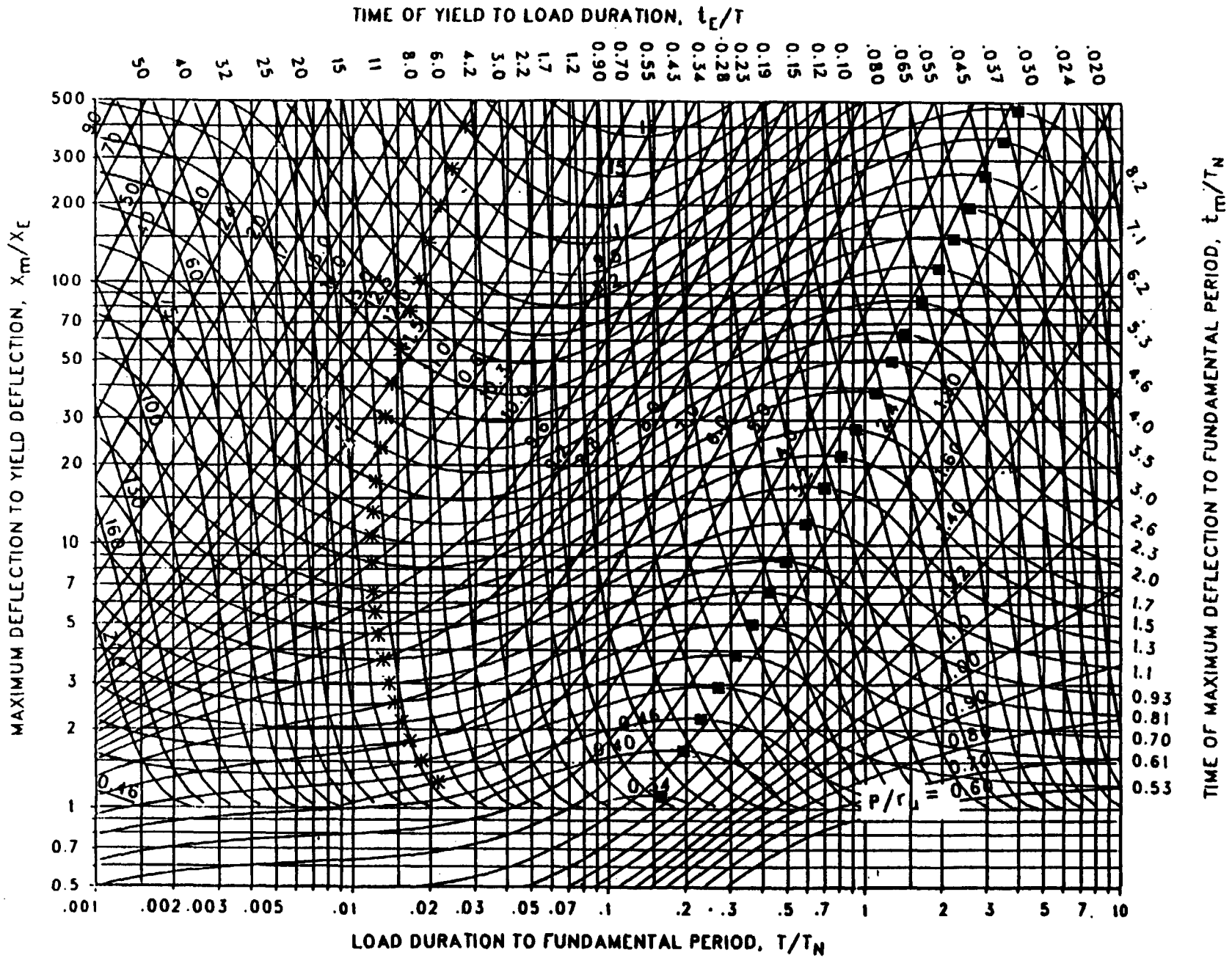


Figure 3-252 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.100$ ,  $C_2 = 1000$ .)

3-311

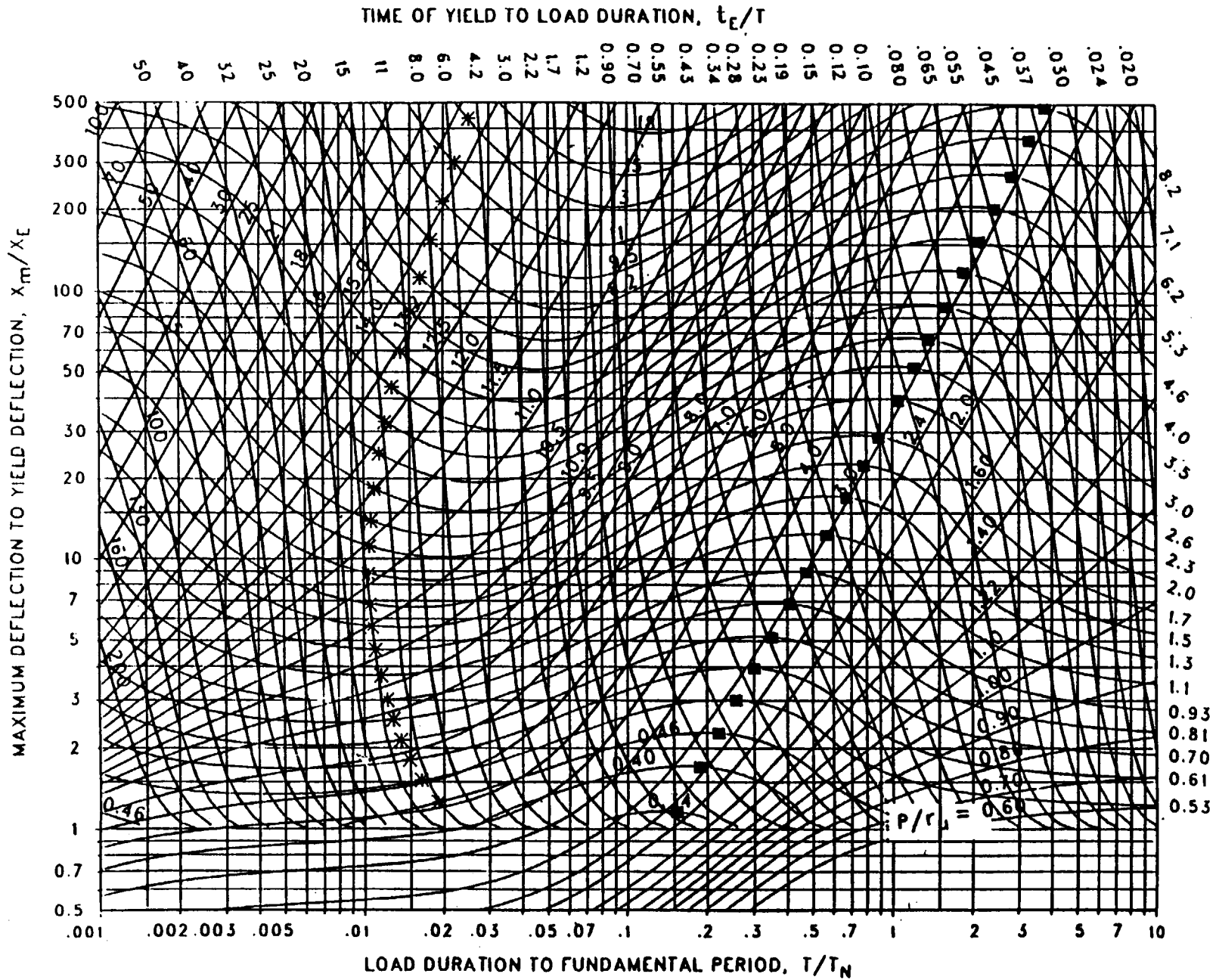


Figure 3-253 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.091$ ,  $C_2 = 1000$ .)

3-312

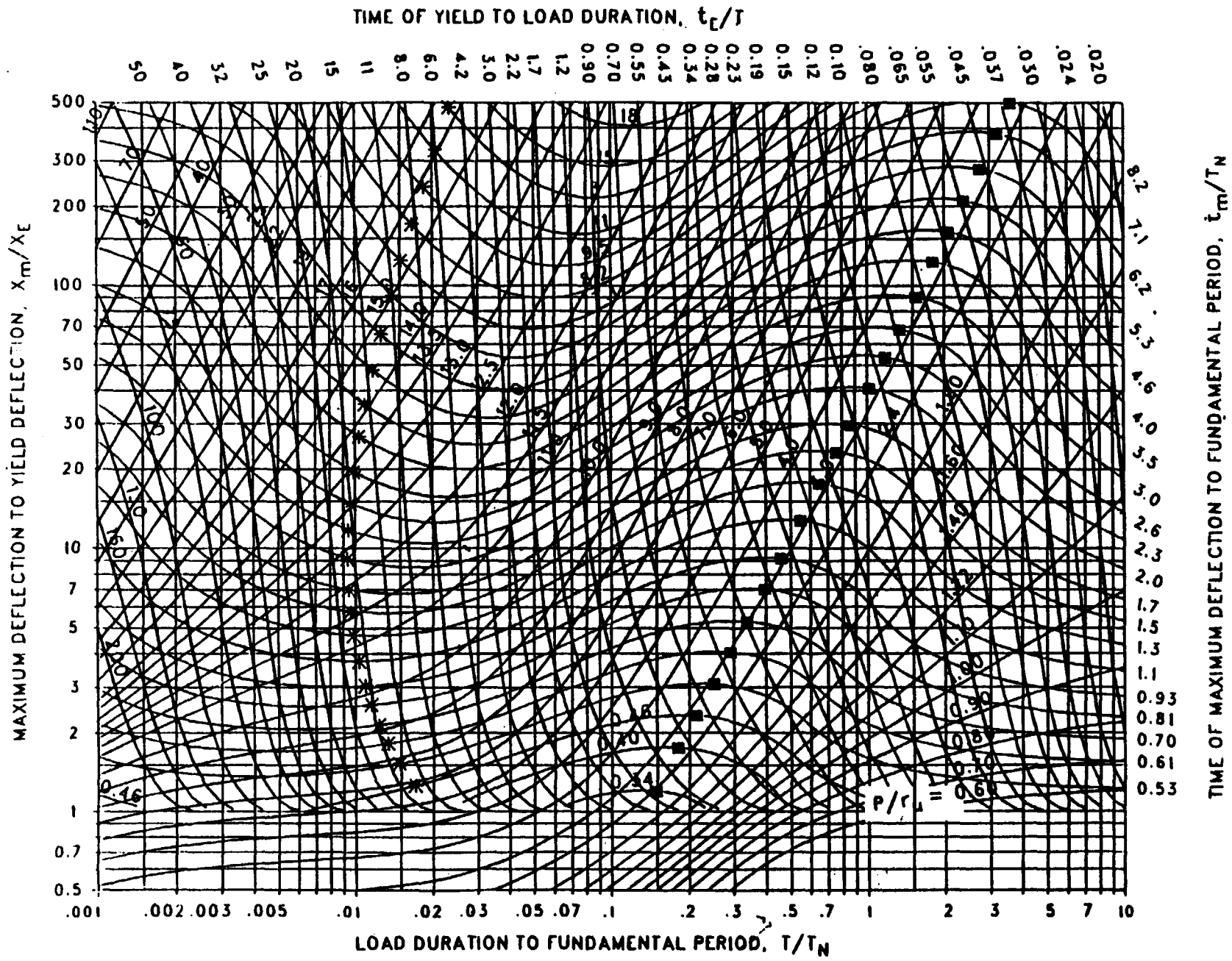


Figure 3-254 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.083$ ,  $C_2 = 1000$ .)

3-313

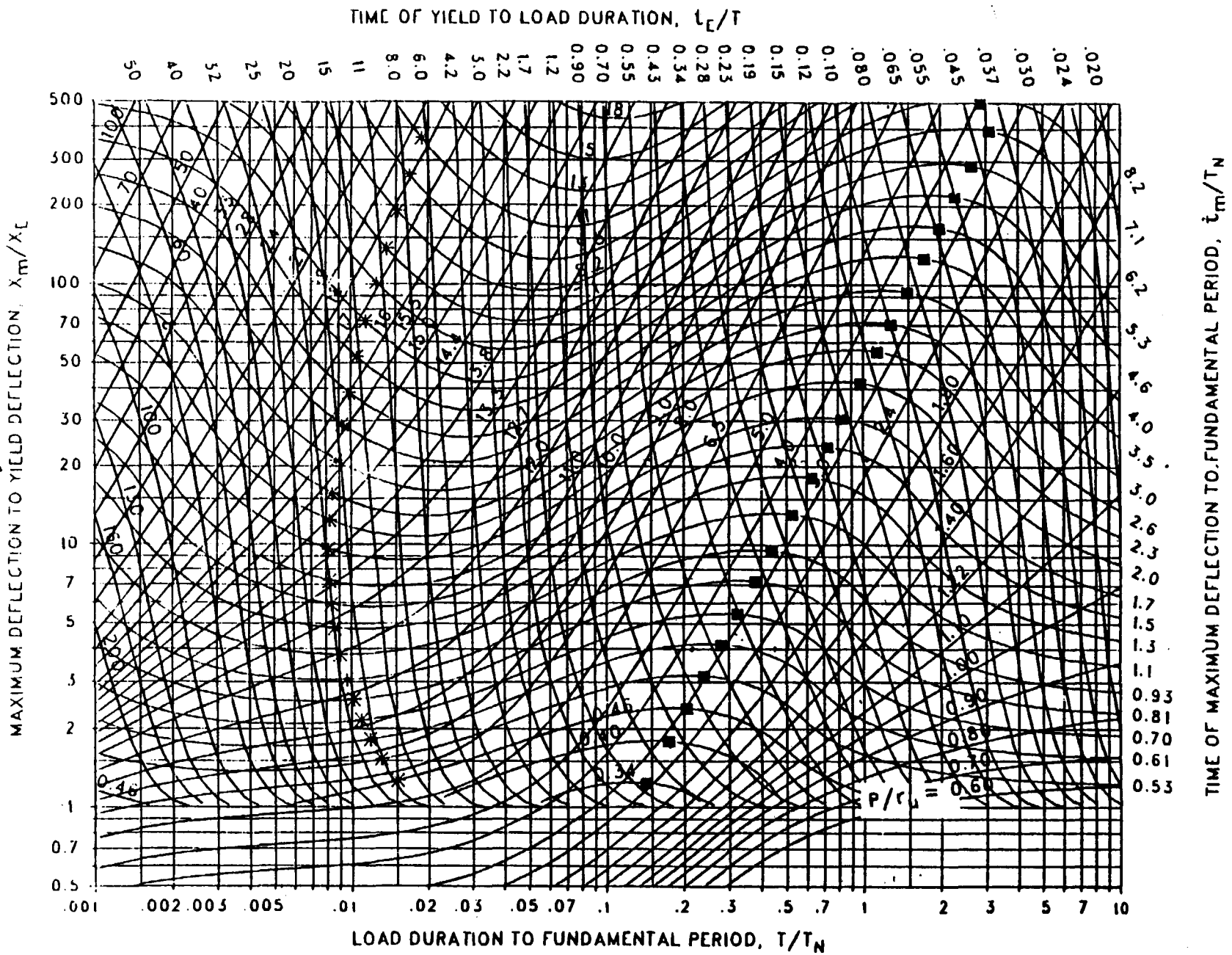


Figure 3-255. Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.075$ ,  $C_2 = 1000$ .)



3-314

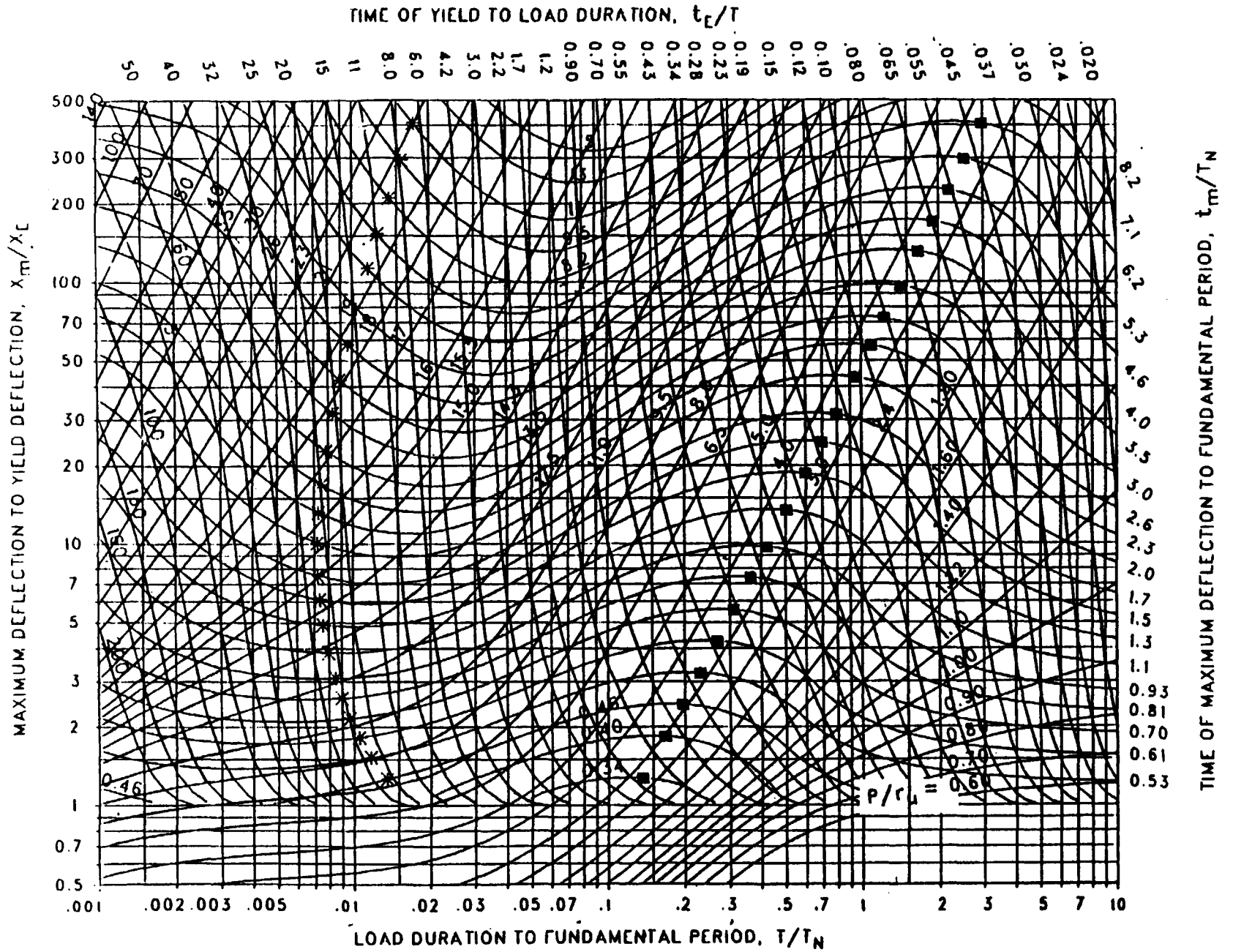


Figure 3-256 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.068$ ,  $C_2 = 1000$ .)

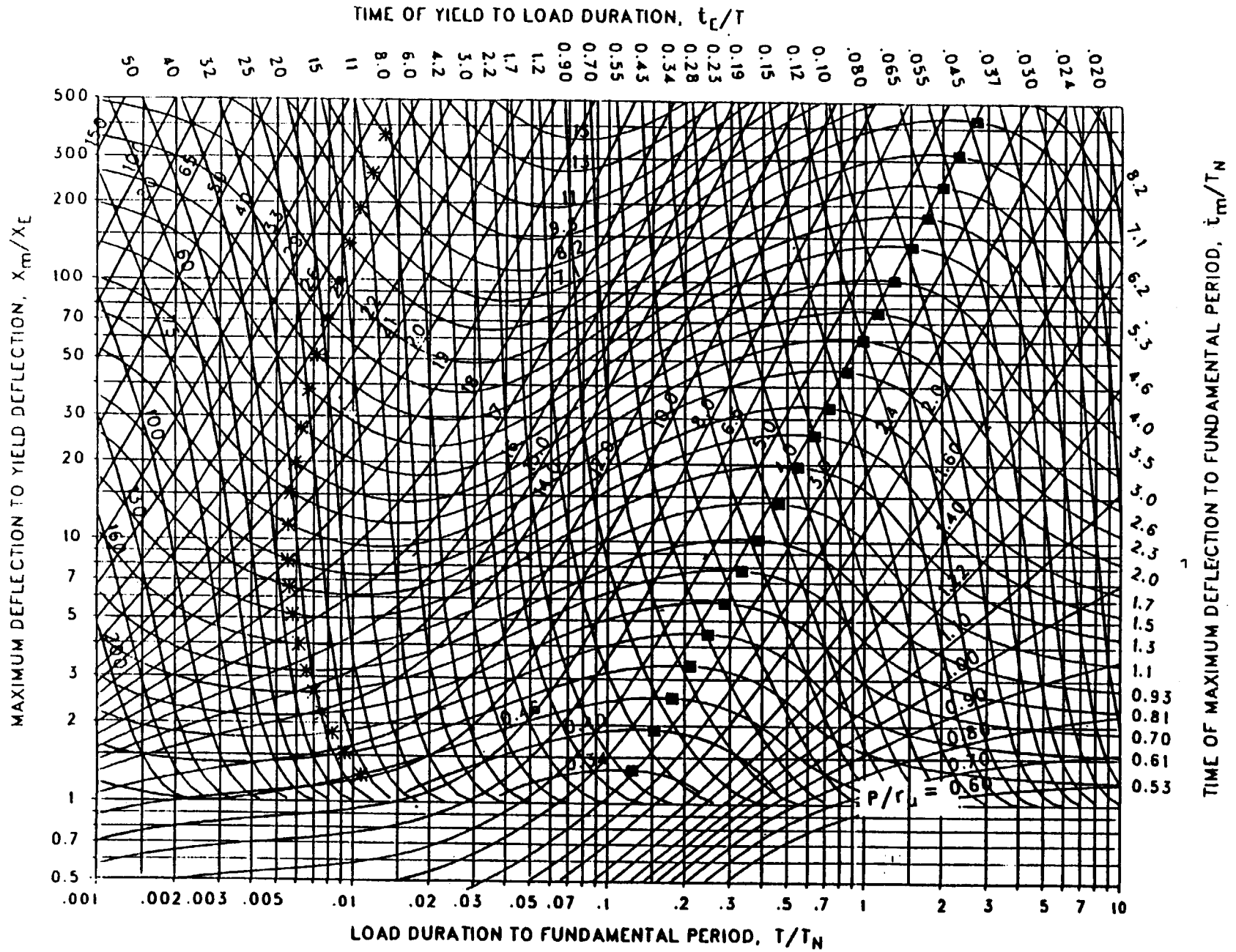


Figure 3-257 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.056$ ,  $C_2 = 1000$ .)

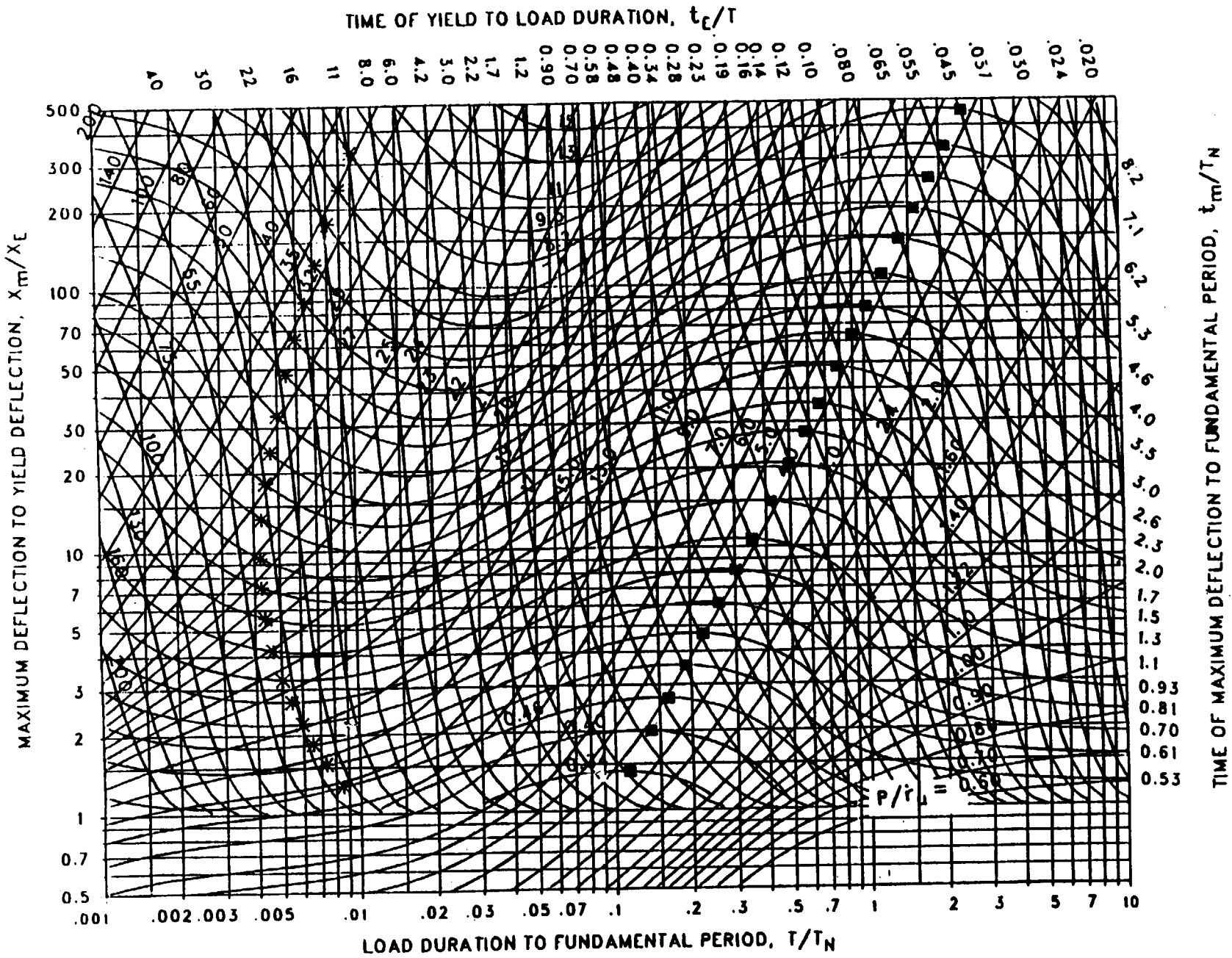


Figure 3-258 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.046$ ,  $C_2 = 1000$ .)

3-317

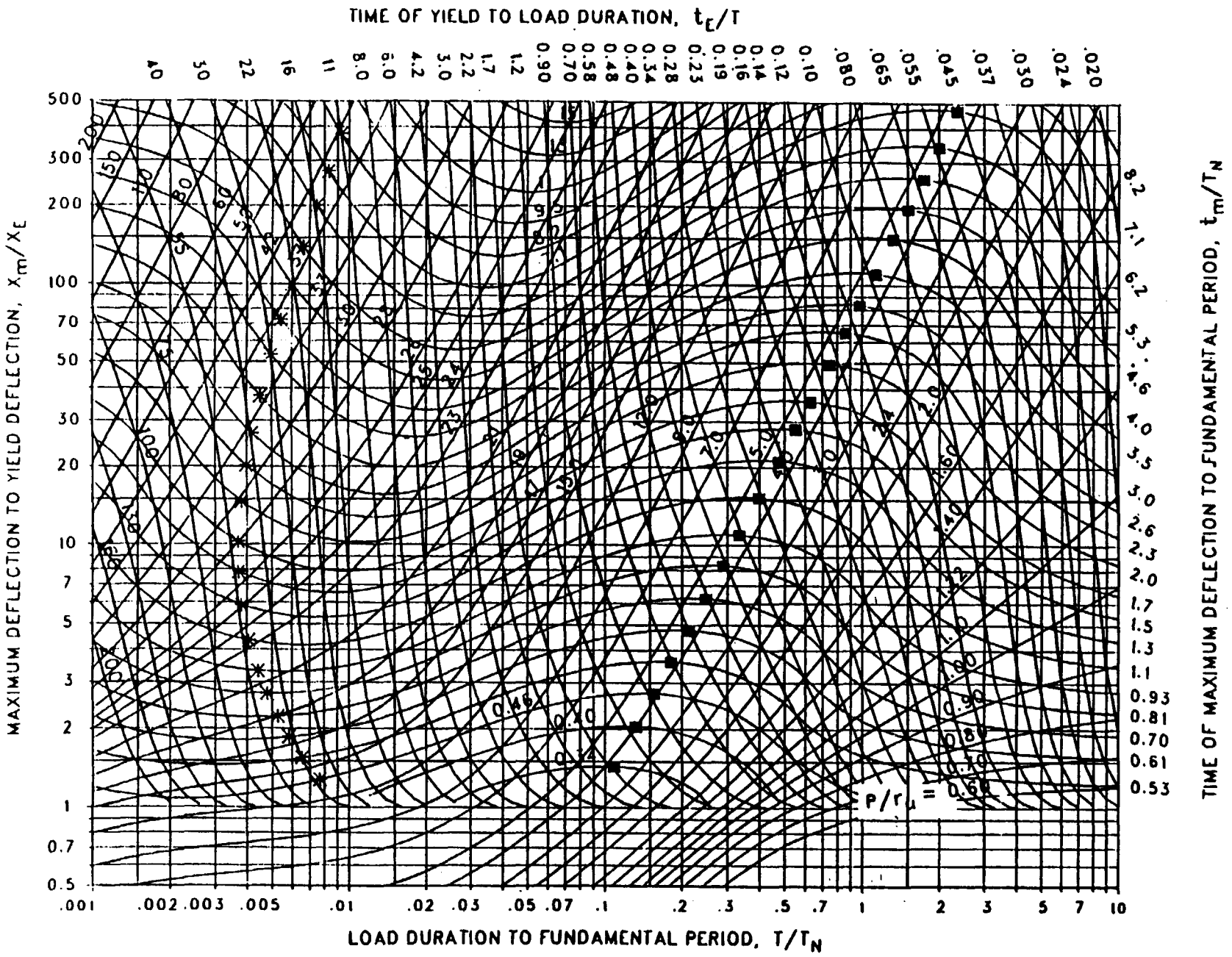


Figure 3-259 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.042$ ,  $C_2 = 1000$ .)

3-318

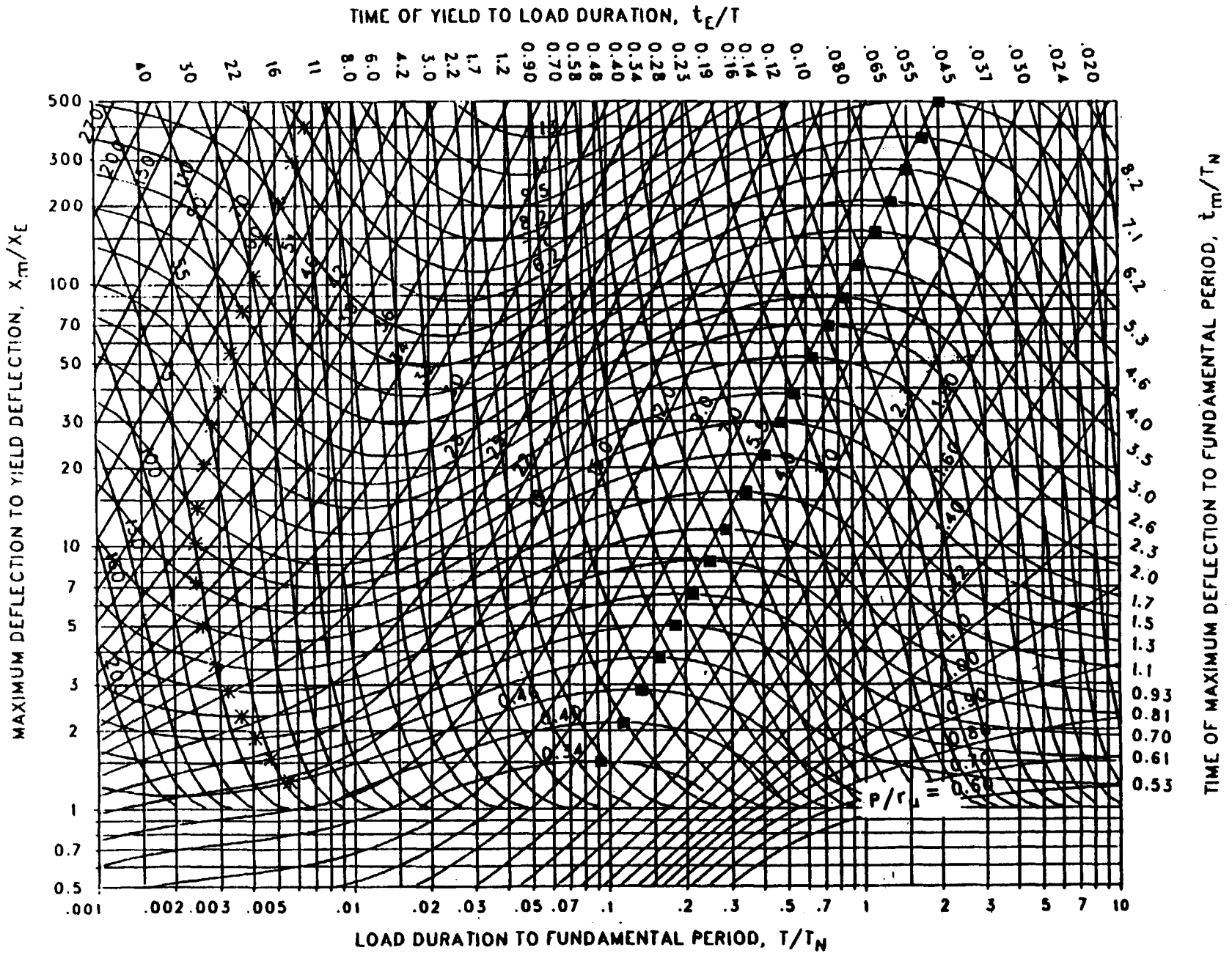


Figure 3-260 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.032$ ,  $C_2 = 1000$ .)

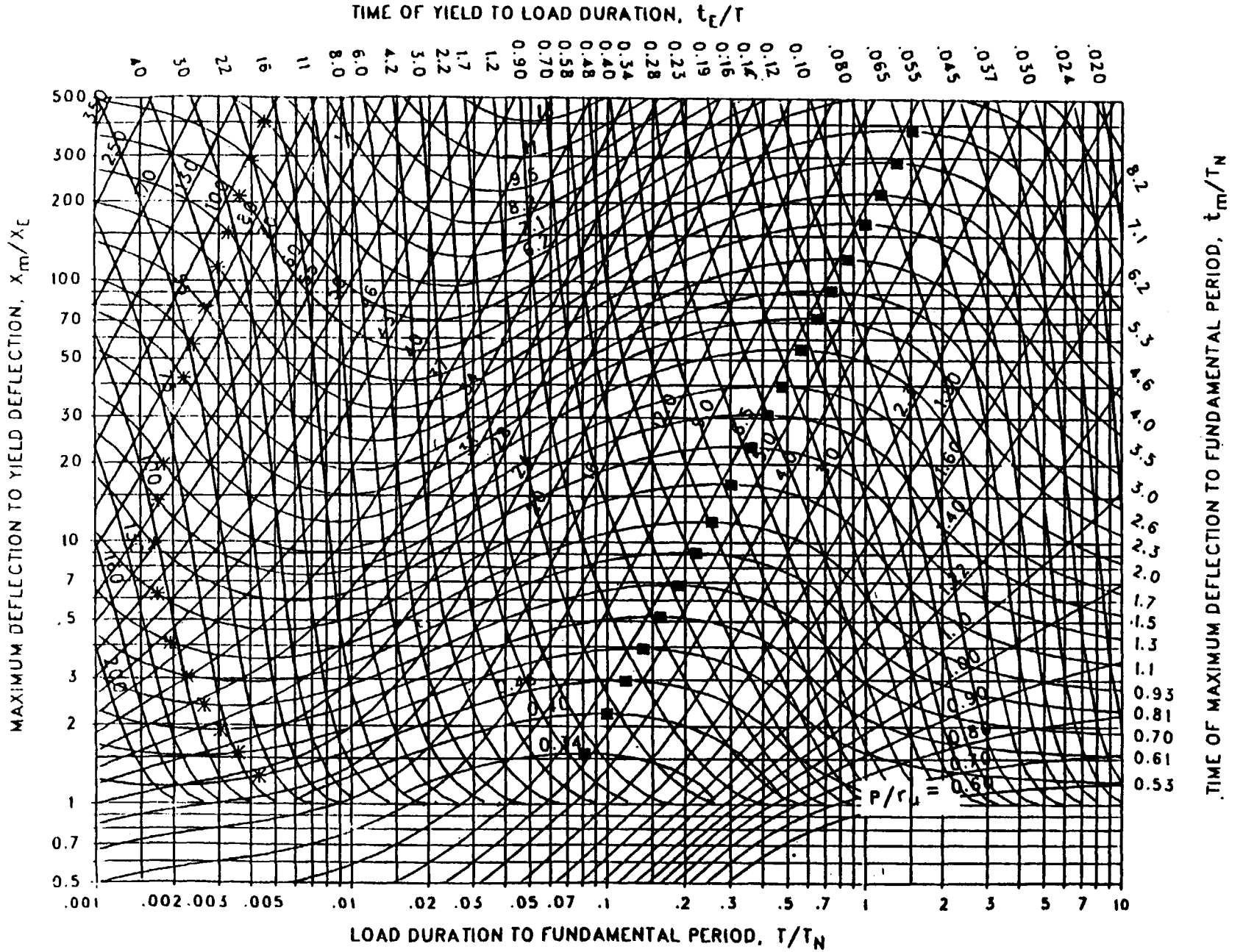


Figure 3-261 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.026$ ,  $C_2 = 1000$ .)

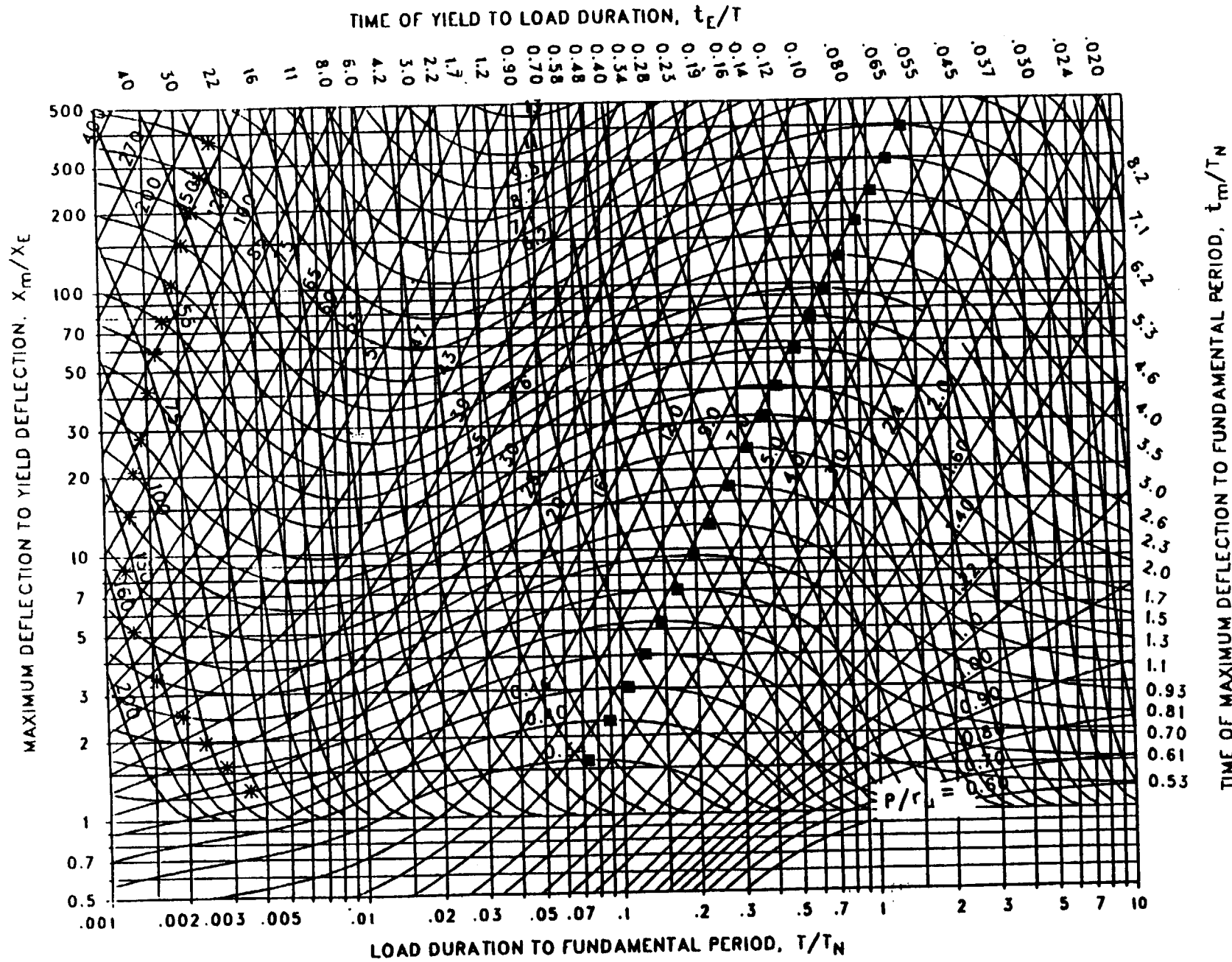


Figure 3-262 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.022, C_2 = 1000.$ )

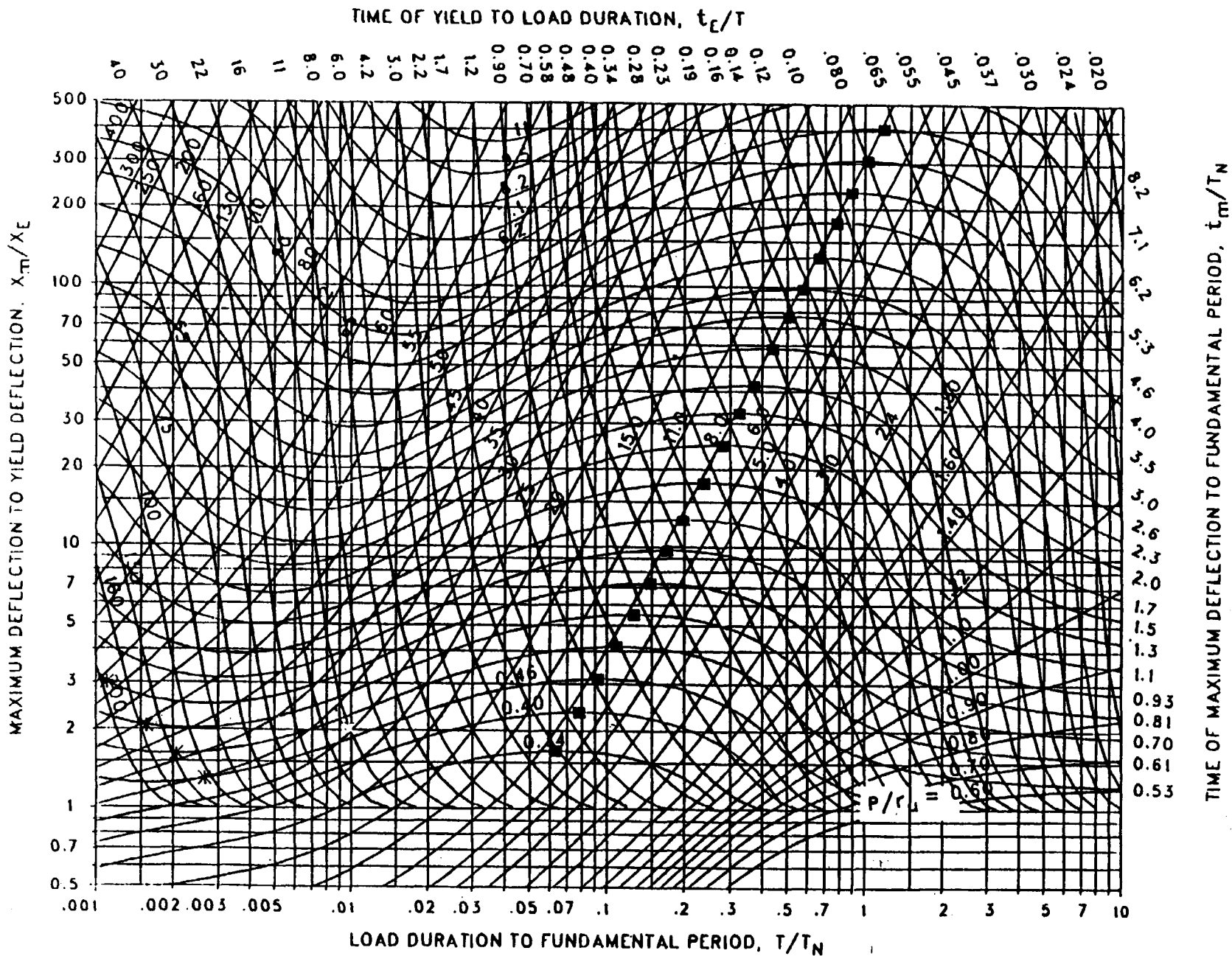


Figure 3-263 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.018$ ,  $C_2 = 1000$ .)



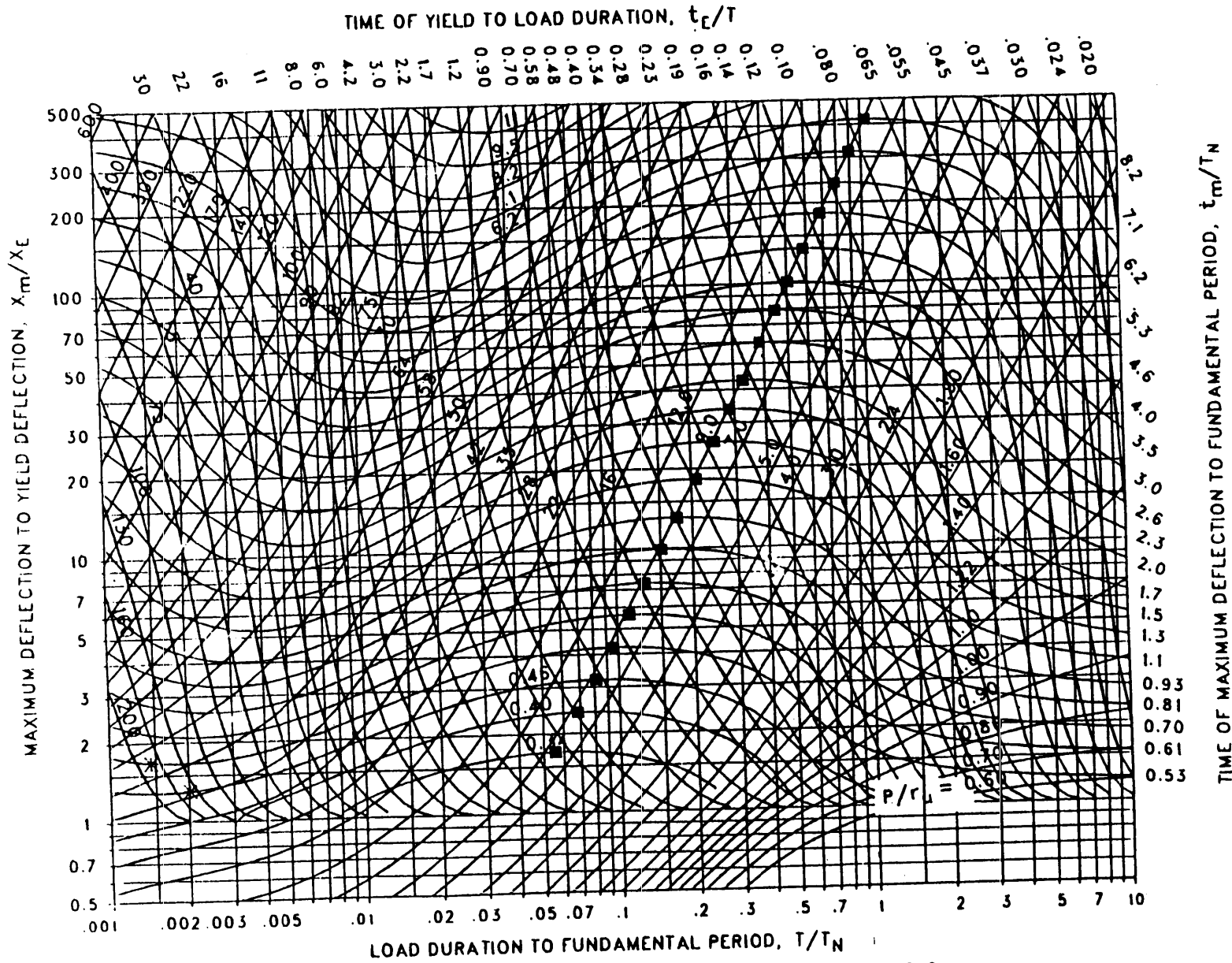


Figure 3-264 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.015$ ,  $C_2 = 1000$ .)

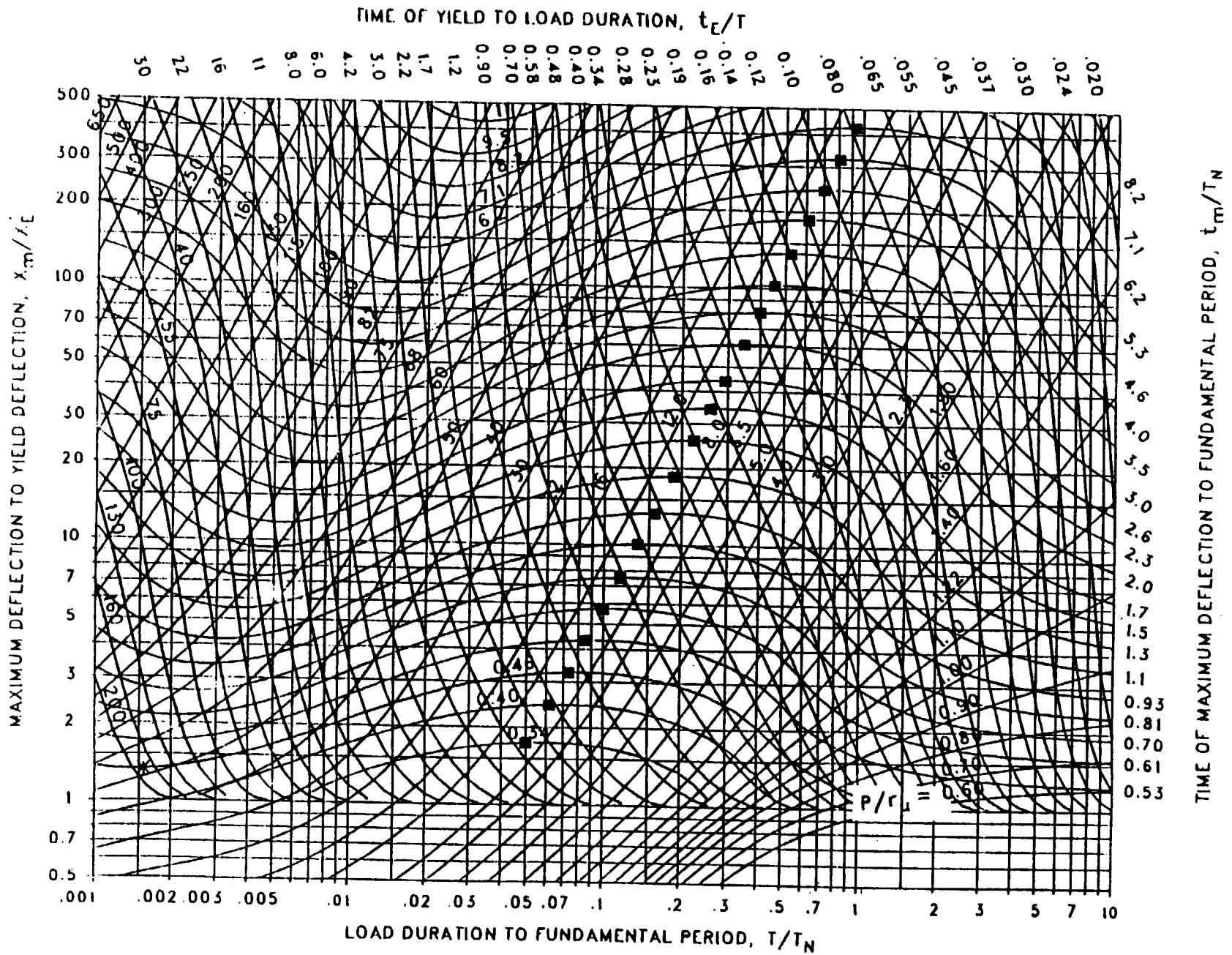


Figure 3-265 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.013, C_2 = 1000.$ )

3-324

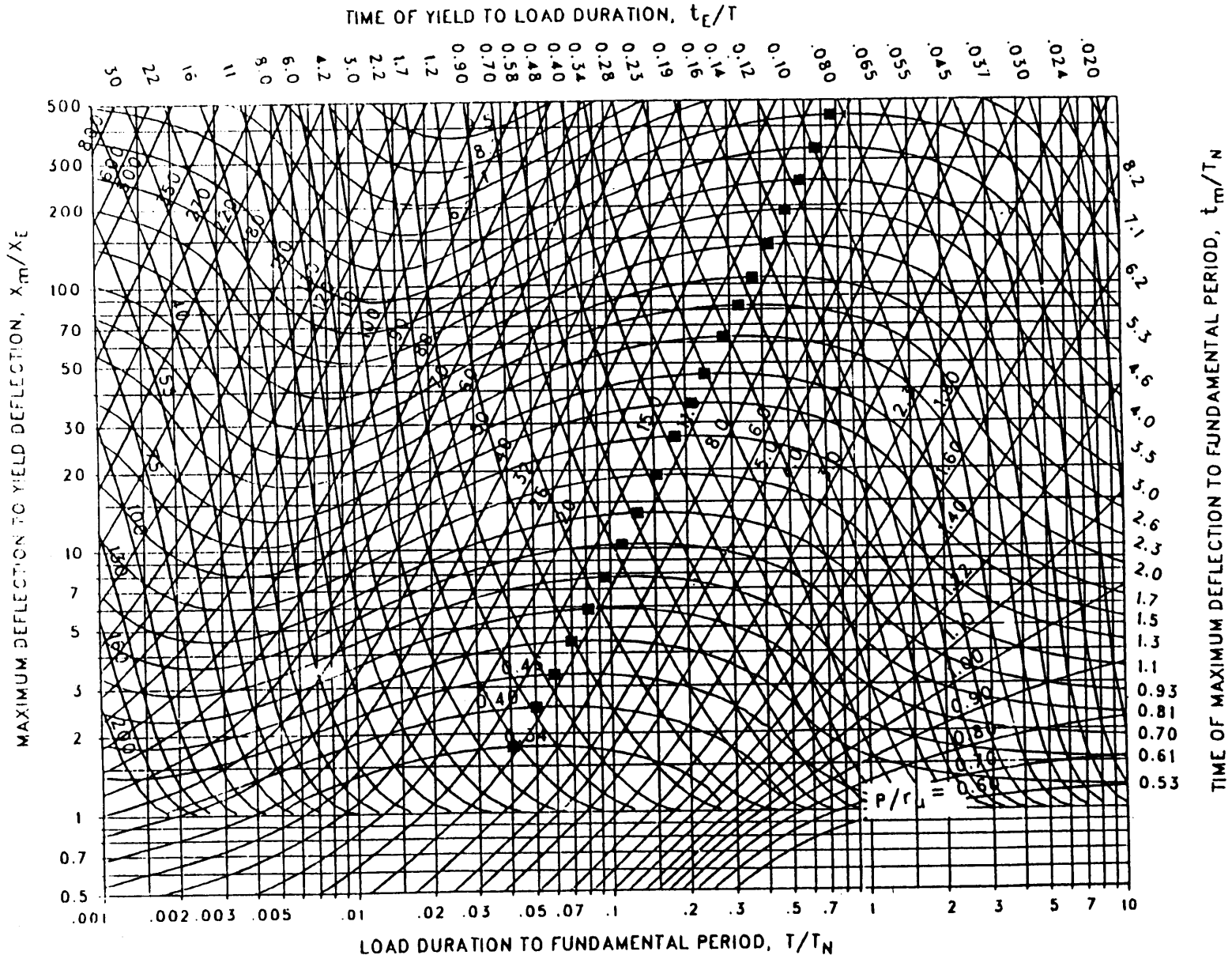


Figure 3-266 Maximum response of elasto-plastic, one-degree-of-freedom system for bilinear-triangular pulse ( $C_1 = 0.010$ ,  $C_2 = 1000$ .)

3-325

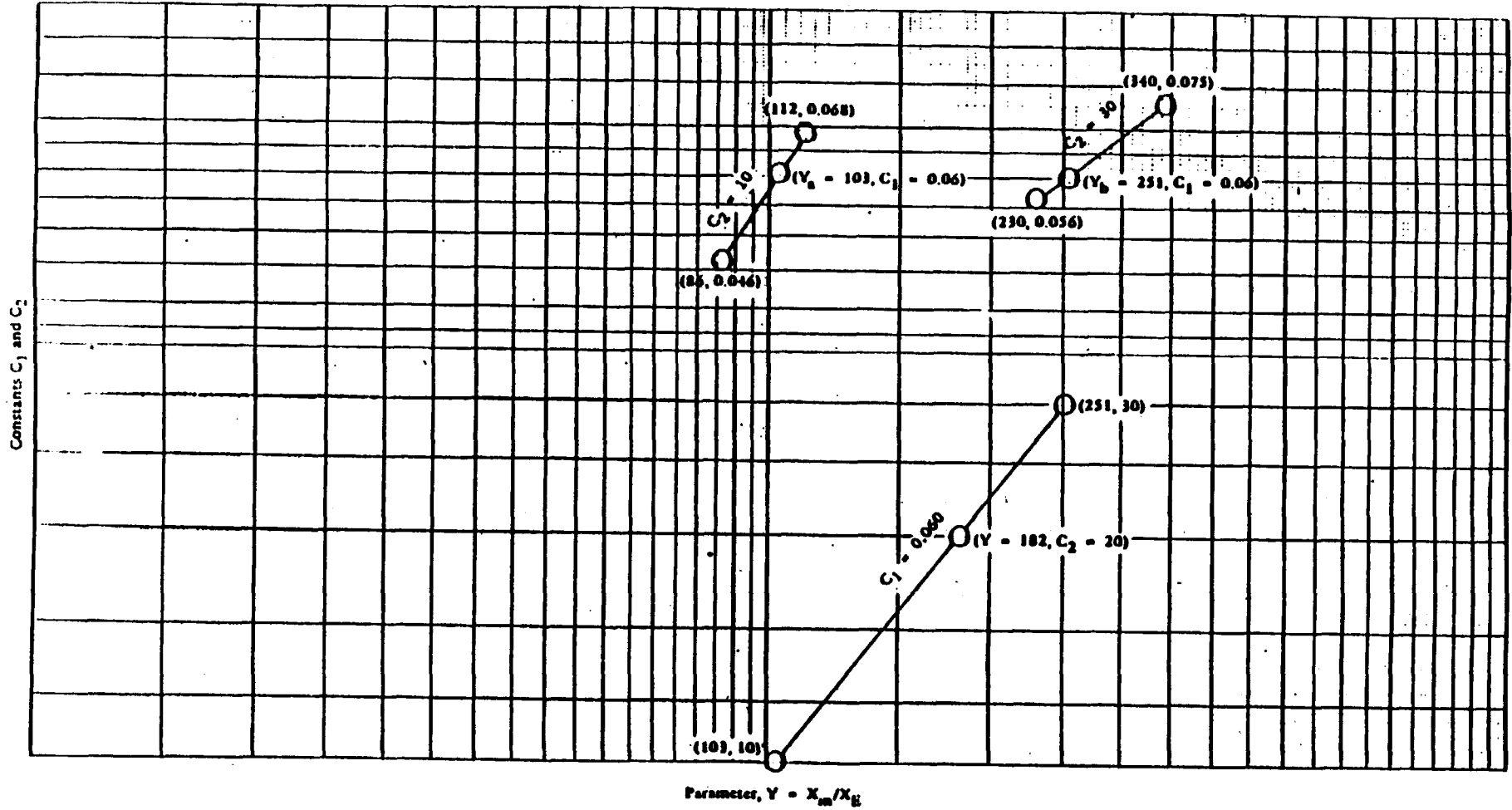
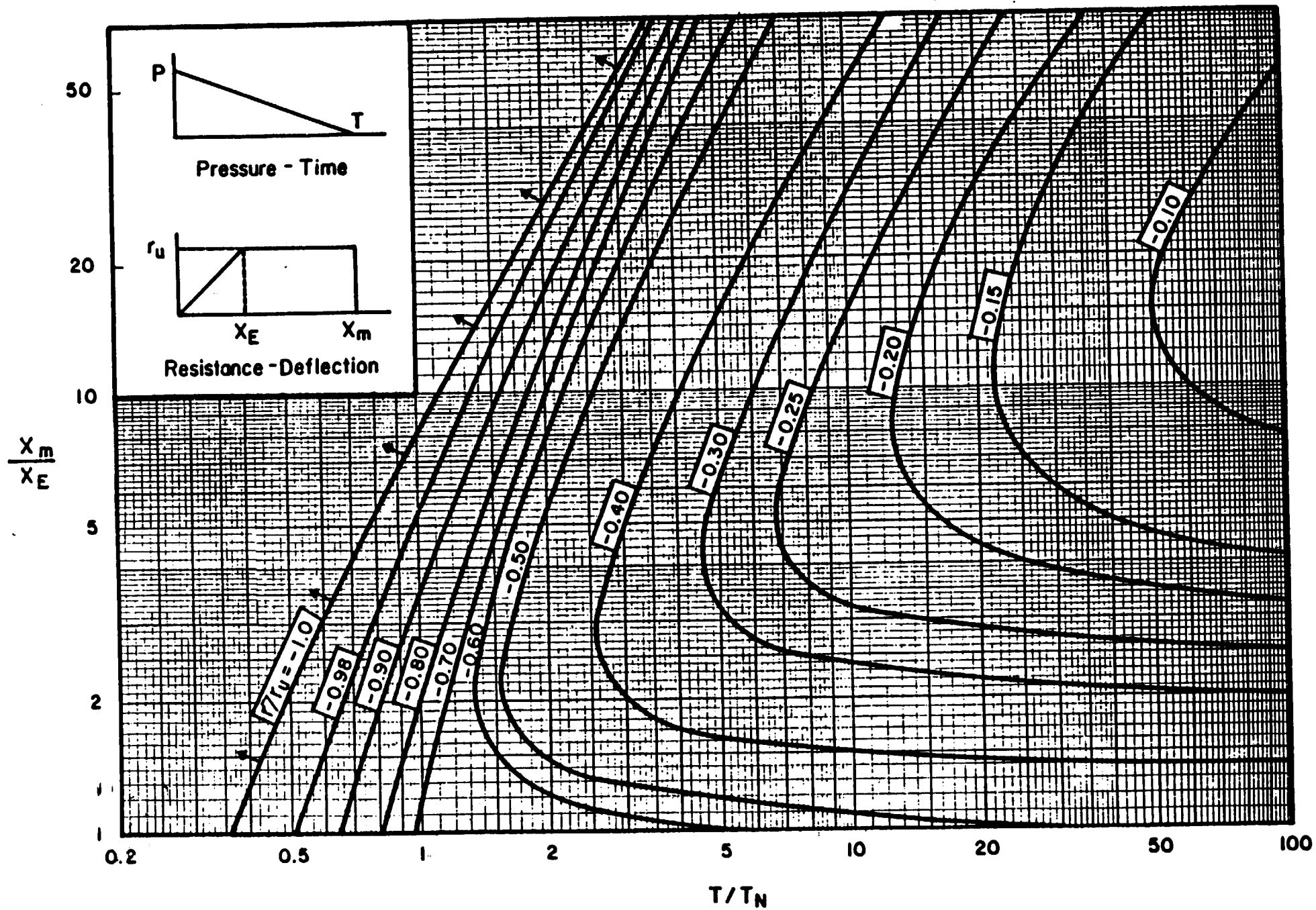


Figure 3-267 Graphical interpolation

3-326



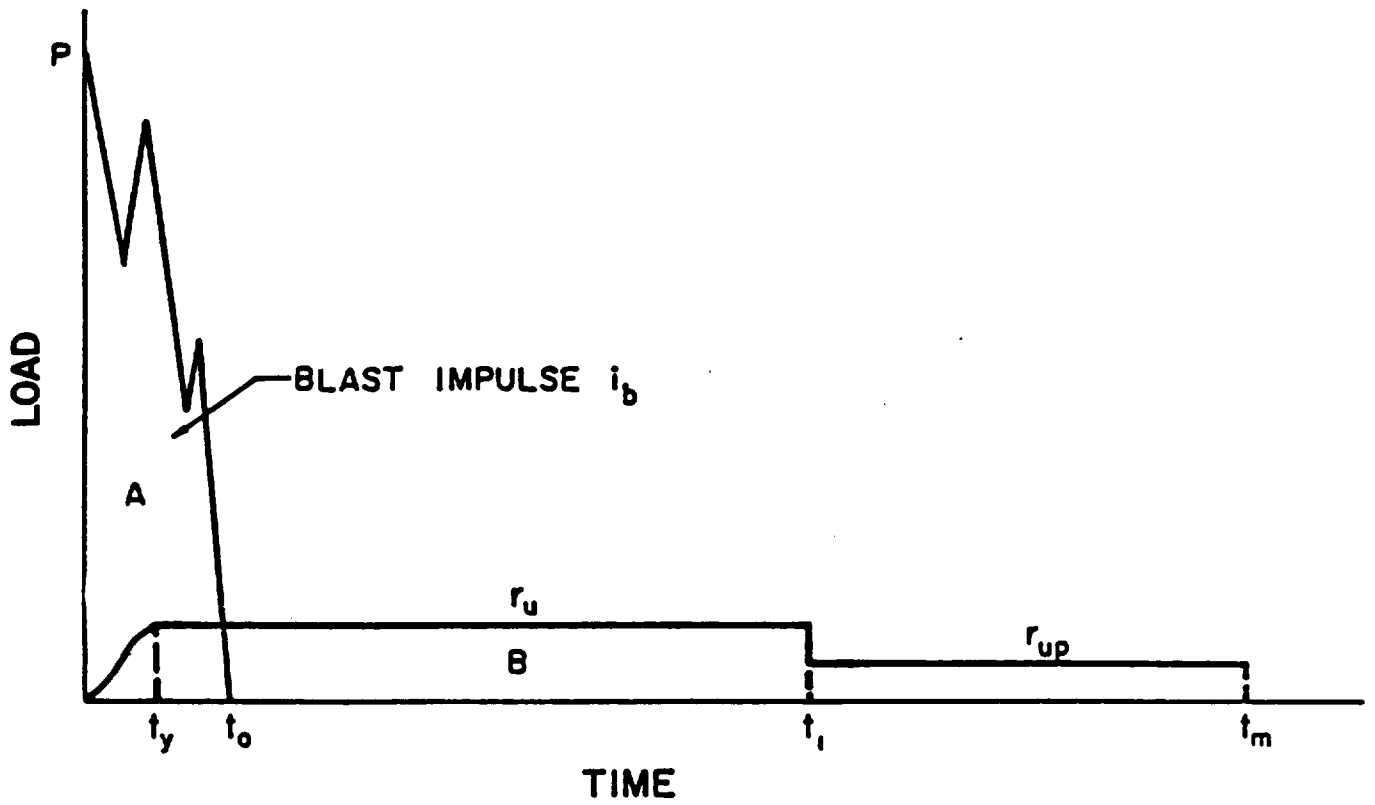


Figure 3-269 Pressure-time and resistance-time curves for elements which respond to impulse

Table 3-14 Details of computation by acceleration impulse extrapolation method

n	t	$P_n$	$R_n$	$P_n - R_n$	$a_n = (P_n - R_n)/m$	$a_n(\Delta t)^2$	$2X_n$	$X_{n-1}$	$X_{n+1}$
0	0	$P_0$	$R_0$	$P_0 - R_0$	$a_0$	$a_0(\Delta t_1)^2$	0	0	$X_1$
1	$\Delta t_1$	$P_1$	$R_1$	$P_1 - R_1$	$a_1$	$a_1(\Delta t_1)^2$	$2X_1$	0	$X_2$
2	$2(\Delta t_1)$	$P_2$	$R_2$	$P_2 - R_2$	$a_2$	$a_2(\Delta t_1)^2$	$2X_2$	$X_1$	$X_3$
.	.	.	.	.	.	.	.	.	.
j-1	$(j-1)\Delta t_1$	$P_{j-1}$	$R_{j-1}$	$P_{j-1} - R_{j-1}$	$a_{j-1}$	$a_{j-1}(\Delta t_1)^2$	$2X_{j-1}$	$X_{j-2}$	$X_j$
j	$j\Delta t_1$	$P_j$	$R_j$	$P_j - R_j$	$a_j$	$a_j(\Delta t_1)^2$	$2X_j$	$X_{j-1}$	$X_{j+1}$
j+1	$j(\Delta t_1) + \Delta t_2$	$P_{j+1}$	$R_{j+1}$	$P_{j+1} - R_{j+1}$	$a_{j+1}$	$a_{j+1}(\Delta t_1)^2$	$2X_{j+1}$	$X_{j-1}$	$X_{j+2}$

Table 3-15 Figure numbers corresponding to various combinations of  $C_1$  and  $C_2$

$C_1 \backslash C_2$	1.00	1.70	3.00	5.50	10.0	30.0	100.	300	1000
1.000	3-64	3-64	3-64	3-64	3-64	3-64	3-64	3-64	3-64
0.909						3-114	3-141	3-173	3-220
0.866						3-115	3-142	3-174	3-221
0.825						3-116	3-143	3-175	3-222
0.787							3-144	3-176	3-223
0.750				3-85	3-99	3-117	3-145	3-177	3-224
0.715						3-118	3-146	3-178	3-225
0.681	3-64	3-65	3-75			3-119	3-147	3-179	3-226
0.648					3-100	3-120	3-148	3-180	3-227
0.619						3-121	3-149	3-181	3-228
0.590						3-150	3-182	3-229	3-229
0.562				3-86	3-101	3-122	3-151	3-183	3-230
0.536								3-184	3-231
0.511						3-123	3-152	3-185	3-232
0.487								3-186	3-233
0.464	3-64	3-66	3-76			3-124	3-153	3-187	3-234
0.422				3-87	3-102		3-154	3-188	3-235
0.383						3-125		3-189	3-236
0.365							3-155	3-190	3-237
0.348								3-191	3-238
0.316	3-64	3-67	3-77	3-88	3-103	3-126	3-156	3-192	3-239
0.287								3-193	3-240
0.274							3-157	3-194	3-241
0.261						3-127	3-158	3-195	3-242
0.237				3-89	3-104		3-159	3-196	3-243
0.215	3-64	3-68	3-78			3-128	3-160	3-197	3-244
0.198								3-198	3-245
0.178				3-90	3-105	3-129	3-161	3-199	3-246
0.162								3-200	3-247
0.147	3-64	3-69	3-79			3-130	3-162	3-201	3-248
0.133				3-91	3-106			3-202	3-249
0.121						3-131	3-163	3-203	3-250
0.110								3-204	3-251
0.100	3-64	3-70	3-80	3-92	3-107	3-132	3-164	3-205	3-252
0.091								3-206	3-253
0.083								3-207	3-254
0.075						3-133	3-165	3-208	3-255
0.068				3-93	3-108			3-209	3-256
0.056	3-64	3-71	3-81			3-134	3-166	3-210	3-257
0.046				3-94	3-109			3-211	3-258
0.042						3-135	3-167	3-212	3-259
0.032	3-64	3-72	3-82	3-95	3-110	3-136	3-168	3-213	3-260
0.026						3-137	3-169	3-214	3-261
0.022				3-96	3-111			3-215	3-262
0.018	3-64	3-73	3-83			3-138	3-170	3-216	3-263
0.015				3-97	3-112			3-217	3-264
0.013						3-139	3-171	3-218	3-265
0.010	3-64	3-74	3-84	3-98	3-113	3-140	3-172	3-219	3-266



Table 3-16 Response Chart Interpolation

Figure Number	$C_1$	$C_2$	Desired Parameter
1	$C_{11}$	$C_{21}$	$Y_1$
	$C_1$	$C_{21}$	
2	$C_{12}$	$C_{22} - C_{21}$	$Y_2$
	$C_1$	$C_2$	
3	$C_{13}$	$C_{23}$	$Y_3$
	$C_1$	$C_{23}$	
4	$C_{14}$	$C_{24} - C_{23}$	$Y_4$

$Y_a$   
 $Y_b$   
 $Y$

**APPENDIX 3A**  
**ILLUSTRATIVE EXAMPLES**

**Problem 3A-1(A) Ultimate Unit Resistance**

**Problem:** Determine the ultimate unit resistance of a two-way structural element using (1) general solution and (2) charts.

**Procedure: Part (a) - General Solution**

- Step 1. Establish design parameters.
- Step 2. Assume yield line locations in terms of x and/or y considering support conditions, presence of openings, etc.
- Step 3. Determine negative and positive moment capacities of sections crossed by assumed yield lines.
- Step 4. Establish distribution of moments across negative and assumed yield lines, considering corner effects and those of openings.
- Step 5. Determine the ultimate unit resistance for each sector in terms of x and/or y considering free body diagram of the sectors (fig. 3-3). Summation of the moments about the axis of rotation (support) of the sector yields equation 3-3.
- Step 6. Equate the ultimate unit resistance of the sectors and solve for the yield line location x and/or y.
- Step 7. With known yield line location, solve for ultimate unit resistance of the element, using equations obtained in Step 6.

Note: For complex problems (three or more different sectors) the solution for the ultimate unit resistance is most easily accomplished through a trial-and-error procedure by determining  $r_u$  for each sector for a given (assumed) yield line location and adjusting the yield lines until the several values of  $r_u$  agree to within a few percent.

**Procedure: Part (b) - Chart Solution**

- Step 1. Same as in step 1 of part a.
- Step 2. Same as in step 2 of part a.
- Step 3. Determine the negative and positive ultimate moment capacities in vertical and horizontal directions.

- Step 4. For given support conditions (and value of  $x_2/x_1$  in the case of an element with three edges supported and fourth free), use the appropriate chart (figs. 3-4 through 3-20) to obtain yield line location ratios  $x/L$  or  $y/H$  for value of quantity obtained in step 4. Then calculate  $x$  or  $y$ .
- Step 5. Using the appropriate equation from table 3-2, determine the ultimate unit resistance of the element.

**Example 3A-1 Ultimate Resistance**

Required: Ultimate unit resistance of two-way structural steel element shown below using (1) general solution and (2) charts.

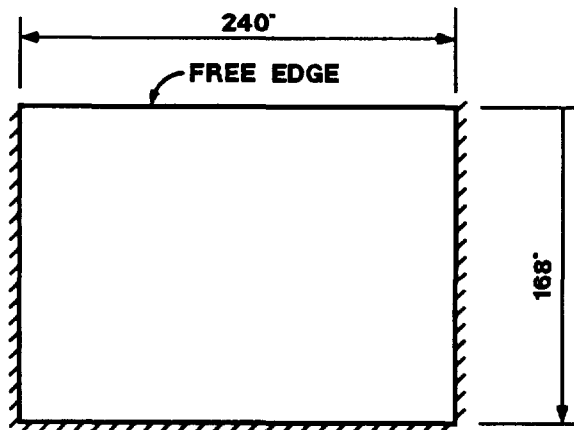
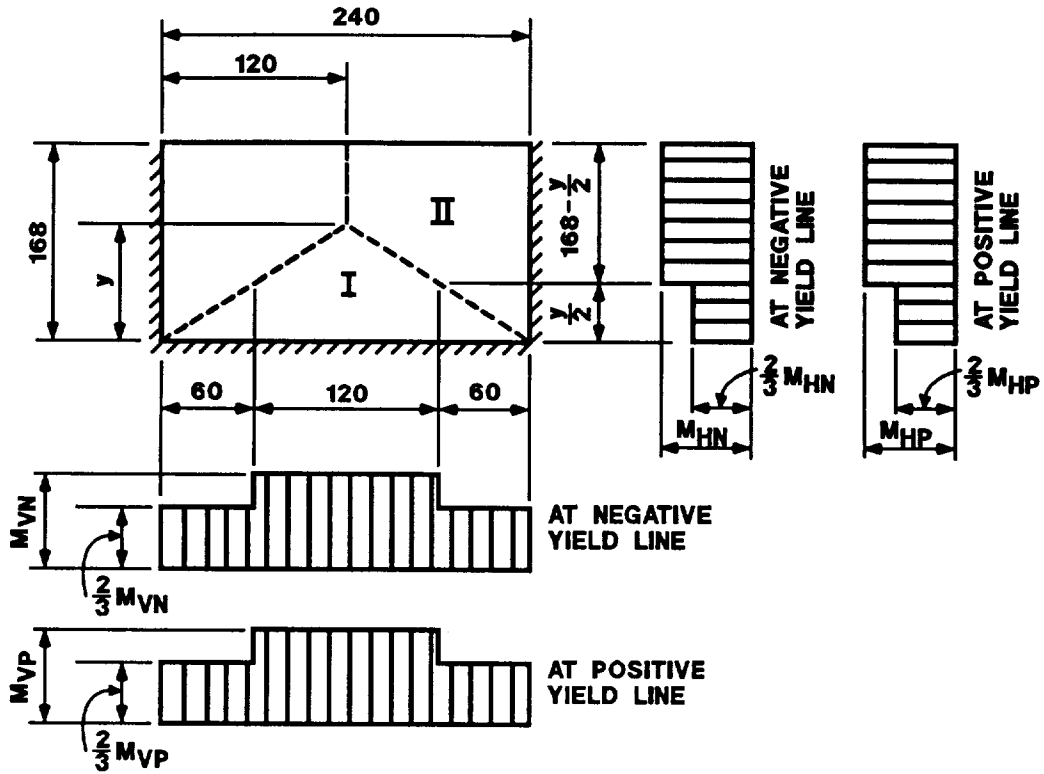


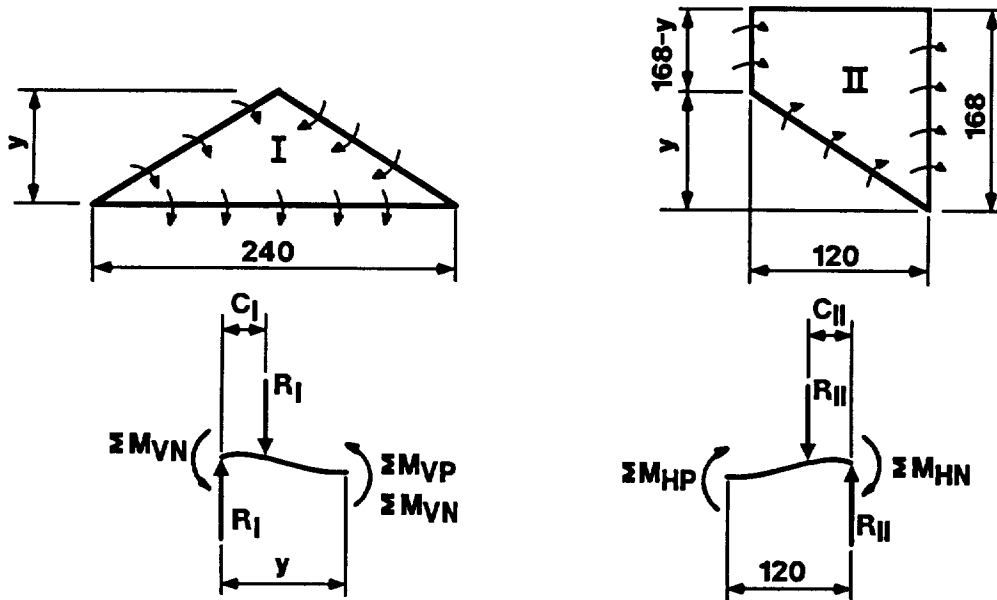
Figure 3A-1

(Solution: Part (a) - General Solution)

- Step 1. Given:
- (a)  $L = 240$  in       $H = 168$  in
  - (b) Fixed on three sides and free at the fourth
- Step 2. Assume yield line location (fig. 3A - 2)
- Step 3. The negative and positive moment capacities in both the horizontal and vertical directions are determined from the properties of the material. For this example, it will be assumed that the moment capacities are equal to  $M = 20,000$  inlbs/in.



**a) ASSUMED YIELD LINES AND DISTRIBUTION OF MOMENTS**



**b) FREE-BODY DIAGRAMS FOR INDIVIDUAL SECTORS**

Figure 3A-2

$$M_{HN} = M_{HP} = M_{VN} = M_{VP} = 20,000 \text{ in lbs/in.}$$

Step 4. For distribution of moments across negative and assumed positive lines, see figure 3A - 2(a).

Step 5. The ultimate unit resistance of each sector is obtained by taking the summation of the moments about its axis of rotation (supports) so that

$$\Sigma M_N + \Sigma M_P = Rc = r_u Ac$$

a. Sector I (fig. 5A - 2)

$$\begin{aligned} \Sigma M_{VN} + \Sigma M_{VP} &= 120(20,000) + 2(2/3)(20,000)(60) + \\ & \quad 120(20,000) + 2(2/3)(20,000)(60) \\ &= 8.0 \times 10^6 \text{ in-lbs.} \end{aligned}$$

$$r_u Ac = r_u \left[ \frac{240(y)}{2} \right] \left[ \frac{y}{3} \right] = 40 r_u y^2$$

therefore,

$$r_u = 400(20,000)/40y^2 = 0.2 \times 10^6 / y^2$$

b. Sector II (fig. 3A - 2)

$$\begin{aligned} \Sigma M_{HN} + \Sigma M_{HP} &= (168-y/2)(20,000) + 2/3(20,000)(y/2) + \\ & \quad (20,000)(168-y/2) + 2/3(20,000)(y/2) \\ &= 336(20,000) - y/3(20,000) \end{aligned}$$

$$\begin{aligned} r_u Ac &= r_u \left[ \frac{120(168+168-y)}{2} \right] \left[ \frac{120[168+2(168-y)]}{3} \right] / (168+168-y) \\ &= 4,800 r_u (252-y) \end{aligned}$$

therefore,

$$r_u = \frac{336(20,000) - y/3(20,000)}{4800 (252-y)}$$

Step 6. Equate the ultimate unit resistance of the sectors.

$$\frac{10(20,000)}{y^2} = \frac{336(20,000) - y/3(20,000)}{4800(252-y)}$$

Simplifying:

$$y^3 - 1008y^2 - 144000y + 36288000 = 0$$

and the desired root is:  $y = 137.6 \text{ ins.}$

Step 7. The ultimate unit resistance is obtained by substituting the value of  $y$  into either equation obtained in step 5, both of which yield:

$$r_u = (20,000)/(137.6)^2 = 10.6 \text{ psi}$$

**Solution: Part (b) - Chart Solution**

Note:

Element conforms to the requirements of section 3-8 since it is fixed on three sides and free on the remaining side and has uniform thickness in the horizontal and vertical directions.

Step 1. Same as step 1 in part a.

Step 2. For illustrative purposes, a different yield pattern (fig. 3A-3) will be assumed.

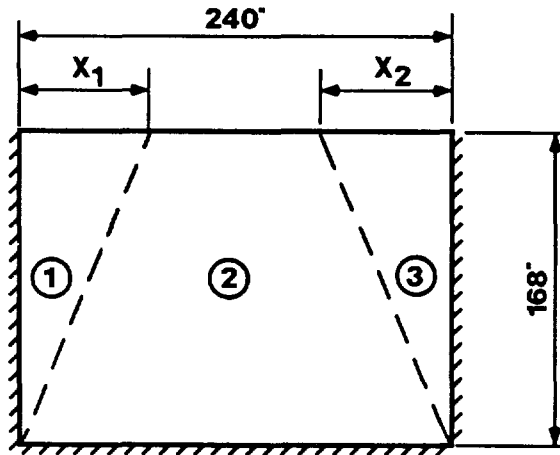


Figure 3A-3

Step 3. For ultimate moment capacities, see step 3 of part a

Step 4. For three sides fixed and the fourth free, calculate the parameter.

$$X_2/X_1 = [(M_{HN3} + M_{HP}) / (M_{HN1} + M_{HP})]^{1/2} = [(20,000)(2) / (20,000)(2)]^{1/2} = 1.0$$

From figure 3-11, ( $X_2/X_1 = 1.0$ ) calculate the parameters:

$$L/H [M_{VP} / (M_{HN1} + M_{HP})] = 240/168 [20,000 / (2)(20,000)]^{1/2} = 1.01$$

and

$$\frac{M_{VP}}{M_{VN2}} = \frac{20,000}{20,000} = 1.0$$

Read yield line location

$X_1/L$  exceeds the maximum possible value of 0.5 therefore, assumed yield line pattern is wrong. Assume alternate yield line pattern as shown in figure 3A-2.

From figure 3-16 calculate the following parameters:

$$\frac{L}{H} = \frac{(M_{VN3} + M_{VP})^{1/2}}{(M_{HN2} + M_{HP})^{1/2} + (M_{HN1} + M_{HP})^{1/2}}$$

$$= \frac{240}{168} \left[ \frac{(20,000 + 20,000)^{1/2}}{(20,000 + 20,000)^{1/2} + (20,000 + 20,000)^{1/2}} \right] = 0.71$$

and

$$X/L = \left[ (M_{HN1} + M_{HP}) / (M_{HN2} + M_{HP}) \right]^{1/2} / 1 + \left[ M_{HN1} + M_{HP} / (M_{HN2} + M_{HP}) \right]^{1/2}$$

$$= \left[ 40,000 / 40,000 \right]^{1/2} / 1 + \left[ 40,000 / 40,000 \right]^{1/2} = 1/2$$

from figure 3-16 read of yield line location:

$$y/H = 0.82; \quad y = 0.8(168) = 137.6 \text{ in}$$

$$X/L = 0.50; \quad X = 0.5(240) = 120.0 \text{ in}$$

Step 5. From table 3-2

NOTE: Both equations given in the table for each edge condition and yield line location, will provide identical values of  $r_u$ .

$$r_u = \frac{5(M_{VN} + M_{VP})}{y^2} = \frac{5(20,000 + 20,000)}{137.6^2} = (0.00055)20,000 = 10.5 \text{ psi}$$

Example 3A-1(B)

Ultimate Unit Resistance

Required: Ultimate unit resistance of the element considered in example 3A-1(A) except there is an opening as shown in figure 3A-4.



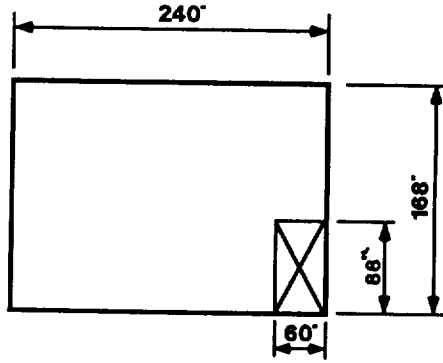


Figure 3A-4

**Solution:**

- Step 1. Given:  
 (a)  $L = 240''$      $H = 168''$     Two additional free edges are formed due to the presence of the opening.
- Step 2. Assumed yield line location is shown in figure 3A-5 (three different sectors are formed).
- Step 3. Same as step 3 of Example 3A-1(A), part a.

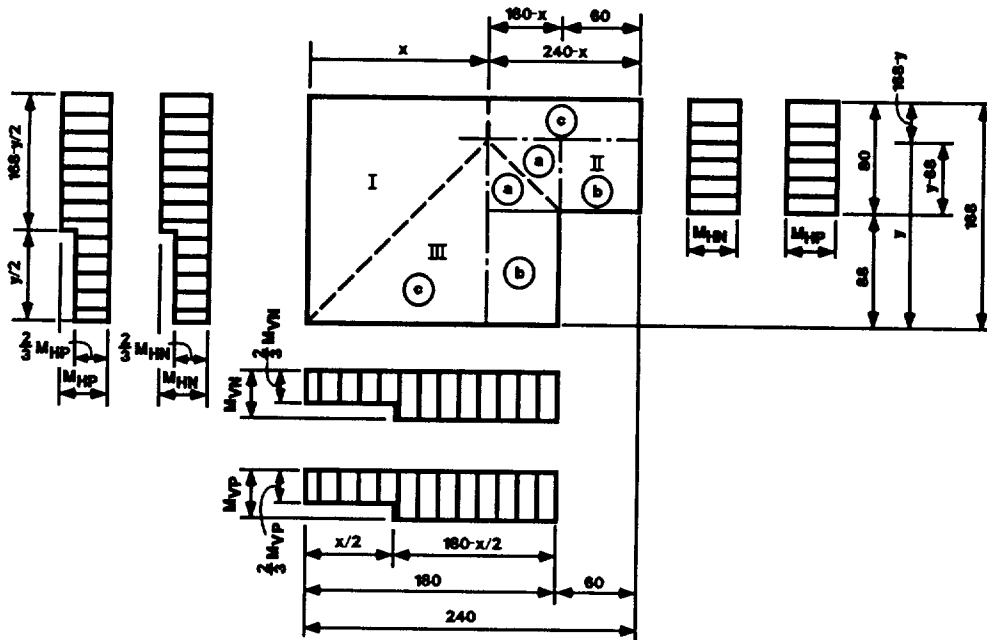


Figure 3A-5

Step 4. For distribution of moments across negative and assumed positively yield lines see figure 3A-5. (Since opening is located at lower right corner, there is no reduced moment capacity in this area.)

Step 5. The ultimate unit resistance is obtained from:

$$\Sigma M_N + \Sigma M_P = R_c = r_u A_c$$

a. Sector I (fig. 3A-5)

$$\begin{aligned} \Sigma M_{HN} + \Sigma M_{HP} &= (20,000)(168-y/2) + 2/3(20,000)(y/2) + \\ &\quad (20,000)(168-y/2) + 2/3(20,000)(y/2) \\ &= 336(20,000) - y/3(20,000) \\ &= (336-y/3)(20,000) \end{aligned}$$

$$\begin{aligned} r_u A_c &= r_u [x(168+168-y)/2] [x(168+2(168-y))/3(168+168-y)] \\ &= r_u x^2 (252-y)/3 \end{aligned}$$

therefore,

$$r_u = \frac{(1008-y)(20,000)}{x^2(252-y)}$$

b. Sector II (fig. 5A-5)

$$\begin{aligned} \Sigma M_{HN} + \Sigma M_{HP} &= (20,000)(80) + (20,000)(80) \\ &= 160(20,000) \end{aligned}$$

Note:

The sector is divided into parts a, b, and c so that the centroid may be obtained (see table below).

Portion of Sector	Area (A')	Distance from Centroid to axis of rotation (c')	A'c'
a	$\frac{(y-88)(180-x)}{2}$	$\frac{(180-x) + 60}{3} = \frac{360-x}{3}$	$\frac{(y-88)(180-x)(360-x)}{6}$
b	$(y-88)(60)$	$\frac{60}{2}$	$\frac{(y-88)(60)^2}{2}$
c	$(168-y)(240-x)$	$\frac{(240-x)}{2}$	$\frac{(168-y)(240-x)^2}{2}$

$$A_c = \Sigma A'c' = \frac{(y-88)(180-x)(360-x)}{6} + \frac{(y-88)(60)^2}{2} + \frac{(168-y)(240-x)^2}{2}$$

$$= \frac{1}{2} \frac{(y-88)}{3} [(180-x)(360-x) + 10800] + (168-y)(240-x)^2$$

$$r_u = \frac{(\Sigma M_{HN} + \Sigma M_{HP})}{A_c}$$

$$= \frac{6,400,000}{\frac{(y-88)}{3} [(180-x)(360-x) + 10,800] + (168-y)(240-x)^2}$$

c. Sector III (fig. 3A-5)

$$\Sigma M_{vn} + \Sigma M_{vp} = (20,000)(180-x/2) + 2/3(20,000)(x/2) + (20,000)(180-x/2) + 2/3(20,000)(x/2)$$

$$= 360(20,000) - (20,000)x/3$$

Portion of Sector	Area (A')	Distance from Centroid to axis of rotation (c')	A'c'
a	$\frac{(108-x)(y-88)}{2}$	$\frac{(y-88) + 88}{3} = \frac{y + 176}{3}$	$\frac{(180-x)(y-88)(y+176)}{6}$
b	$(180-x)(88)$	$\frac{88}{2}$	$\frac{(180-x)(88)^2}{2}$
c	$\frac{xy}{2}$	$\frac{y}{3}$	$\frac{xy^2}{6}$

$$A_c = \Sigma A'c' = (y-88)(180-x)(y+176)/6 + (180-x)(88)^2/2 + xy^2/6$$

$$= 1/6 (180-x)[(y-88)(y+176)+23,232] + xy^2/6$$

$$r_u = \frac{\Sigma M_{HN} + \Sigma M_{HP}}{A_c}$$

$$= \frac{(2160-2x)(20,000)}{(180-x)[(y-88)(y+176)+23,232]+xy^2}$$

Step 6. Due to the complexity of obtaining a direct solution for ultimate unit resistance, a trial-and-error solution will be used ( see table below):

x	y	r <sub>I</sub>	r <sub>II</sub>	r <sub>III</sub>
125	130	9.21	7.67	9.33
125	135	9.55	7.92	8.77
125	140	9.92	8.19	8.25
125	145	10.32	8.48	7.78
125	150	10.77	8.79	7.35
130	130	8.52	8.29	9.50
130	135	8.83	8.55	8.92
131	135	8.70	8.68	8.85

Therefore:

$$\begin{aligned}
 x &= 131 \text{ ins} \\
 y &= 135 \text{ ins} \\
 r_u &= 8.68 \text{ psi}
 \end{aligned}$$

**Problem 3A-2 Resistance - Deflection Function**

**Problem:** Determine the actual and equivalent resistance deflection function in the elasto-plastic region for a two-way structural element.

**Procedure:**

- Step 1. Establish design parameters
  - a. Geometry of element.
  - b. Support conditions
- Step 2. Determine ultimate positive and negative moment capacities.
- Step 3. Determine static properties:
  - a. Modules of elasticity for the element.
  - b. Moment of inertia of the element.
- Step 4. Establish points of interest and their ultimate moment capacities (fig. 3-23)
- Step 5. Compute properties at first yield.
  - a. Location of first yield
  - b. Resistance at first yield  $r_e$
  - c. Moments at remaining points consistent with  $r_e$
  - d. Maximum deflection at first yield.
- Step 6. Compute properties at second yield
  - a. Remaining moment capacity at other points
  - b. Location of second yield.
  - c. Change in unit resistance  $\Delta r$  between first and second yield.
  - d. Unit resistance at second yield  $r_{ep}$ .
  - e. Moment at remaining point consistent with  $r_{ep}$ .

- f. Change in maximum deflection.
- g. Total maximum deflection.

Note:

An element with unsymmetrical support conditions may exhibit three or four support yields. Therefore, repeat Step 6 as many times as necessary to obtain properties at the various yield points.

- Step 7. Compute properties at final yield (ultimate unit resistance)
  - a. Ultimate unit resistance.
  - b. Change in resistance between ultimate unit resistance and resistance at prior yield.
  - c. Change in maximum deflection (for elements supported on two, three, or four sides, use stiffness obtained from figure 3-26, 3-30 and 3-36, respectively).
  - d. Total maximum deflection.
- Step 8. Draw the actual resistance-deflection curve (fig. 3-39).
- Step 9. Calculate equivalent maximum elastic deflection of the element.

**Example: 3A-2 Resistance-Deflection Function**

Required: The actual and equivalent resistance-deflection function (curve) in the elasto-plastic region for the two-way structural steel element.

Solution:

- Step 1. Given:
  - a.  $L = 240$  in.                       $H = 168$  in.
  - b. Fixed on three sides and free at the fourth.
- Step 2. Same as step 3 of example 3A-1(A), part a.
- Step 3. Static properties.
  - a. Modulus of elasticity,  $E_s$  for steel  
 $E_s = 29 \times 10^6$  psi
  - b. Considering a 1-inch strip ( $b = 1$  inch)  
 Assume  $I = 144$  in<sup>4</sup>
- Step 4. For points of interest, see figure 3A-6.
- Step 5. Properties at first yield.

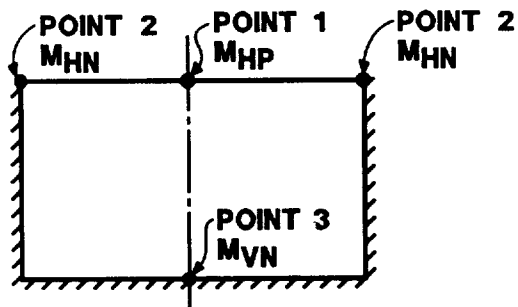


Figure 3A-6

From figure 3-27 for  $H/L = 0.7$

$$\beta_1 = 0.077 \quad \beta_2 = 0.160 \quad \beta_3 = 0.115$$

$$\gamma_1 = 0.012 \quad \nu = 0.3$$

a.  $M_{HP} = M_{HN} = M_{VP} = M_{VN} = 20,000 \text{ in-lbs/in}$

$$M_P = \beta r H^2$$

$$r = M/\beta H^2$$

$$r_1 = 20,000/[(0.077)(168)^2] = 9.20 \text{ psi}$$

$$r_2 = 20,000/[(0.160)(168)^2] = 4.43 \text{ psi}$$

$$r_3 = 20,000/[(0.115)(168)^2] = 6.16 \text{ psi}$$

First yield at point 2 (smallest  $r$ )

b.  $r_e = 4.43 \text{ psi}$

c.  $M_P$  (Point 1) =  $(0.077)(4.43)(168)^2 = 9,627 \text{ in-lbs/in}$

$$M_N$$
 (Point 3) =  $(0.115)(4.43)(168)^2 = 14,379 \text{ in-lbs/in}$

d.  $D = EI/b(1-\nu^2)$

$$= 29 \times 10^6 \times 144/1[1-(0.3)^2] = 45.9 \times 10^8 \text{ in-lbs}$$

$$X_e = \gamma_1 r_e H^4/D = (0.0120)(4.43)(168)^4 / 43(10^8) = 0.0092 \text{ in}$$

Step 6. Properties at second yield.

After first yield element assumes a simple-simple-fixed-free stiffness, therefore from figure 3-29 for H/L = 0.7.

$$\beta_1 = 0.120 \quad \beta_3 = 0.220$$

$$\gamma_1 = 0.045 \quad \nu = 0.3$$

$$\begin{aligned} \text{a. } M_P \text{ (Point 1)} &= M_{HP} - M_P \text{ (at } r_e) \\ &= 20,000 - 9627 = 10373 \text{ in-lbs/in} \end{aligned}$$

$$\begin{aligned} M_N \text{ (Point 3)} &= M_{VN} - M_P \text{ (at } r_e) \\ &= 20,000 - 14,379 = 5621 \text{ in-lbs/in} \end{aligned}$$

$$\begin{aligned} \text{b. } M_P \text{ (Point 1)} &= 10373 \text{ in-lbs/in} = \beta_1 \Delta r H^2 \\ \Delta r &= 10373 / (0.120)(168)^2 = 3.06 \text{ psi} \end{aligned}$$

$$\begin{aligned} M_N \text{ (Point 3)} &= 5,621 \text{ in-lbs/in} = \beta_3 \Delta r H^2 \\ \Delta r &= 5,621 / [(0.220)(168)^2] = 0.90 \text{ psi} \end{aligned}$$

Second yield at Point 3 (smaller  $\Delta r$ )

$$\text{c. } \Delta r = 0.90 \text{ psi}$$

$$\text{d. } r_{ep} = r_e + \Delta r = 4.43 + 0.90 = 5.33 \text{ psi}$$

$$\begin{aligned} \text{e. } M_P \text{ (Point 1)} &= 0.120(0.90)(168)^2 \\ &= 3.048 \text{ in-lbs/in} \end{aligned}$$

$$\begin{aligned} \text{f. } D &= EI/b(1-\nu^2) = (29)(10^6)(144)/1[1-(0.3)^2] \\ &= 45.9 \times 10^8 \text{ in-lbs/in} \end{aligned}$$

$$\begin{aligned} \Delta x &= \gamma_1 \Delta r H^4 / D = 0.030(0.90)(168)^4 / 45.9(10)^8 \\ &= 0.0047 \text{ in} \end{aligned}$$

$$\text{g. } X_{ep} = X_e + \Delta X = 0.0092 + 0.0047 = 0.014 \text{ in}$$

Step 7. Properties at final yield (ultimate unit resistance). After second yield element assumes a simple-simple-simple-free stiffness, therefore from figure 3-30 for H/L = 0.7.

$$\gamma_1 = 0.045 \quad \nu = 0.3$$

$$\text{a. } r_u = 10.6 \text{ psi (part a, example 3A-1(A))}$$

$$\text{b. } \Delta r = r_u - r_{ep} = 10.6 - 5.33 = 5.27 \text{ psi}$$

$$\begin{aligned} \text{c. } D &= EI/b(1-\nu^2) = 29(10^6)(144)/1[1-(0.3)^2] \\ &= 45.9 \times 10^8 \text{ in-lbs} \end{aligned}$$

$$\begin{aligned} \Delta x &= \gamma_1 r H^4 / D = (0.045)(5.27)(168)^4 / 45.9 \times 10^8 \\ &= 0.041 \text{ in} \end{aligned}$$

$$\text{d. } X_p = X_{ep} + \Delta X = 0.014 + 0.041 = 0.055 \text{ in}$$

Step 8. For actual resistance-deflection curve, see figure 3A-7.

$$\text{Step 9. } X_E = X_e(r_{ep}/r_u) + X_{ep}[1-(r_e/r_u)] + X_p[1-(r_{ep}/r_u)] \quad \text{Equation 3-35}$$

$$\begin{aligned} X_E &= 0.0092(5.33/10.6) + 0.014 [1-(4.43/10.6)] \\ &+ 0.055[1-(5.33/10.6)] \\ &= 0.00463 + 0.0081 + 0.0273 \\ &= 0.040 \text{ in} \end{aligned}$$

The equivalent resistance-deflection curve is shown in figure 3A-7.

### Problem 3A-3 Dynamic Design Factors For A One Way Element

**Problem:** Determine the plastic load, mass and load-mass factors for a one-way element.

**Procedure:**

- Step 1. Establish design parameters.
- Step 2. Determine deflected shape.
  - a. geometry of element
  - b. support conditions
  - c. type of load and mass
- Step 3. Determine maximum deflection
- Step 4. Determine deflection function
  - a. For distributed load and/or continuous mass determine the deflection at any point.



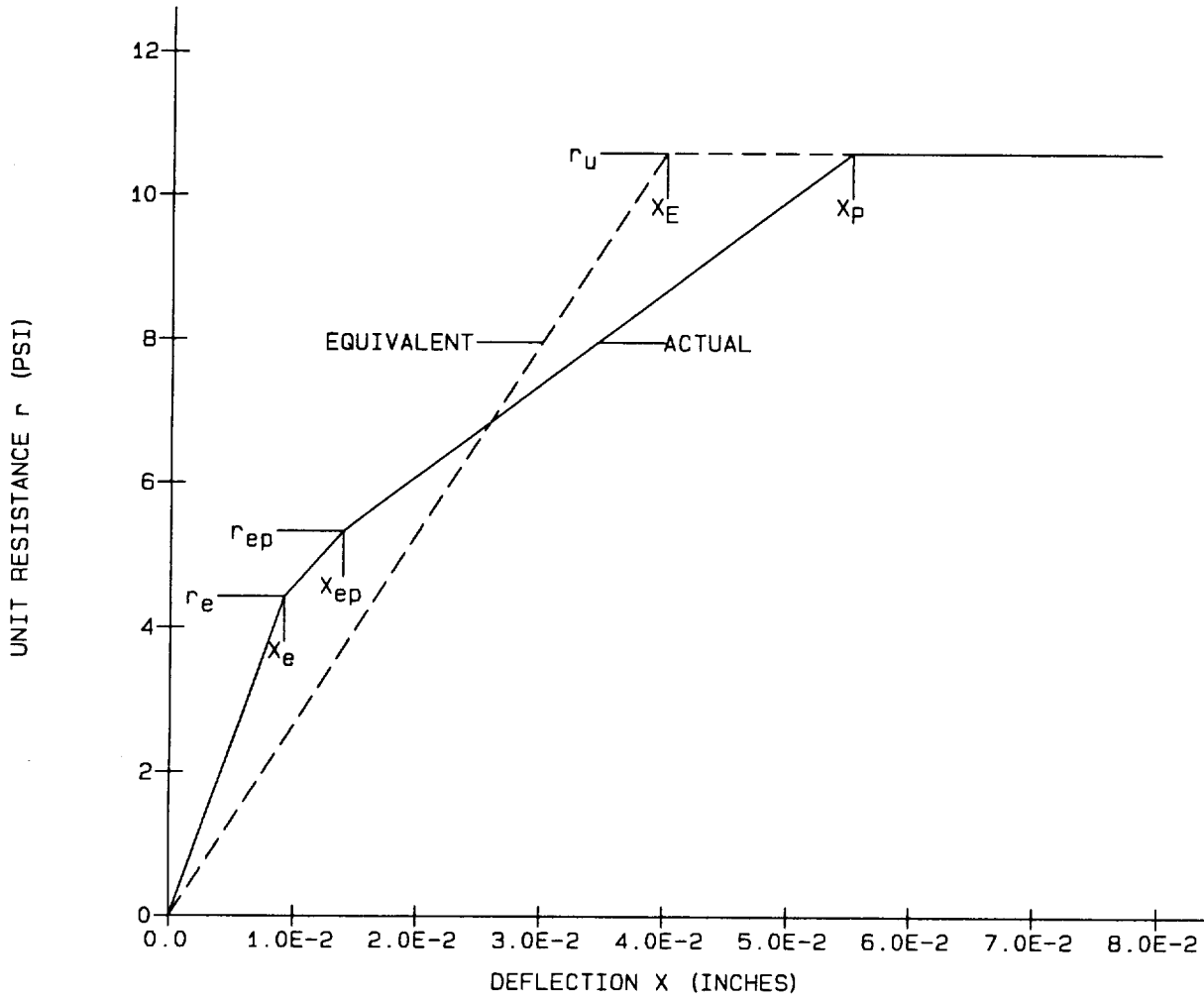


Figure 3A-7

- b. For concentrated loads and concentrated mass determine the deflection at the load.
- Step 5. Calculate the shape function
- a. For distributed load and/or continuous mass calculate  $\phi(x)$ , equation 3-43.
  - b. For concentrated load and concentrated mass calculate  $\phi_r$ , equation 3-46.
- Step 6. Calculate the load factor,  $K_L$ .
- a. Use equations 3-41 and 3-42 for a distributed load.
  - b. Use equations 3-41 and 3-45 for a concentrated load
- Step 7. Calculate the mass factor,  $K_M$ .
- a. Use equations 3-47 and 3-48 for a continuous mass
  - b. Use equations 3-44 and 3-49 for concentrated mass
- Step 8. Calculate the load-mass factor  $K_{LM}$ , from equation 3-53.

**Example 3A-3(A) Dynamic Design Factors For A One-Way Element**

Required: The load, mass and load-mass factors for a structural steel beam in the elastic range, with a distributed load.

Solution:

Step 1: Given structural steel beam shown in figure 3A-8

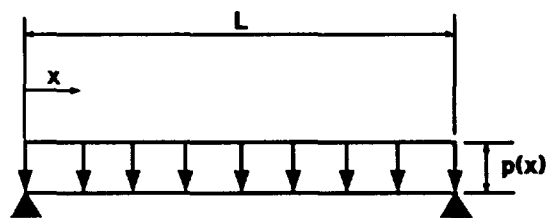


Figure 3A-8

- a.  $L = 120$  in.
- b. Simply supported on both edges
- c.  $p(x) = 2,000$  lb/in  
 $m(x) = 0.0055(\text{lb}\cdot\text{s}^2/\text{in}^4)/\text{in}$

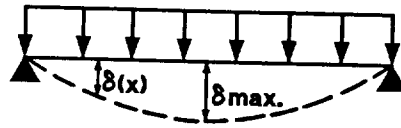


Figure 3A-9

Step 2: Assumed deflected shape for elastic range is shown in figure 3A-9

Step 3: The maximum deflection at the center is

$$\delta_{\max} = \frac{5p(x)L^4}{384 EI}$$

Step 4: Determine deflection function

$$\delta(x) = \frac{p(x)}{24EI} (L^3 - 2Lx^2 + x^3)$$

Step 5: Calculate the shape function using equation 3-43

$$\begin{aligned} \phi &= \frac{\delta(x)}{\delta_{\max}} = \frac{p(x)x}{24EI} (L^3 - 2Lx^2 + x^3) \frac{384EI}{5p(x)L^4} \\ &= \frac{16}{5L^4} (L^3x - 2Lx^3 + x^4) \end{aligned}$$

Step 6: a. Using equation 3-42, determine equivalent force

$$\begin{aligned} F_E &= \int_0^L p(x)\phi(x)dx = \int_0^{120} (2,000 \text{ lb/in}) \frac{16}{5L^4} (L^3x - 2Lx^3 + x^4)dx \\ &= \frac{6,400}{L^4} \left[ \frac{L^3x^2}{2} - \frac{2Lx^4}{4} + \frac{x^5}{5} \right]_0^{120} = 1,280L \\ &= 153,600 \text{ lb.} \end{aligned}$$

b. From equation 3-41, find the load factor

$$K_L = \frac{F_E}{F} = \frac{153,600 \text{ lb.}}{(2,000 \text{ lb/in} \times 120 \text{ in.})}$$

$K_L = 0.64$  in the elastic range

Step 7: a. Find the equivalent mass from equation 3-48

$$\begin{aligned}
 M_E &= \int_0^L m(x)\phi(x)dx = .0055 \frac{256}{25L^8} \int_0^{120} (L^3x - 2Lx^3 + x^4)^2 dx \\
 &= \frac{1.408}{25L^8} \int_0^{120} (L^6x^2 - 4L^4x^4 + 2L^3x^5 + 4L^2x^6 - 4Lx^7 + x^8)dx \\
 &= \frac{1.408}{25L^8} \left[ \frac{L^6x^3}{3} - \frac{4L^4x^5}{5} + \frac{2L^3x^6}{6} + \frac{4L^2x^7}{7} - \frac{4Lx^8}{8} + \frac{x^9}{9} \right]_0^{120} \\
 &= .00277L \\
 &= 0.3325 \text{ lb}^2 \cdot \text{s}^3/\text{in}
 \end{aligned}$$

b. From equation 3-47, calculate the mass factor

$$K_M = \frac{M_E}{M} = \frac{0.3325 \text{ lb} \cdot \text{s}^2 / \text{in}^3}{(0.0055 \text{ lb} \cdot \text{s}^2 / \text{in}^4 \times 120 \text{ in})}$$

$$K_M = 0.50 \text{ in the elastic range}$$

Step 8: Calculate the load-mass factor as defined by equation 3-51

$$\begin{aligned}
 K_{LM} &= K_M/K_L \\
 &= 0.50/0.64
 \end{aligned}$$

$$K_{LM} = 0.78 \text{ in the elastic range}$$

**Example 3A-3(B) Dynamic Design Factors For A One-Way Element**

Required: The load, mass and load-mass factors for a structural steel beam in the plastic range with a distributed load.

Solution:

Step 1. Given the structural steel beam shown in figure 3A-8

- a. L = 120 in.
- b. Simply-supported on both ends
- c. p(x) = 2,000 lb/in
- d. m(x) = 0.0055 (lb · s<sup>2</sup>/in<sup>4</sup>)/in

Step 2. Assume deflected shape for the plastic range is shown in figure 3A-10

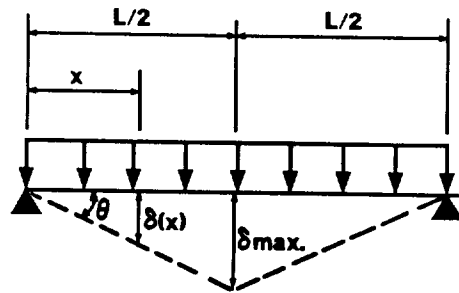


Figure 3A-10

Step 3. Determine maximum deflection

$$\delta_{\max} = (L/2)\tan\theta$$

Step 4. Determine the deflection at any point.

$$\delta(x) = x\tan\theta \quad x < L/2$$

Step 5. Calculate the shape function, equation 3-43

$$\begin{aligned} \phi(x) &= \frac{\delta(x)}{\delta_{\max}} = \frac{x\tan\theta}{(L/2)\tan\theta} \\ &= 2x/L \quad x < L/2 \end{aligned}$$

Step 6:

a. Find  $F_E$  using equation 3-42.

$$\begin{aligned} F_E &= \int_0^L p(x)\phi(x)dx = 2 \int_0^{60} 2,000 \text{ lb/in} (2x/L)dx \\ &= 4,000 \text{ lb/in} \left[ \frac{x^2}{L} \right]_0^{60} \\ &= 120,000 \text{ lb} \end{aligned}$$

b. From equation 3-41

$$K_L = \frac{F_E}{F} = \frac{120,000 \text{ lb.}}{(2,000 \text{ lb/in}) 120 \text{ in}}$$

$$K_L = 0.5 \text{ in the plastic range}$$

Step 7:

a. Use equation 3-48 to find the equivalent mass

$$\begin{aligned} M_E &= \int_0^L m(x)\phi^2(x)dx = 2 \int_0^{60} (0.0055) (4 x^2/L^2)dx \\ &= 0.044 \left[ \frac{x^3}{3L^2} \right]_0^{60} \\ &= 0.22 \text{ lb} \cdot \text{s}^2/\text{in}^3 \end{aligned}$$

b. As defined by equation 3-47

$$K_M = \frac{M_E}{M} = \frac{0.22 \text{ lb} \cdot \text{s}^2 / \text{in}^3}{(0.0055 \text{ lb} \cdot \text{s}^2 / \text{in}^4) 120 \text{ in}}$$

$$K_M = 0.33 \text{ in the plastic range}$$

Step 8. Calculate  $K_{LM}$  using equation 3-53

$$K_{LM} = K_M / K_L$$

$$= 0.33 / 0.5$$

$$K_{LM} = 0.66 \text{ in the plastic range}$$

**Example 3A-3(C) Dynamic Design Factors For A One-Way Element**

Required: The load, mass and load-mass factors for a structural steel beam in the elastic range, with a concentrated load.

Solution:

Step 1: Given structural steel beam shown in figure 3A-11

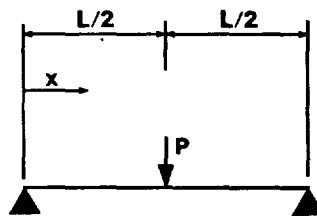


Figure 3A-11

- a.  $L = 120 \text{ in.}$
- b. Simply supported on both sides
- c.  $F = 240 \text{ kips}$

$$m(x) = 0.0055(\text{lb} \cdot \text{s}^2 / \text{in}^4) / \text{in.}$$

Step 2: Assume deflected shape for elastic range is shown in figure 3A-12

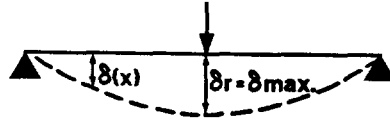


Figure 3A-12

Step 3: Determine maximum deflection

$$\delta_{\max} = \frac{PL^3}{48EI}$$

Step 4: Determine deflection functions

a. for continuous mass,

$$\delta(x) = \frac{Px}{48EI} (3L^2 - 4x^2)$$

b. for concentrated load

$$\delta_r = \frac{PL^3}{48EI}$$

Step 5: Calculate shape functions

a. for continuous mass use equation 3-43

$$\begin{aligned} \phi(x) &= \frac{\delta(x)}{\delta_{\max}} = \frac{Px(3L^2 - 4x^2)}{48EI} \frac{48EI}{PL^3} \\ &= (3L^2x - 4x^3)/L^3 \end{aligned}$$

b. for concentrated load, use equation 3-46

$$\begin{aligned} \phi_r &= \frac{PL^3}{48EI} \frac{48EI}{PL^3} \\ &= 1.0 \end{aligned}$$

Step 6:

a. Find equivalent force from equation 3-45

$$F_E = \sum_i F_r \phi_r = Px1 = 240 \text{ kips}$$

b. Using equation 3-41, calculate the load factor

$$K_L = \frac{F_E}{F} = \frac{240 \text{ kips}}{240 \text{ kips}}$$

$$K_L = 1.0 \text{ for the elastic range}$$

Step 7: a. Equation 3-48 gives the equivalent mass.

$$\begin{aligned} M_E &= \int_0^L m(x)\phi^2(x) dx = \int_0^L \frac{120 (0.0055)}{L^6} (9L^4x^2 - 24L^2x^4 + 16x^6) dx \\ &= \frac{0.0055}{L^6} \left[ \frac{3L^4 x^3}{3} - \frac{24L^2 x^5}{5} + \frac{16x^7}{7} \right]_0^{120} \\ &= 0.0027L \\ &= 0.321b - s^2/in^3 \end{aligned}$$

b. From equation 3-47, calculate the mass factor

$$K_M = \frac{M_E}{M} = \frac{0.321b - s^2/in^3}{(0.00551b - s^2/in^4 \times 120in)}$$

$$K_M = 0.49 \text{ in the elastic range}$$

Step 8: Calculate the load-mass factor, from equation 3-53

$$\begin{aligned} K_{LM} &= K_M/K_L \\ &= 0.49/1.0 \end{aligned}$$

$$K_{LM} = 0.49 \text{ for the elastic range}$$

**Example 3A-3(D) Dynamic Design Factors For A One-Way Element**

**Required:** Determine the load, mass and the load-mass factors for a structural steel beam, in the plastic range, with a concentrated load.

**Solution:**

- Step 1. Given structural steel beams shown in figure 3A-11.
- a. L = 120 in
  - b. Simply-supported at both edges



c.  $F = 240$  kips

$$m(x) = 0.0055 \text{ (lb} \cdot \text{s}^2/\text{in}^4\text{)}/\text{in}$$

Step 2. Assumed deflected shape for the plastic range is shown in figure 3A-13

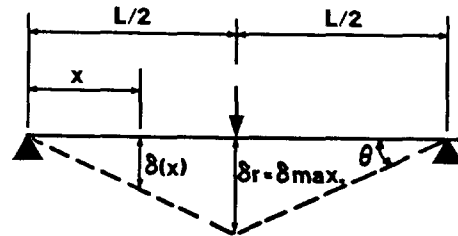


Figure 3A-13

Step 3: Determine maximum deflection

$$\delta_{max} = (L/2)\tan\theta$$

Step 4: Determine deflection function.

a. for continuous mass

$$\delta(x) = x\tan\theta \quad x < L/2$$

b. for a concentrated load

$$\delta_r = (L/2)\tan\theta$$

Step 5: Calculate shape factors using

a. equation 3-43 for continuous mass

$$\begin{aligned} \phi(x) &= \frac{\delta(x)}{\delta_{max}} = \frac{x\tan\theta}{(L/2)\tan\theta} \\ &= 2x/L \quad x < L/2 \end{aligned}$$

b. equation 3-46 for concentrated load

$$\begin{aligned} \phi_r &= \frac{\delta_r}{\delta_{max}} = \frac{(L/2)\tan\theta}{(L/2)\tan\theta} \\ &= 1.0 \end{aligned}$$

Step 6: a. The equivalent force is found using equation 3-45

$$F_E = \sum_{r=1}^i F_r \phi_r = P x_1 = 240 \text{ kips}$$

b. Equation 3-41 gives the load factor

$$K_L = \frac{F_E}{F} = \frac{240 \text{ kips}}{240 \text{ kips}}$$

$$K_L = 1.0 \text{ for plastic range}$$

Step 7: a. The equivalent mass is found using equation 3-48

$$M_E = \int_0^L m(x) \phi^2(x) dx = 2 \int_0^{60} (0.0055) (4 x^2/L^2) dx$$

$$= 0.044 \left[ \frac{x^3}{3L^2} \right]_0^{60} = 0.22 \text{ lb} \cdot \text{s}^2/\text{in}^3$$

b. Solve for  $K_M$  using equation 3-47

$$K_M = \frac{M_E}{M} = \frac{0.22 \text{ lb} \cdot \text{s}^2/\text{in}^3}{(0.0055 \text{ lb} \cdot \text{s}^2/\text{in}^4) 120 \text{ in}}$$

$$K_M = 0.33 \text{ in the plastic range}$$

Step 8: From equation 3-53, calculate  $K_{LM}$

$$K_{LM} = K_M/K_L$$

$$= 0.33 \text{ in the plastic range}$$

#### Problem 3A-4 Plastic Load-Mass Factor

**Problem:** Determine the plastic load-mass factor  $K_{LM}$  for a two-way element using (1) general solution and (2) chart solution.

**Note:** The determination of the plastic load-mass factor follows the calculations for the ultimate resistance, hence the structural configuration and the location of the plastic yield lines will be known.

**Procedure:** Part (a) - General Solution

Step 1. See part a, problem 3A-1 for the structural configuration and location of plastic yield lines. Denote sectors formed by yield lines.

- Step 2. Determine the load-mass factors properties  $I$ ,  $c$ , and  $L'$  for all sectors.
- Step 3. Determine the factor  $I/cL'$  for all sectors.
- Step 4. Calculate the total area of the element.
- Step 5. With values obtained above, calculate the plastic load-mass factor for the element using equation 3-57.

Note: In the above problem, an element of uniform thickness was considered. For non-uniform elements, the load-mass factor is calculated using equation 3-53 where the mass of the individual sectors must be considered.

Procedure: Part (b) - Chart Solution

- Step 1. See part b, problem 3A-1 for structural configuration and location of plastic yield lines in terms of  $x/L$  or  $y/H$ .
- Step 2. For known value of  $X/L$  or  $y/H$  and support condition, determine the load-mass factor for the element from figure 3-44.

Note: Chart solution may be used only if the element conforms to the requirements listed in section 3-17.3

**Example 3A-4 Plastic Load-Mass Factor**

Required: Plastic load-mass factor for the element considered in example 3A-1(A) using (1) general solution and (2) chart solution.

Solution: Part (a) - General Solution

- Step 1. Given structural configuration and location of yield lines shown below (see part a, example 3A-1(A)) in figure 3A-14.

- $L = 240$  in
- $H = 168$  in
- $X = 120$  in
- $y = 137.6$  in
- $T_c = \text{constant}$

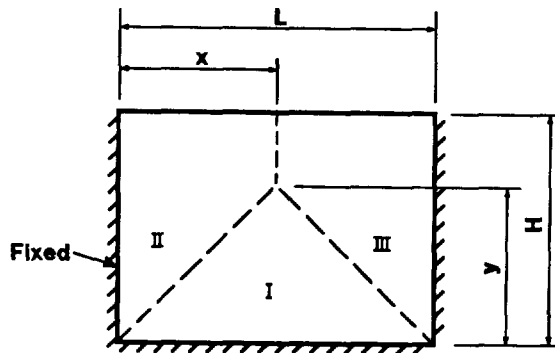


Figure 3A-14

Step 2. Load-mass factor properties.

a. Sector 1.

$$L' = y = 137.6 \text{ in}$$

$$c = y/3 = 137.6/3$$

$$I = L(L')^3 / 12 = 240(137.6)^3 / 12.$$

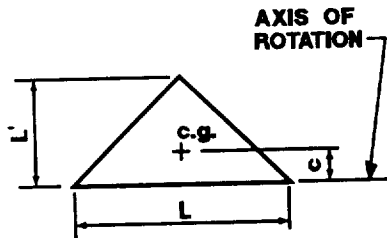


Figure 3A-15

b. Sector II.

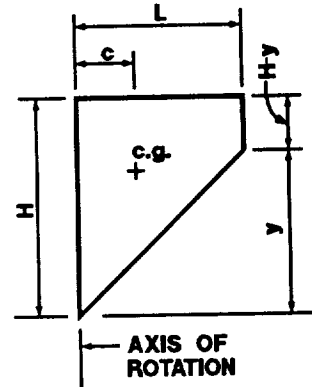


Figure 3A-16

$$L' = x = 120 \text{ in}$$

$$H - y = 168 - 137.6 = 30.4 \text{ in}$$

$$c = \frac{L' [H + 2 (H-y)]}{3 [H + (H-y)]} = \frac{120 [168 + 2 (30.4)]}{3 (168 + 30.4)}$$

$$c = 120 (0.384)$$

$$I = \frac{(H-y)(L')^3}{3} + \frac{y(L')^3}{12}$$

$$= \frac{30.4 (120)^3}{3} + \frac{137.6(120)^3}{12} = 21.60(120)^3$$

Step 3. Calculate factor  $I/cL'$  for each sector:

$$\text{Sector I. } \frac{I}{cL'} = \frac{240(137.6)^3 / 12}{(133.4/3)(133.4)} = 8,256 \text{ in}^2$$

$$\text{Sector II. } \frac{I}{cL'} = \frac{21.60(120)^3}{(0.390 \times 120)(120)} = 6,646 \text{ in}^2$$

$$\text{Sector III. } \frac{I}{cL'} = 6,646 \text{ in}^2$$

Step 4. Area of panel

$$A = LH = 240 (168) = 40,320 \text{ in}^2$$

Step 5. Load-mass factor

$$K_{LM} = \frac{I/cL'}{A} \quad (\text{eq. 6-14})$$

$$K_{LM} = \frac{8,256 + 2(6,646)}{40,320} = 0.534$$

Solution: Part (b) - Chart Solution

Step 1. Given: Panel fixed on 3 edges, 1 free and  $y/H = 0.803$  (see part b, example 3A-1(A)).

Step 2. From figure 3-44, read load-mass factor

$$K_{LM} = 0.543$$

**Problem 3A-5 Response of a Single-Degree-of Freedom System subject to Dynamic Load**

**Problem:** Determine the maximum response and the corresponding time it occurs of a single-degree-of-freedom system subjected to dynamic load using (a) numerical methods and (b) design charts.

**Procedure: Part (a) - Numerical Methods**

- Step 1. Establish dimensional parameters of the system.
- Step 2. Determine the natural period of vibration and integration time interval.
- Step 3. Construct a table similar to table 3-14 of section 3-19.2. Note: For the first interval  $n=1$ , Equation 3-59 is used and subsequent intervals, the recurrence formula (eqn. 3-56) is used.

**Procedure: Part (b) - Chart solution**

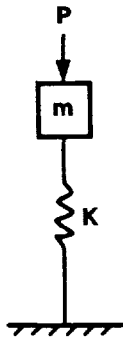
- Step 1. Same as step 1 of example 3A-5, part a.
- Step 2. Determine the non-dimensional parameters.
- Step 3. Determine the ratio of the maximum displacement to the elastic displacement  $X_m/X_E$  and the ratio of the time at which this maximum displacement occurs to the duration of the blast load.

**Example 3A-5 Maximum Response of Single-Degree-of-Freedom System Subjected to a Triangular Load.**

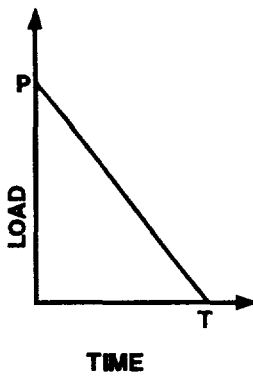
**Required:** The maximum response and the time it occurs, of a single-degree-of-freedom system subjected to blast loads, using (a) numerical methods and (b) design charts.

Solution: Part (a) - Numerical Methods

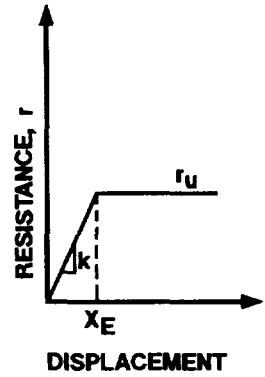
Step 1. Given:



(a) Single-Degree-of-Freedom System



(b) External Load



(c) Resistance Function

Figure 3A-17

$$\begin{aligned}
 m &= 2.5 \text{ Kips-sec}^2/\text{ft} \\
 K &= 9,860 \text{ Kips/ft} \\
 r_u &= 750 \text{ Kips} \\
 X_E &= 0.076 \text{ ft} \\
 T &= 0.10 \text{ sec} \\
 P &= 1000 \text{ Kips}
 \end{aligned}$$

Step 2. Natural period of vibration and integration time interval.

$$T_N = 2\pi [m/K]^{1/2} = 2\pi [2.5/9,860]^{1/2} = 0.10 \text{ sec}$$

$$\tau = T_N/10 = 0.01 \text{ sec}$$

Step 3. Construct table as shown below.

1	2	3	4	5	6	7	8	9	10
n	t	P <sub>n</sub>	R <sub>n</sub>	P <sub>n</sub> -R <sub>n</sub>	A <sub>n</sub> =(P <sub>n</sub> -R <sub>n</sub> /m	a <sub>n</sub> (Δt) <sup>2</sup>	2X <sub>n</sub>	X <sub>n-1</sub>	X <sub>n+1</sub>
	(sec)	Kips	(Kips)	(Kips)	(ft/sec <sup>2</sup> )	(ft)	(ft)	(ft)	(ft)
0	0	1000	0	1000	400	0.040	0.0	0.0	0.020
1	0.01	900	197.200	702.800	281.120	0.028112	0.0400	0.0	0.068112
2	0.02	800	671.684	128.426	51.366	0.05137	0.13622	0.020	0.121357
3	0.03	700	750	-50.0	-20.0	-0.00200	0.242714	0.06811	0.17261
4	0.04	600	750	-150.0	-60.0	-0.00600	0.34522	0.121357	0.21786
5	0.05	500	750	-250.0	-100.0	-0.0100	0.43673	0.17261	0.25312
6	0.06	40	750	-350.0	-140.0	-0.0140	0.050623	0.21786	0.27437
7	0.07	300	750	-450.0	-180.0	-0.0180	0.54874	0.25312	0.27762
8	0.08	200	750	-550.0	-220.0	-0.0220	0.55525	0.27437	0.25880
9	0.09								

Note:  $P_n = f(t_n) = 1000 [1 - n(\Delta t/T)]$   $R_n = \begin{cases} KX_n & \text{for } X_n < X_e \\ r_u = 750 & \text{for } X_n > X_e \end{cases}$

$$X_{n+1} = 2X_n - X_{n-1} + a_n (\Delta t)^2$$

Note:

$$\begin{aligned} \text{For } n=0, X_{n+1} \text{ (Column 10)} &= X_{0+1} = X_1 = (1/2)a_0(\Delta t)^2 \\ &= (1/2)(0.040) \\ &= 0.02 \text{ ft} \end{aligned}$$

$$\begin{aligned} \text{For } n=1, 2X_n \text{ (Column 8)} &= 2X_1 = 2\{(1/2)a_0(\Delta t)^2\} \\ &= 2(0.02) = 0.04 \text{ ft} \end{aligned}$$

$$X_{n-1} \text{ (Column 9)} = X_0 = 0.0$$

$$\begin{aligned} X_{n+1} \text{ (Column 10)} &= X_2 = 2X_1 - X_0 + a_1(\Delta t)^2 \\ &= 2(0.02) - 0 + 0.02811 \\ &= 0.06811 \text{ ft} \end{aligned}$$

$$\begin{aligned} \text{For } n=2, 2X_n \text{ (Column 8)} &= 2X_2 = 2(0.06811) \\ &= 0.13622 \text{ ft.} \end{aligned}$$

$$\begin{aligned}
 X_{n-1} \text{ (Column 9)} &= X_1 = 0.02 \text{ ft.} \\
 X_{n+1} \text{ (Column 10)} &= X_3 = 2X_2 - X_1 + a_2 (\Delta t)^2 \\
 &= (2)(0.06811) - 0.02 + 0.005137 \\
 &= 0.121357 \text{ ft.}
 \end{aligned}$$

For  $n=3, 4, \dots$ , repeat the above procedure.

**Solution: Part (b) - Design Charts**

Step 1. Same as step 1 of example 3A-5, part a

Step 2. Non-dimensional parameters

a. Natural period of vibration,  $T_n$

$$T_n = 2\pi[m/K]^{1/2} = 2\pi[2.5/9,860]^{1/2} = 0.10 \text{ sec}$$

b. Ratio of duration of blast load  $T$  to natural period  $T_n$

$$T/T_n = 0.10/0.10 = 1.0$$

c. Ratio of peak resistance  $r_u$  to peak load  $P$

$$r_u/P = 750/1000 = 0.75$$

Step 3. Using the ratios calculated in step 2 and figures 3-54 and 3-55, determine the value of  $X_m/X_E$  and  $t_m/T_n$ .

$$\text{For } T/T_n = 1 \quad \text{and} \quad r_u/P = 0.75$$

$$X_m/X_E = 3.7 \quad \text{from figure 3-54}$$

$$t_m/T = 0.77 \quad \text{from figure 3-55}$$

Step 4. Determine  $X_m$  and  $t_m$

$$X_m/X_E = 3.7$$

$$\begin{aligned}
 X_m &= (3.7)X_E = (3.7)(r_u/K_E) \\
 &= (3.7)(750/9,860) = 0.28144 \text{ ft.}
 \end{aligned}$$

$$t_m/T = 0.77$$

$$t_m = (0.77)T = 0.77(0.10) = 0.077 \text{ sec}$$



**Problem 3A-6 Maximum Response of a Single-Degree-of-Freedom System to Bilinear Blast Loads**

**Problem:** Determine  $X_m/X_E$ ,  $t_m/T_N$  and  $t_E/T$  (when applicable) for a single-degree-of-freedom system subject to various bilinear blast loads.

**Procedure: Part (a) - Solution in Region D**

- Step 1. Establish normalized parameters
- Step 2. Enter table 3-15 with the given C parameters and determine which figures have to be used.
- Step 3. Enter each of the figures determined in step 2, with the given values of the other two parameters and determine the region where the intersection points are located.
- Step 4. Based on the region where the intersection points are located, enter the appropriate figure and find  $X_m/X_E$ ,  $t_m/T_N$  and  $t_E/T$ .

**Procedure: Part (b) - Solution in Region C - Graphical Interpolation.**

- Step 1. Same as step 1 in part a.
- Step 2. Same as step 2 in part a.
- Step 3. Same as step 3 in part a.
- Step 4. Set up a table as shown in table 3A-1. Post each figure number and the corresponding values of  $C_1$  and  $C_2$ , leaving a space between each line of information. Post in the spaces the appropriate values of  $C_1$  and  $C_2$  needed for interpolation. Enter each of the figures determined in Step 2 with the given parameters and find the values of  $X_m/X_E$ . Post these values in table 3A-1.
- Step 5. Use log-log graph paper to plot the points obtained in Step 4. Post these values in table 3A-1, using linear interpolation where necessary.
- Step 6. Plot on log-log graph paper the points which represent  $(X_m/X_E, C_2)$  for the given value of C. Use linear interpolation to find  $X_m/X_E$  for given value of  $C_2$ .

**Procedure: Part (c) - Solution in Region C - Mathematical Interpolation**

- Step 1. Same as step 1 in part a.
- Step 2. Same as step 2 in part a.
- Step 3. Same as step 3 in part a.



Step 4. Same as step 4 in part b.

Step 5. Solve for  $\ln Y_a$  and  $\ln Y_b$  using equations 3-83 and 3-84.

Step 6. Solve for  $\ln Y$  using equation 3-85.

Step 7. Solve for  $Y$  using equation 3-86.

**Example 3A-6      Maximum Response of a Single-Degree-of-Freedom System to Bilinear Blast Loads**

Required: Determine  $X_m/X_E$ ,  $t_m/T_N$  and  $t_E/T$  (when applicable) for a single-degree-of-freedom system subject to various bilinear blast loads.

Solution: Part (a) - Solution in Region D

Step 1. Given:  $P/r_u = 1.0$

$T/T_N = 3.0$

$C_1 = 0.66$

$C_2 = 50$

Step 2. Enter table 3-15 with  $C_1 = 0.66$  and  $C_2 = 50$ . Note figures 3-119, 3-120, 3-147, and 3-148 apply.

Step 3. Enter each of the figures determined in step 2, with  $P/r_u = 1.0$  and  $T/T_N = 3.0$ . Note that the intersection point is located to the right of the line of solid squares, defined as region D. In region D, the maximum dynamic response depends only on the shock load described by  $P/r_u$  and  $T/T_N$ ; the gas load described by  $C_1 P/r_u$  and  $C_2 T/T_N$  does not influence the maximum dynamic response. Consequently, figures 3-64a and 3-64b for a single triangular load pulse apply. Enter figure 3-64a with  $P/r_u = 1.0$  and  $T/T_N = 3.0$  and find  $X_m/X_E = 3.55$ ,  $t_m/T_N = 0.98$ ,  $t_E/T = 0.086$ .

Solution: Part (b) - Solution in Region C - Graphical Interpolation

Step 1. Given:  $P/r_u = 32$

$T/T_N = 0.10$

$C_1 = 0.06$

$C_2 = 20$

Step 2. Enter table 3-15 with  $C_1 = 0.06$  and  $C_2 = 20$ . Note figures 3-108, 3-109, 3-133 and 3-134 apply.

- Step 3. Enter each of these figures with  $P/r_u = 32$  and  $T/T_n = 0.10$ . Note that the intersection point is not located in regions A, B or D. Therefore the intersection points lie in region C and interpolation between charts is required to obtain a solution.
- Step 4. Set up table as shown in table 3A-1 below. Post each chart number and the corresponding values of  $C_1$  and  $C_2$  leaving a space between each line of information. Post in the spaces the appropriate values of  $C_1$  and  $C_2$  needed for interpolation. Enter figure 3-108 with  $P/r_u = 32$  and  $T/T_n = 0.10$  and find  $X_m/X_E = 112$ . Post this value in the table. Enter figure 3-109 with  $P/r_u = 32$  and  $T/T_n = 0.10$  and find  $X_m/X_E = 86$ . Post this value in the table. Repeat this process for figures 3-133 and 3-134, and post values for  $X_m/X_E$  in the table.
- Step 5. Use log-log graph paper to plot the points (112,0.068) and (86,0.046) which represent  $(X_m/X_E, C_1)$  for  $C_2 = 10$  as shown in figure 3-267. Use straight-line interpolation to find  $X_m/X_E = 103$  for  $C_1 = 0.060$ . Post this value in the table. Repeat this process for  $C_2 = 30$ , and find  $X_m/X_E = 251$  for  $C_1 = 0.06$  as shown in figure 3-267.
- Step 6. Plot on log-log graph paper the points (103,10) and (251,30) which represent  $(X_m/X_E, C_2)$  for  $C_1 = 0.060$ . Use straight-line interpolation for finding  $X_m/X_E = 182$  for  $C_2 = 50$  as shown in figure 3-267. Thus the solution is  $X_m/X_E = 182$ .

Table 3A-1

Figure No.	$C_1$	$C_2$	$X_m/X_E$
3-108	0.068		112
	0.060	10	--
3-109	0.046	10	86
	0.060	20	--
3-133	0.075	30	340
	0.060	30	--
3-134	0.056	30	230

**Solution: Part (c) - Solution in region C - Mathematical Interpolation**

Step 1. Same as step 1 of part (b).

Step 2. Same as step 2 of part (b).

Step 3. Same as step 3 of part (b).

Step 4. Same as step 4 of part (b).

Step 5. Using equation 3-83 and 3-84, find  $\ln Y_a$  and  $\ln Y_b$ .

$$\begin{aligned} \ln Y_a &= \ln Y_1 + \frac{\ln[Y_2/Y_1] \ln[C_1/C_{11}]}{\ln[C_{12}/C_{11}]} \\ &= \ln 112 + \frac{\ln(86/112)\ln(0.060/0.068)}{\ln(0.046/0.068)} \end{aligned}$$

$$\ln Y_a = 4.6339$$

$$\begin{aligned} \ln Y_b &= \ln Y_3 + \frac{\ln[Y_4/Y_3] \ln[C_1/C_{13}]}{\ln[C_{14}/C_{13}]} \\ &= \ln 340 + \frac{\ln(230/340)\ln(0.06/0.075)}{\ln(0.056/0.075)} \end{aligned}$$

$$\ln Y_b = 5.5304$$

Step 6. Find  $\ln Y$  from equation 3-85

$$\begin{aligned} \ln Y &= \ln Y_a + \frac{(\ln Y_b - \ln Y_a)\ln(C_2/C_{21})}{\ln(C_{23}/C_{21})} \\ &= 4.6339 + \frac{(5.5304 - 4.6339)\ln(20/10)}{\ln(30/10)} \end{aligned}$$

$$\ln Y = 5.1995$$

Step 7. Solve for Y using equation 3-86

$$\begin{aligned} Y &= e^{\ln Y} \\ &= e^{5.1995} \\ Y &= 181 \end{aligned}$$

**APPENDIX 3B**  
**LIST OF SYMBOLS**

a	(1) acceleration (in./ms <sup>2</sup> ) (2) depth of equivalent rectangular stress block (in.)
A	area (in. <sup>2</sup> )
A <sub>a</sub>	area of diagonal bars at the support within a width b (in. <sup>2</sup> )
A <sub>O</sub>	area of openings (ft <sup>2</sup> )
A <sub>S</sub>	area of tension reinforcement within a width b (in. <sup>2</sup> )
A' <sub>S</sub>	area of compression reinforcement within a width b (in. <sup>2</sup> )
A <sub>SH</sub>	area of flexural reinforcement within a width b in the horizontal direction on each face (in. <sup>2</sup> )*
A <sub>SV</sub>	area of flexural reinforcement within a width b in the vertical direction on each face (in. <sup>2</sup> )*
A <sub>V</sub>	total area of stirrups or lacing reinforcement in tension within a distance, s <sub>s</sub> or s <sub>l</sub> and a width b <sub>s</sub> or b <sub>l</sub> (in. <sup>2</sup> ).
A <sub>I</sub> , A <sub>II</sub>	area of sector I and II, respectively (in. <sup>2</sup> )
b	(1) width of compression face of flexural member (in.) (2) width of concrete strip in which the direct shear stresses at the supports are resisted by diagonal bars (in.)
b <sub>s</sub>	width of concrete strip in which the diagonal tension stresses are resisted by stirrups of area A <sub>V</sub> (in.)
b <sub>l</sub>	width of concrete strip in which the diagonal tension stresses are resisted by lacing of area A <sub>V</sub> (in.)
B	constant defined in paragraph
c	(1) distance from the resultant applied load to the axis of rotation (in.) (2) damping coefficient
c <sub>I</sub> , c <sub>II</sub>	distance from the resultant applied load to the axis of rotation for sectors I and II, respectively (in.)
c <sub>s</sub>	dilatational velocity of concrete (ft/sec)
C	shear coefficient
C <sub>cr</sub>	critical damping

$C_d$	shear coefficient for ultimate shear stress of one-way elements
$C_f$	post-failure fragment coefficient ( $lb^2\text{-ms}^4/in.^8$ )
$C_{r\alpha}$	peak reflected pressure coefficient at angle of incidence $\alpha$
$C_s$	shear coefficient for ultimate support shear for one-way elements
$C_{sH}$	shear coefficient for ultimate support shear in horizontal direction for two-way elements*
$C_{sV}$	shear coefficient for ultimate support shear in vertical direction for two-way elements*
$C_D$	drag coefficient
$C_{Dq}$	drag pressure (psi)
$C_{Dq_0}$	peak drag pressure (psi)
$C_E$	equivalent load factor
$C_H$	shear coefficient for ultimate shear stress in horizontal direction for two-way elements*
$C_L$	leakage pressure coefficient
$C_M$	maximum shear coefficient
$C_u$	impulse coefficient at deflection $X_u$ ( $psi\text{-ms}^2/in.^2$ )
$C_u'$	impulse coefficient at deflection $X_m$ ( $psi\text{-ms}^2/in.^2$ )
$C_v$	shear coefficient for ultimate shear stress in vertical direction for two-way elements*
$C_1$	(1) impulse coefficient at deflection $X_1$ ( $psi\text{-ms}^2/in.^2$ ) (2) parameter defined in figure (3) ratio of gas load to shock load
$C_1'$	impulse coefficient at deflection $X_m$ ( $psi\text{-ms}^2/in.^2$ )
$C_2$	ratio of gas load duration to shock load duration
$d$	distance from extreme compression fiber to centroid of tension reinforcement (in.)
$d'$	distance from extreme compression fiber to centroid of compression reinforcement (in.)
$d_c$	distance between the centroids of the compression and tension reinforcement (in.)

\* See note at end of symbols

$d_e$	distance from support and equal to distance $d$ or $d_c$ (in.)
$d_i$	inside diameter of cylindrical explosive container (in.)
$d_l$	distance between center lines of adjacent lacing bends measured normal to flexural reinforcement (in.)
$d_{co}$	diameter of steel core (in.)
$d_1$	diameter of cylindrical portion of primary fragment (in.)
$D$	(1) unit flexural rigidity (lb-in.) (2) location of shock front for maximum stress (ft) (3) minimum magazine separation distance (ft)
$D_o$	nominal diameter of reinforcing bar (in.)
$D_E$	equivalent loaded width of structure for non-planar wave front (ft)
DIF	dynamic increase factor
DLF	dynamic load factor
$e$	base of natural logarithms and equal to 2.71828...
$(2E')^{1/2}$	Gurney Energy Constant (ft/sec)
$E$	modulus of elasticity
$E_c$	modulus of elasticity of concrete (psi)
$E_s$	modulus of elasticity of reinforcement (psi)
$f$	unit external force (psi)
$f'_c$	static ultimate compressive strength of concrete at 28 days (psi)
$f'_{dc}$	dynamic ultimate compressive strength of concrete (psi)
$f_{ds}$	dynamic design stress for reinforcement (psi)
$f_{du}$	dynamic ultimate stress of reinforcement (psi)
$f_{dy}$	dynamic yield stress of reinforcement (psi)
$f_s$	static design stress for reinforcement (a function of $f_y$ , $f_u$ and $\theta$ ) (psi)
$f_u$	static ultimate stress of reinforcement (psi)
$f_y$	static yield stress of reinforcement (psi)



F	(1) total external force (lbs) (2) coefficient for moment of inertia of cracked section (3) function of $C_2$ and $C_1$ for bilinear triangular load
$F_o$	force in the reinforcing bars (lbs)
$F_E$	equivalent external force (lbs)
g	variable defined in table 4-3
h	charge location parameter (ft)
H	(1) span height (in.) (2) distance between reflecting surface(s) and/or free edge(s) in vertical direction (ft)
$H_c$	height of charge above ground (ft)
$H_c$	scaled height of charge above ground (ft/lb <sup>1/3</sup> )
$H_s$	height of structure (ft)
$H_T$	scaled height of triple point (ft/lb <sup>1/3</sup> )
i	unit positive impulse (psi-ms)
$i^-$	unit negative impulse (psi-ms)
$\bar{i}_a$	sum of scaled unit blast impulse capacity of receiver panel and scaled unit blast impulse attenuated through concrete and sand in a composite element (psi-ms/lb <sup>1/3</sup> )
$i_b$	unit blast impulse (psi-ms)
$\bar{i}_b$	scaled unit blast impulse (psi-ms/lb <sup>1/3</sup> )
$\bar{i}_{bt}$	total scaled unit blast impulse capacity of composite element (psi-ms/lb <sup>1/3</sup> )
$\bar{i}_{ba}$	scaled unit blast impulse capacity of receiver panel of composite element (psi-ms/lb <sup>1/3</sup> )
$\bar{i}_{bd}$	scaled unit blast impulse capacity of donor panel of composite element (psi-ms/lb <sup>1/3</sup> )
$i_e$	unit excess blast impulse (psi-ms)
$i_r$	unit positive normal reflected impulse (psi-ms)
$i_r^-$	unit negative normal reflected impulse (psi-ms)
$i_s$	unit positive incident impulse (psi-ms)
$i_s^-$	unit negative incident impulse (psi-ms)

\* See note at end of symbols

I	moment of inertia (in. <sup>4</sup> )
I <sub>a</sub>	average of gross and cracked moments of inertia of width b (in. <sup>4</sup> )
I <sub>c</sub>	moment of inertia of cracked concrete section of width b (in. <sup>4</sup> )
I <sub>g</sub>	moment of inertia of gross concrete section of width b (in. <sup>4</sup> )
I <sub>m</sub>	mass moment of inertia (lb-ms <sup>2</sup> -in.)
j	ratio of distance between centroids of compression and tension forces to the depth d
k	constant defined in paragraph
K	(1) unit stiffness (psi-in for slabs) (lb/in/in for beams) (2) constant defined in paragraph
K <sub>e</sub>	elastic unit stiffness (psi-in for slabs) (lb/in/in for beams)
K <sub>ep</sub>	elasto-plastic unit stiffness (psi-in for slabs) (psi for beams)
K <sub>E</sub>	equivalent elastic unit stiffness (psi-in for slabs) (psi for beams)
K <sub>L</sub>	load factor
K <sub>LM</sub>	load-mass factor
(K <sub>LM</sub> ) <sub>u</sub>	load-mass factor in the ultimate range
(K <sub>LM</sub> ) <sub>up</sub>	load-mass factor in the post-ultimate range
K <sub>M</sub>	mass factor
K <sub>R</sub>	resistance factor
K <sub>1</sub>	factor defined in paragraph
KE	kinetic energy
l	charge location parameter (ft)
l <sub>p</sub>	spacing of same type of lacing bar (in.)
L	(1) span length (in.) except in chapter 4 (ft)* (2) distance between reflecting surface(s) and/or free edge(s) in horizontal direction (ft)
L <sub>1</sub>	length of lacing bar required in distance s <sub>1</sub> (in.)

$L_o$	embedment length of reinforcing bars (in.)
$L_w$	wave length of positive pressure phase (ft)
$L_w^-$	wave length of negative pressure phase (ft)
$L_{wb}, L_{wd}$	wave length of positive pressure phase at points b and d, respectively (ft)
$L_1$	total length of sector of element normal to axis of rotation (in.)
$m$	unit mass (psi-ms <sup>2</sup> /in.)
$m_a$	average of the effective elastic and plastic unit masses (psi-ms <sup>2</sup> /in.)
$m_e$	effective unit mass (psi-ms <sup>2</sup> /in.)
$m_u$	effective unit mass in the ultimate range (psi-ms <sup>2</sup> /in.)
$m_{up}$	effective unit mass in the post-ultimate range (psi-ms <sup>2</sup> /in.)
$M$	(1) unit bending moment (in-lbs/in.) (2) total mass (lb-ms <sup>2</sup> /in.)
$M_e$	effective total mass (lb-ms <sup>2</sup> /in.)
$M_u$	ultimate unit resisting moment (in-lbs/in.)
$M_c$	moment of concentrated loads about line of rotation of sector (in.-lbs)
$M_A$	fragment distribution parameter
$M_E$	equivalent total mass (lb-ms <sup>2</sup> /in.)
$M_{HN}$	ultimate unit negative moment capacity in horizontal direction (in.-lbs/in.)*
$M_{HP}$	ultimate unit positive moment capacity in horizontal direction (in.-lbs/in.)*
$M_N$	ultimate unit negative moment capacity at supports (in.-lbs/in.)
$M_P$	ultimate unit positive moment capacity at midspan (in.-lbs/in.)
$M_{VN}$	ultimate unit negative moment capacity in vertical direction (in.-lbs/in.)*
$M_{VP}$	ultimate unit positive moment capacity in vertical direction (in.-lbs/in.)*

\* See note at end of symbols

n	(1) modular ratio (2) number of time intervals
N	number of adjacent reflecting surfaces
$N_f$	number of primary fragments larger than $W_f$
p	reinforcement ratio equal to $\frac{A_s}{bd}$ or $\frac{A_s}{bd_c}$
p'	reinforcement ratio equal to $\frac{A_s'}{bd}$ or $\frac{A_s'}{bd_c}$
$P_b$	reinforcement ratio producing balanced conditions at ultimate strength
$P_m$	mean pressure in a partially vented chamber (psi)
$P_{mo}$	Peak mean pressure in a partially vented chamber (psi)
$P_H$	reinforcement ratio in horizontal direction on each face*
$P_T$	reinforcement ratio equal to $P_H + P_V$
$P_V$	reinforcement ratio in vertical direction on each face*
$p(x)$	distributed load per unit length
P	(1) pressure (psi) (2) concentrated load (lbs)
$P^-$	negative pressure (psi)
$P_i$	interior pressure within structure (psi).
$\Delta P_i$	interior pressure increment (psi)
$P_f$	fictitious peak pressure (psi)
$P_o$	peak pressure (psi)
$P_r$	peak positive normal reflected pressure (psi)
$P_r^-$	peak negative normal reflected pressure (psi)
$P_{r\alpha}$	peak reflected pressure at angle of incidence $\alpha$ (psi)
$P_s$	positive incident pressure (psi)
$P_{sb}, P_{se}$	positive incident pressure at points b and e, respectively (psi)

$P_{so}$	peak positive incident pressure (psi)
$P_{so}^-$	peak negative incident pressure
$P_{sob}, P_{sod}, P_{soe}$	peak positive incident pressure at points b, d, and e, respectively (psi)
$q$	dynamic pressure (psi)
$q_b, q_e$	dynamic pressure at points b and e, respectively (psi)
$q_o$	peak dynamic pressure (psi)
$q_{ob}, q_{oe}$	peak dynamic pressure at points b and e, respectively (psi)
$r$	(1) unit resistance (psi) (2) radius of spherical TNT (density equals 95 lb/ft <sup>3</sup> charge (ft))
$r^-$	unit rebound resistance (psi)
$\Delta r$	change in unit resistance (psi)
$r_e$	elastic unit resistance
$r_{ep}$	elasto-plastic unit resistance (psi)
$r_u$	ultimate unit resistance (psi, for slabs) (lb/in for beams)
$r_{up}$	post-ultimate unit resistant (psi)
$r_1$	radius of hemispherical portion of. primary fragment (in.)
$R$	(1) total internal resistance (lbs) (2) slant distance (ft)
$R_f$	distance traveled by primary fragment (ft)
$R_l$	radius of lacing bend (in.)
$R_A$	normal distance (ft)
$R_E$	equivalent total internal resistance (lbs)
$R_G$	ground distance (ft)
$R_u$	total ultimate resistance
$R_I, R_{II}$	total internal resistance of sectors I and II, respectively (lbs)
$s_s$	spacing of stirrups in the direction parallel to the longitudinal reinforcement (in.)

$s_1$	spacing of lacing in the direction parallel to the longitudinal reinforcement (in.)
S	height of front wall or one-half its width, whichever is smaller (ft)
SE	strain energy
t	time (ms)
$\Delta t$	time increment (ms)
$t_a$	any time (ms)
$t_b, t_e, t_f$	time of arrival of blast wave at points b, e, and f, respectively (ms)
$t_c$	(1) clearing time for reflected pressures (ms) (2) container thickness of explosive charges (in.)
$t_d$	rise time (ms)
$t_E$	time to reach maximum elastic deflection
$t_m$	time at which maximum deflection occurs (ms)
$t_o$	duration of positive phase of blast pressure (ms)
$t_o^-$	duration of negative phase of blast pressure (ms)
$t_{of}$	fictitious positive phase pressure duration (ms)
$t_{of}^-$	fictitious negative phase pressure duration (ms)
$t_r$	fictitious reflected pressure duration (ms)
$t_u$	time at which ultimate deflection occurs (ms)
$t_y$	time to reach yield (ms)
$t_A$	time of arrival of blast wave (ms)
$t_1$	time at which partial failure occurs (ms)
T	duration of equivalent triangular loading function (ms)
$T_C$	thickness of concrete section (in.)
$T_C$	scaled thickness of concrete section (ft/lb <sup>1/3</sup> )
$T_N$	effective natural period of vibration (ms)
$T_r$	rise time (ms)
$T_s$	thickness of sand fill (in.)

\* See note at end of symbols

$T_s$	scaled thickness of sand fill ( $\text{ft}/\text{lb}^{1/3}$ )
$u$	particle velocity (ft/ms)
$u_u$	ultimate flexural or anchorage bond stress (psi)
$U$	shock front velocity (ft/ms)
$v$	velocity (in./ms)
$v_a$	instantaneous velocity at any time (in./ms)
$v_b$	boundary velocity for primary fragments (ft/sec)
$v_c$	ultimate shear stress permitted on an unreinforced web (psi)
$v_f$	maximum post-failure fragment velocity (in./ms)
$v_f(\text{avg.})$	average post-failure fragment velocity (in./ms)
$v_i$	velocity at incipient failure deflection (in./ms)
$v_o$	initial velocity of primary fragment (ft/sec)
$v_r$	residual velocity of primary fragment after perforation (ft/sec)
$v_s$	striking velocity of primary fragment (ft/sec)
$v_u$	ultimate shear stress (psi)
$v_{uH}$	ultimate shear stress at distance $d_e$ from the horizontal support (psi)*
$v_{uV}$	ultimate shear stress at distance $d_e$ from the vertical support (psi)*
$V$	volume of partially vented chamber ( $\text{ft}^3$ )
$V_d$	ultimate direct shear capacity of the concrete of width $b$ (lbs)
$V_{dH}$	shear at distance $d_e$ from the vertical support on a unit width (lbs./in.)*
$V_{dV}$	shear at distance $d_e$ from the horizontal support on a unit width (lbs/in.)*
$V_o$	volume of structure ( $\text{ft}^3$ )
$V_s$	shear at the support on a unit width (lbs/in.)*
$V_{sH}$	shear at the vertical support on a unit width (lbs/in.)*

\* See note at end of symbols

$V_{sV}$	shear at the horizontal support on a unit width (lbs/in.)*
$V_u$	total shear on a width b (lbs)
w	weight density of concrete (lbs/ft <sup>3</sup> )
$w_s$	weight density of sand (lbs/ft <sup>3</sup> )
W	charge weight (lbs)
$W_c$	total weight of explosive containers (lbs)
$W_f$	weight of primary fragment (oz)
$W_{co}$	total weight of steel core (lbs)
$W_{c1}, W_{c2}$	total weight of plates 1 and 2, respectively (lbs)
$W_s$	width of structure (ft)
WD	work done
x	yield line location in horizontal direction (in.)*
X	deflection (in.)
$X_a$	any deflection (in.)
$X_e$	elastic deflection (in.)
$X_{ep}$	elasto-plastic deflection (in.)
$X_f$	maximum penetration into concrete of armor-piercing fragments (in.)
$X_f'$	maximum penetration into concrete of fragments other than armor-piercing (in.)
$X_m$	maximum transient deflection (in.)
$X_p$	plastic deflection (in.)
$X_s$	(1) maximum penetration into sand of armor-piercing fragments (in.) (2) static deflection
$X_u$	ultimate deflection (in.)
$X_E$	equivalent elastic deflection (in.)
$X_1$	partial failure deflection (in.)
y	yield line location in vertical direction (in.)*

\* See note at end of symbols



Z	scaled slant distance (ft/lb <sup>1/3</sup> )
Z <sub>A</sub>	scaled normal distance (ft/lb <sup>1/3</sup> )
Z <sub>G</sub>	scaled ground distance (ft/lb <sup>1/3</sup> )
α	(1) angle formed by the plane of stirrups, lacing, or diagonal reinforcement and the plane of the longitudinal reinforcement (deg)
α	(2) angle of incidence of the pressure front (deg)
β	(1) coefficient for determining elastic and elasto-plastic resistances
	(2) particular support rotation angle.(deg)
γ	coefficient for determining elastic and elasto-plastic deflections
γ	deflections increase in support rotation angle after partial failure (deg)
θ	support rotation angle (deg)
θ	angular acceleration (rad/ms <sup>2</sup> )
θ <sub>max</sub>	maximum support rotation angle (deg)
θ <sub>H</sub>	horizontal rotation angle (deg)*
θ <sub>V</sub>	vertical rotation angle (deg)*
Σo	effective perimeter of reinforcing bars (in.)
ΣM	summation of moments (in.-lbs)
ΣM <sub>N</sub>	sum of the ultimate unit resisting moments acting along the negative yield lines (in.-lbs)
ΣM <sub>P</sub>	sum of the ultimate unit resisting moments acting along the positive yield lines (in.-lbs)
μ	ductility factor
ν	Poisson's ratio
φ	(1) capacity reduction factor
	(2) bar diameter (in.)
φ <sub>r</sub>	assumed shape function for concentrated loads
φ(x)	assumed shape function for distributed loads

—	free edge
==	simple support
////	fixed support
XXXXXX	either fixed, restrained, or simple support

\* Note. This symbol was developed for two-way elements which are used as walls. When roof slabs or other horizontal elements are under consideration, this symbol will also be applicable if the element is treated as being rotated into a vertical position.

**APPENDIX 3C**

**BIBLIOGRAPHY**

1. Blast Resistant Design, NAVFAC Design Manual 2.8, Department of the Navy, Naval Facilities Engineering Command, Alexandria, VA 22332, April 1982.
2. Design of Structures to Resist Nuclear Weapons Effects, ASCE Manual of Engineering Practice, No. 42, American Society of Civil Engineers, New York, NY, 1961. (1983 Edition under preparation)
3. Designing Facilities to Resist Nuclear Weapons Effects. Structures, TM5-858-3, Headquarters, Department of the Army, Washington, DC.
4. Structures to Resist the Effects of Accidental Explosions, Technical Manual TM5-1300, Navy Publication NAVFAC P-397, Air Force Manual AFM 88-22, Department of the Army, the Navy, and the Air Force, Washington, DC, June 1969.
5. Crawford, R. E., et al, The Air Force Manual for Design and Analysis of Hardened Structures, AFWL-TR-74-102, Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico, 87117.
6. Healey, J., et at, Design of Steel Structures to Resist the Effects of HE Explosions, by Ammann and Whitney, Consulting Engineers, New York, NY, Technical Report 4837, Picatinny Arsenal, Dover, NJ, August 1975.
7. Hopkins, J., Charts for Predicting Response of a Simple Spring-Mass System to a Bilinear Blast Load, Technical Note N-1669, Naval Civil Engineering Laboratory, Port Hueneme, CA, April 1983.
8. Stea, W., et al, Nonlinear Analysis of Frame Structures Subjected to Blast Overpressures, by Ammann and Whitney, Consulting Engineers, New York, NY, Contractor report ARLCD-CR-77008, U.S. Army Armament Research and Development Command, Large Caliber Weapon Systems Laboratory, Dover, NJ, May 1977.
9. Stea, W. Weissman, S., and Dobbs, N., Overturning and Sliding Analysis of Reinforced Concrete Protective Structures, by Ammann and Whitney, consulting Engineers, New York, NY, Technical Report 4921, Picatinny Arsenal, Dover, NJ, February 1976.
10. Javornicky, J. and Van Amerongen, C., Tables for the Analysis of Plates, Slabs and Diaphragms Based on the Elastic Theory, Bauverlag GmbH., Wiesbaden/Germany (2nd Edition 1971).
11. Roark, R.J. and Young, W. C., Formulas for Stress and Strain, McGraw Hill, New York (5th Edition 1975).
12. Cohen, E. and Dobbs, N., Design Procedures and Details for Reinforced Concrete Structures Utilized in Explosive Storage and Manufacturing Facilities, Ammann and Whitney, Consulting Engineers, New York, NY, Annals of the New York Academy of Sciences, Conference on Prevention of and Protection against Accidental Explosion of Munitions, Fuels and Other Hazardous Mixtures, Volume 152, Art. 1, October 1968.